Nuclear Structure Theory I

Nuclear Properties

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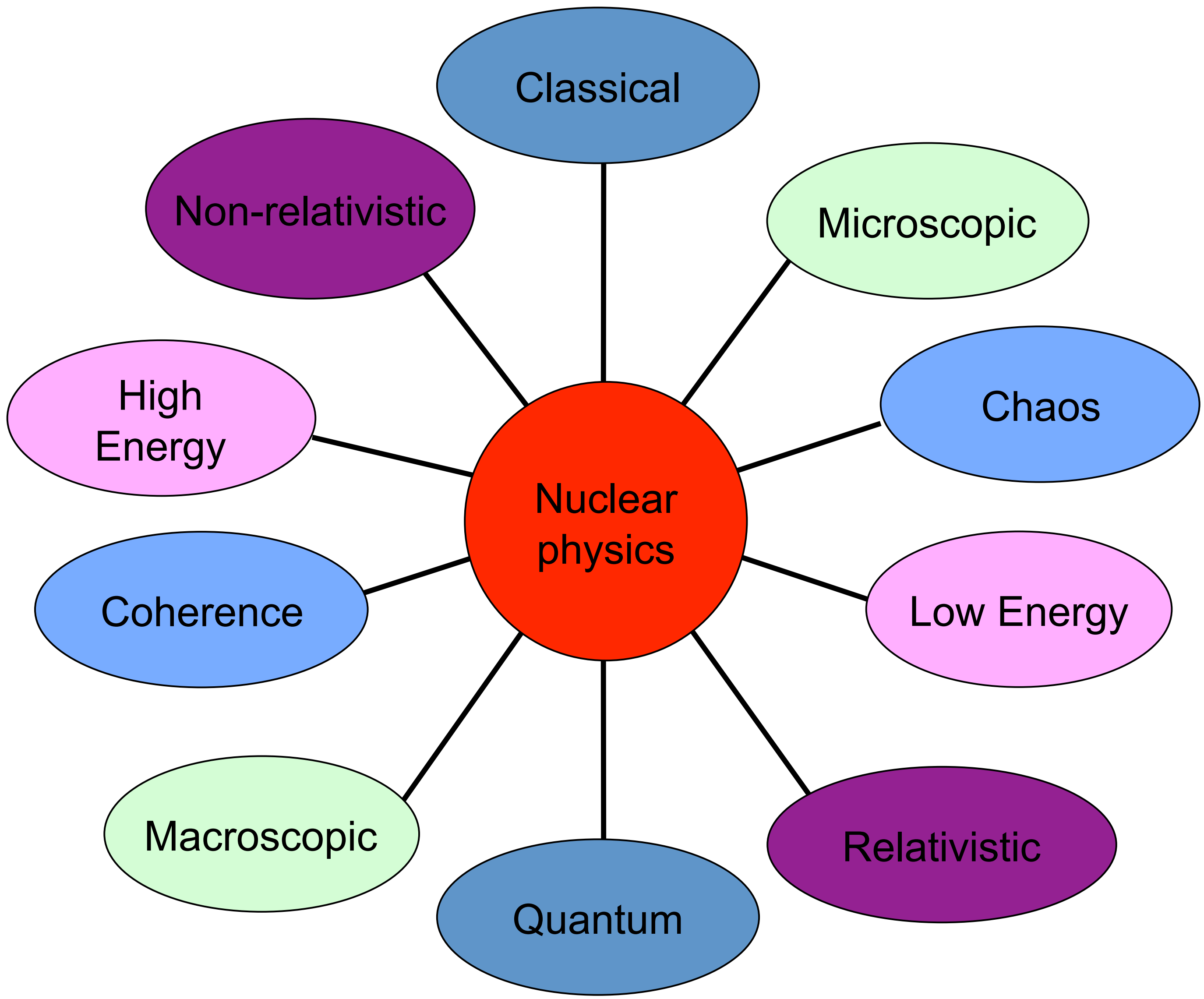
10th Exotic Beam Summer School - EBSS2011
East Lansing, Michigan. 25-30 July, 2011


Classical
Non-relativistic
Microscopic
High Energy
Chaos
Nuclear physics
Low Energy
Coherence
Macroscopic
Quantum
Relativistic

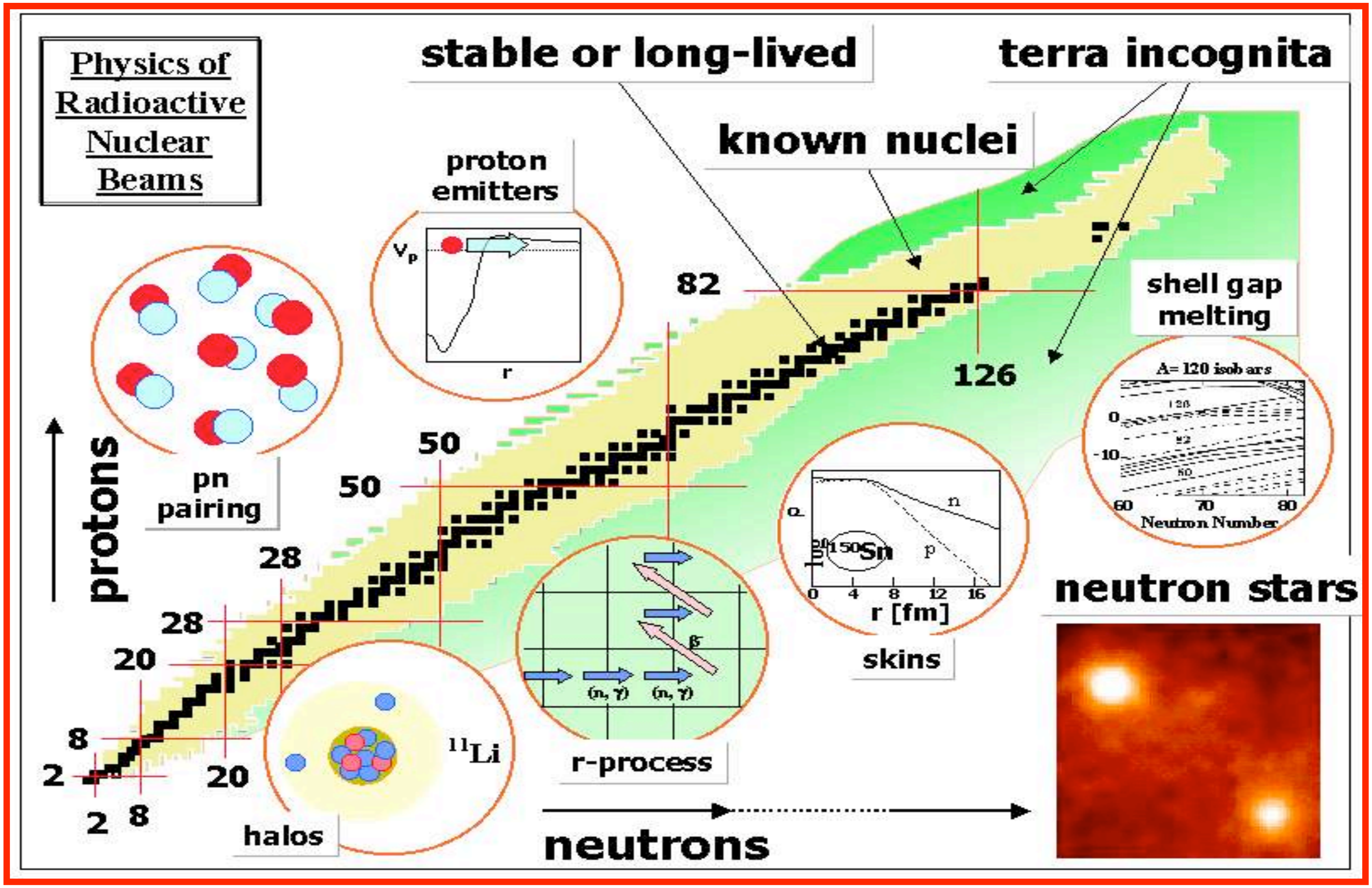
The nuclear world: the rich variety of natural mesoscopic phenomena

- Predicted: 6000 - 7000 particle-stable nuclides
- Observed: 2932
- even-even 737; odd-A 1469; odd-odd 726.
- Lightest $^{2}_{1}\mathrm{H}_{1}$ (deuteron), Heaviest $^{294}_{118}(?)_{176}$
- No gamma-rays known 785.
- Largest number of levels known (578) $^{40}_{20}\mathrm{Ca}_{20}$
- Largest number of transitions known 1319 $^{53}_{25}\mathrm{Mn}_{28}$
- Highest multipolarity of electromagnetic transition E6 in

  $^{53}_{26}\mathrm{Fe}_{27},\ 19/2^{-}\,(3040\,\mathrm{keV}) \rightarrow 7/2^{-}(\mathrm{g.s.});\, 2.58\ \mathrm{min}$
- Resut of 100 years of reasearch 182000 citations in Brookhaven database, 4500 new entries per year.

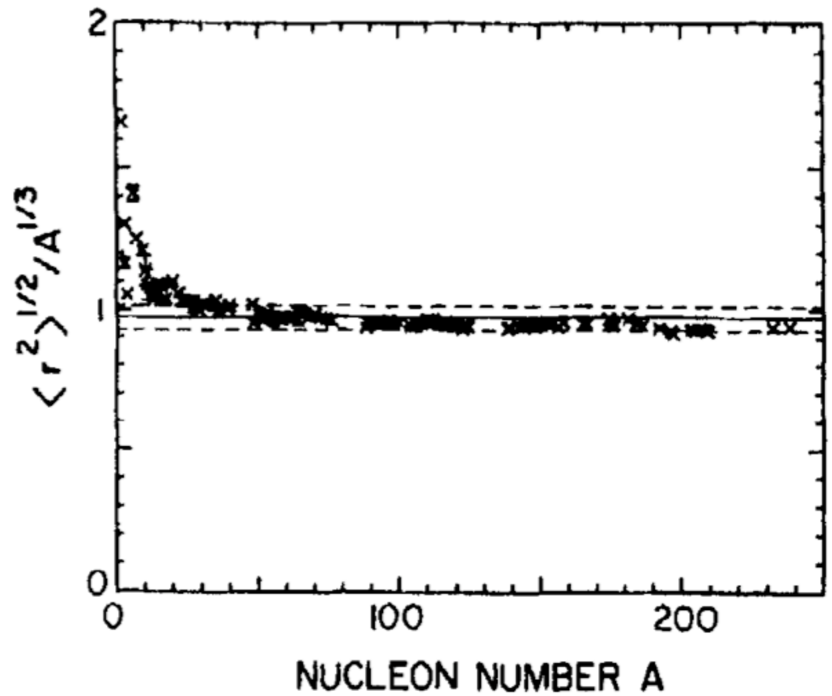
Data collected by V. Zelevinsky

Nuclear Chart

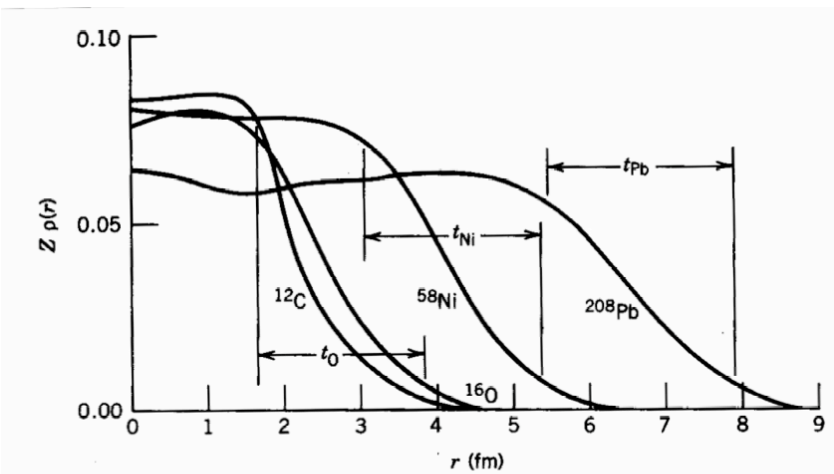


Nuclear Sizes

Radius $R = r_0 A^{1/3}$



Electron scattering data,
H.D. Vries et.al Atom Data,
Nucl Data Tab. 36 (1987) 495



Barrett and Jackson Nuclear sizes and structure

Nuclear Binding
Weizacker mass formula

$$E_B(Z,N) = \alpha_1 A - \alpha_2 A^{2/3} - \alpha_3 \frac{Z(Z-1)}{A^{1/3}} - \alpha_4 \frac{(N-Z)^2}{A} + \Delta$$

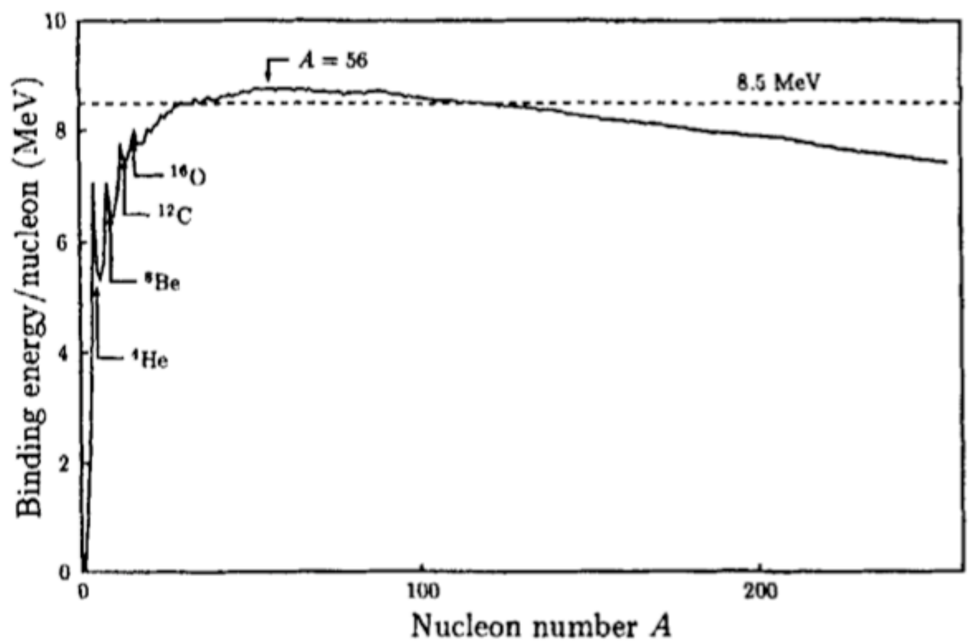
Volume term $\alpha_1 = 16$ MeV

Surface term $\alpha_2 = 17$ MeV

Coulomb term $\alpha_3 = 0.6$ MeV

Symmetry term $\alpha_4 = 25$ MeV

Pairing term



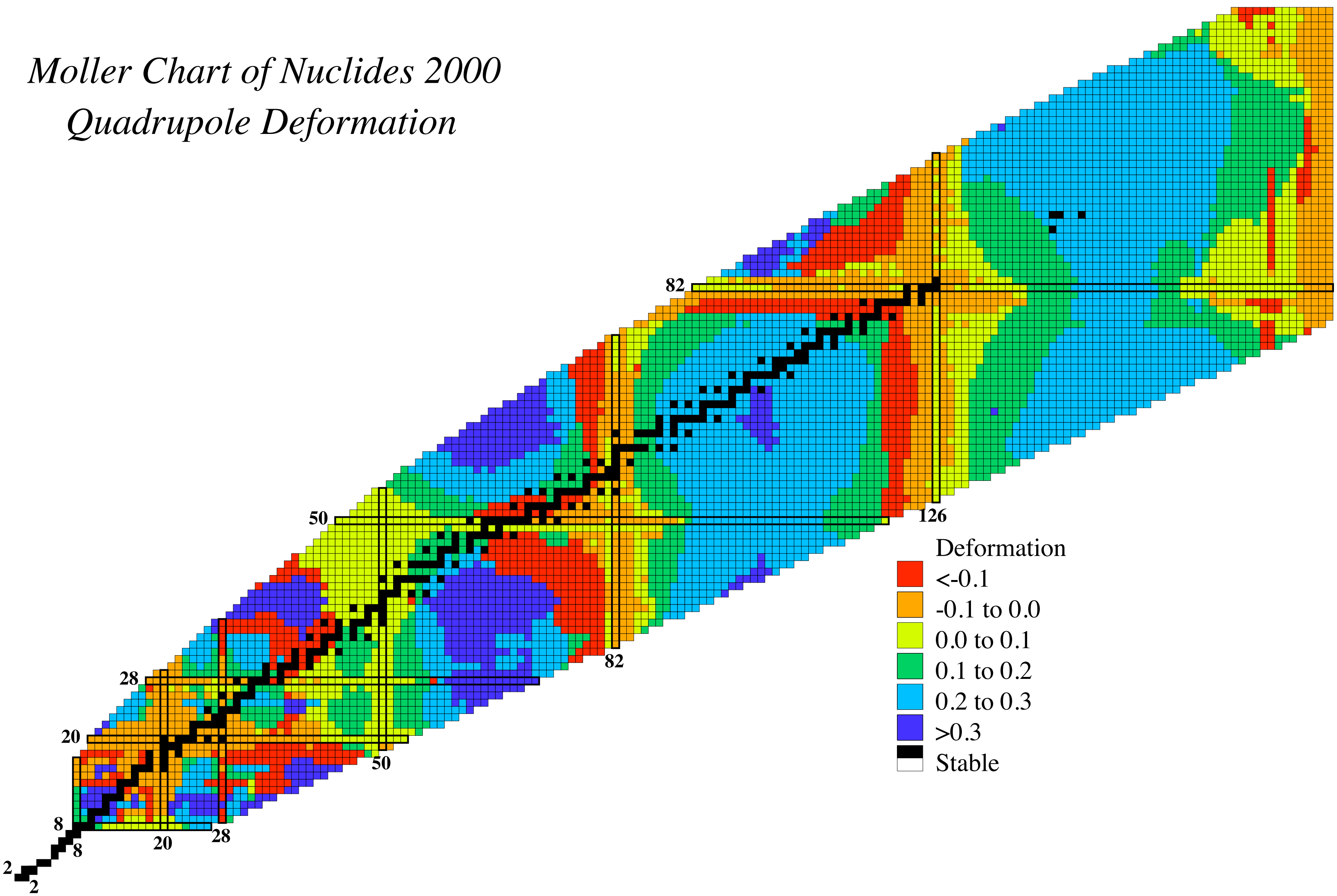
$$\Delta = \begin{cases} \delta & \text{for even-even nuclei} \\ 0 & \text{for odd-mass nuclei} \\ -\delta & \text{for odd-odd nuclei} \end{cases}$$

$$\delta = \frac{25}{A} \text{ MeV}$$

Nuclear Shapes

- Origins of nuclear deformation
  - Core polarization
  - Proton-neutron interaction
- Physics of Nuclear rotations
- Nuclear vibrations
- Evidence for nuclear superfluidity

Moller Chart of Nuclides 2000
Quadrupole Deformation



Describing nuclear shapes

Expand nuclear shapes $R(\theta,\phi) = R_0\left(1 + \sum_{\lambda=0}^{\infty}\sum_{\mu=-\lambda}^{\lambda} \alpha^*_{\lambda\mu} Y_{\lambda\mu}(\theta,\phi)\right)$

$\lambda = 0$ Compression

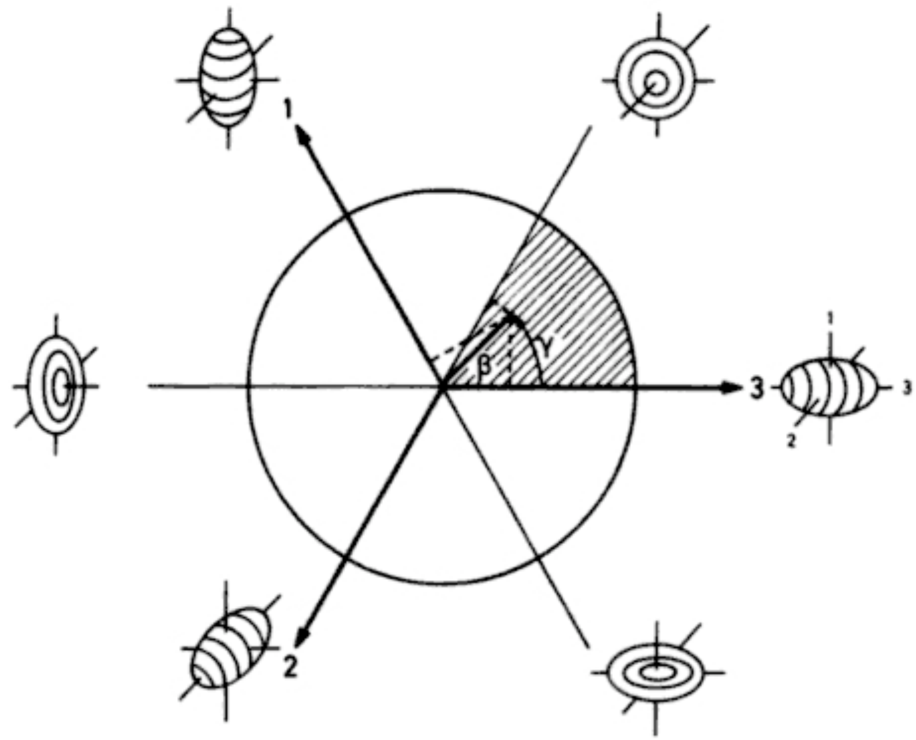
$\lambda = 1$ Center-of-mass translation

$\lambda = 2$ Quadrupole deformation

Hill-Wheeler Parameters

$$\alpha_{22} = \alpha_{2-2} = \beta \sin\gamma / \sqrt{2}$$

$$\alpha_{20} = \beta \cos\gamma$$

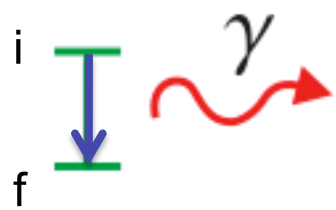


From Ring and Schuck,
The nuclear many-body problem

Multipole moments

$$\mathcal{M}_{\lambda\mu} \sim \alpha_{\lambda\mu} \qquad \mathcal{M}_{\lambda\mu} = \int d^3r \rho(\mathbf{r}) r^\lambda Y_{\lambda\mu}(\hat{\mathbf{r}})$$

Reduced transition probability $$B(E2, J_i \to J_f) = \sum_{\mu, M_f} |\langle J_f M_f | \mathcal{M}_{2\mu} | J_i M_i \rangle|^2$$



EM decay rate $$\tau^{-1} \sim B(E\lambda, i \to f)\,(E_i - E_f)^{2\lambda+1}$$

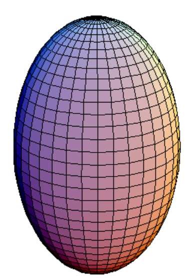
See EM width calculator: http://www.volya.net/

Quadrupole moment $$\mathcal{Q}(J) = \sqrt{\frac{16\pi}{5}} \langle JJ | \mathcal{M}_{20} | JJ \rangle$$

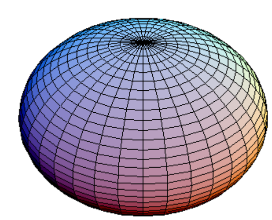
Note that:

$$\sqrt{\frac{16\pi}{5}} r^2 Y_{20} = 3z^2 - r^2$$

Prolate Q>0

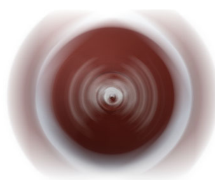


Oblate Q<0



warning: lab frame and body-fixed are different

Quantum Mechanics of Rotations

	Laboratory frame	**Body-fixed frame**
Angular Momentum	J_k k=x,y,z	I_k k=1,2,3
Shape:		
	$\mathcal{Q}(J)$	$\beta \;\; \gamma \;\; Q$

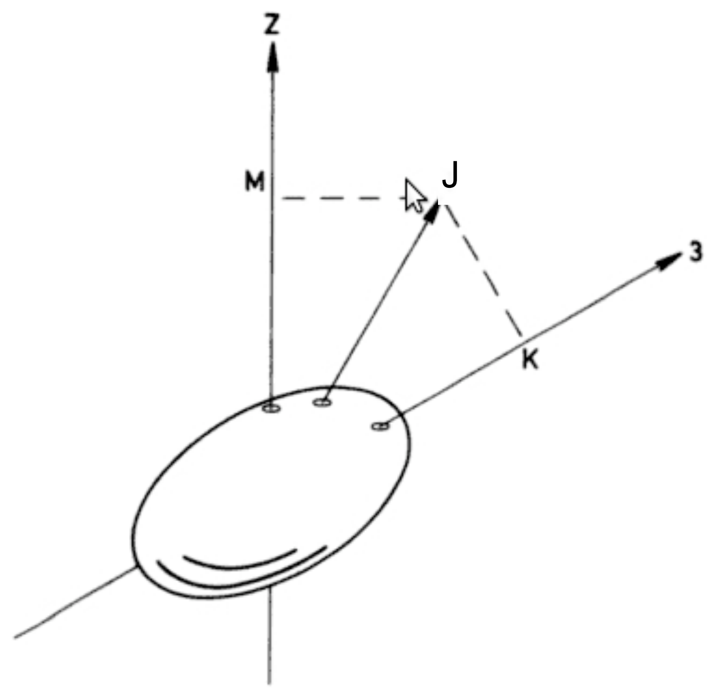
Note that J^2 and all I_k are scalars

Collective Rotor Hamiltonian

$$H_{rot} = \sum_{i=123} A_i I_i^2$$

Three parameters $A_1,\ A_2,\ A_3$

$$A_k = \frac{1}{2\mathcal{L}_k}$$



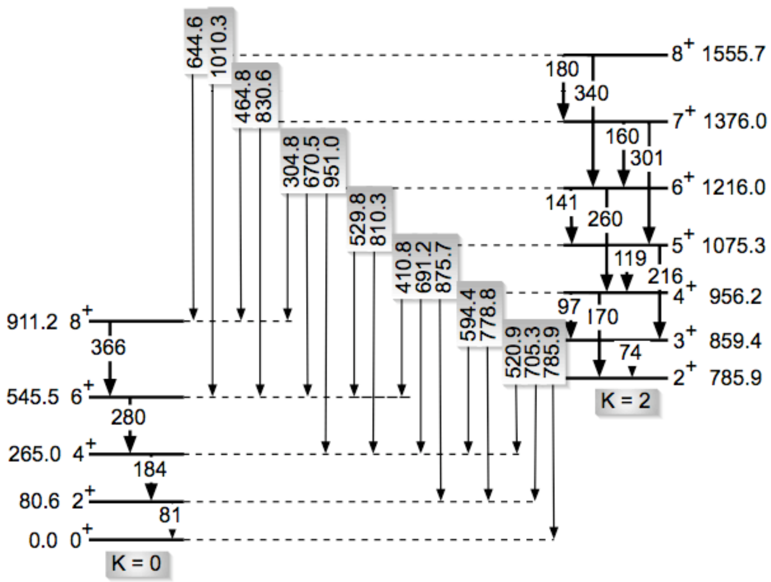
From A. Bohr and B. R. Mottelson. *Nuclear structure*, volume 2

Rotational Spectrum

Spherical $A_1 = A_2 = A_3$ Rotations are not possible

Axially symmetric rotor

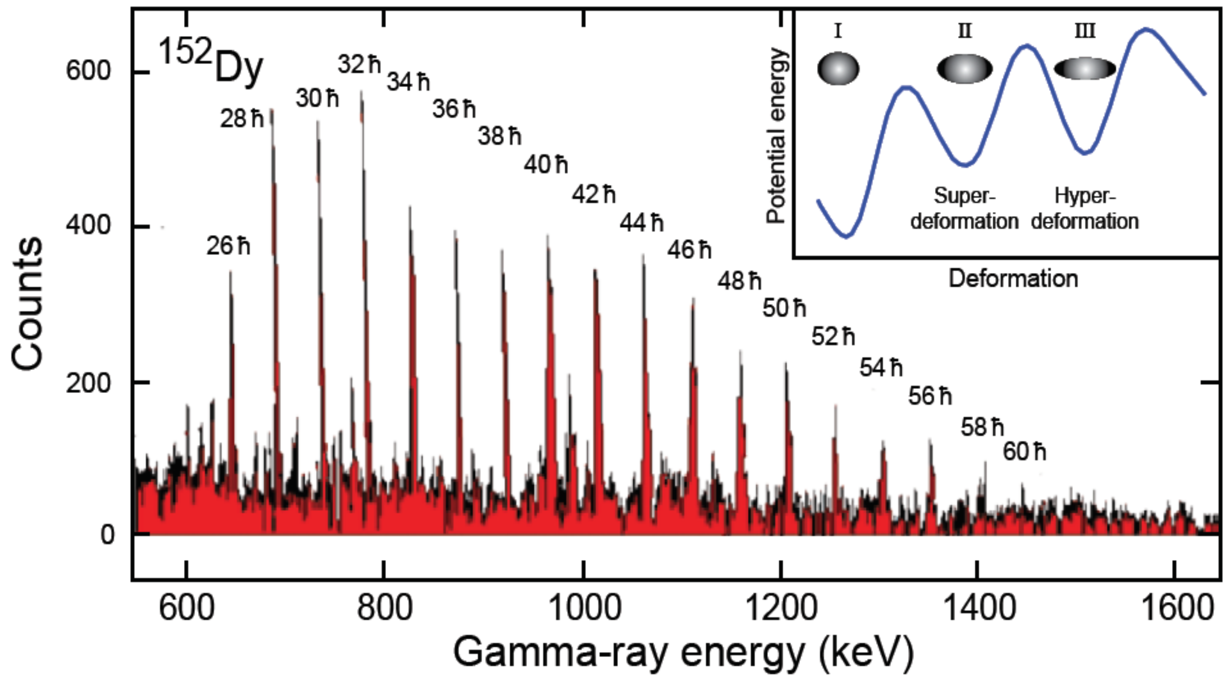
$$A_1 = A_2 = A \neq A_3 \qquad H_{\rm rot} = AJ^2 + (A_3 - A)K^2$$



Energy level diagram for 166Er.
From W.D. Kulp et. al, Phys. Rev.
C 73, 014308 (2006).

Properties:
- -Band structures E~J(J+1)
- -Band head J=K
- -K good quantum number (transitions etc)

Rotation and gamma rays



Observed reduced rates and moments

$$\mathcal{Q}(J) = Q\, C^{JJ}_{20,JJ} C^{JK}_{20,JK}$$

$$\mathcal{Q}(0^+) = 0,\ \mathcal{Q}(2^+_1) = -\frac{2}{7} Q \ldots$$

$$B(E2, J_i \to J_f) = \frac{5}{16\pi} Q^2 \left| C^{J_f K}_{20,J_i K} \right|^2$$

Alaga rules

$$\frac{\mathcal{Q}^2(2^+_1)}{B(E2, 0^+ \to 2^+)} = \frac{16\pi}{5} \left(\frac{2}{7}\right)^2 \approx 0.82$$

Triaxial rotor

Spectrum and states

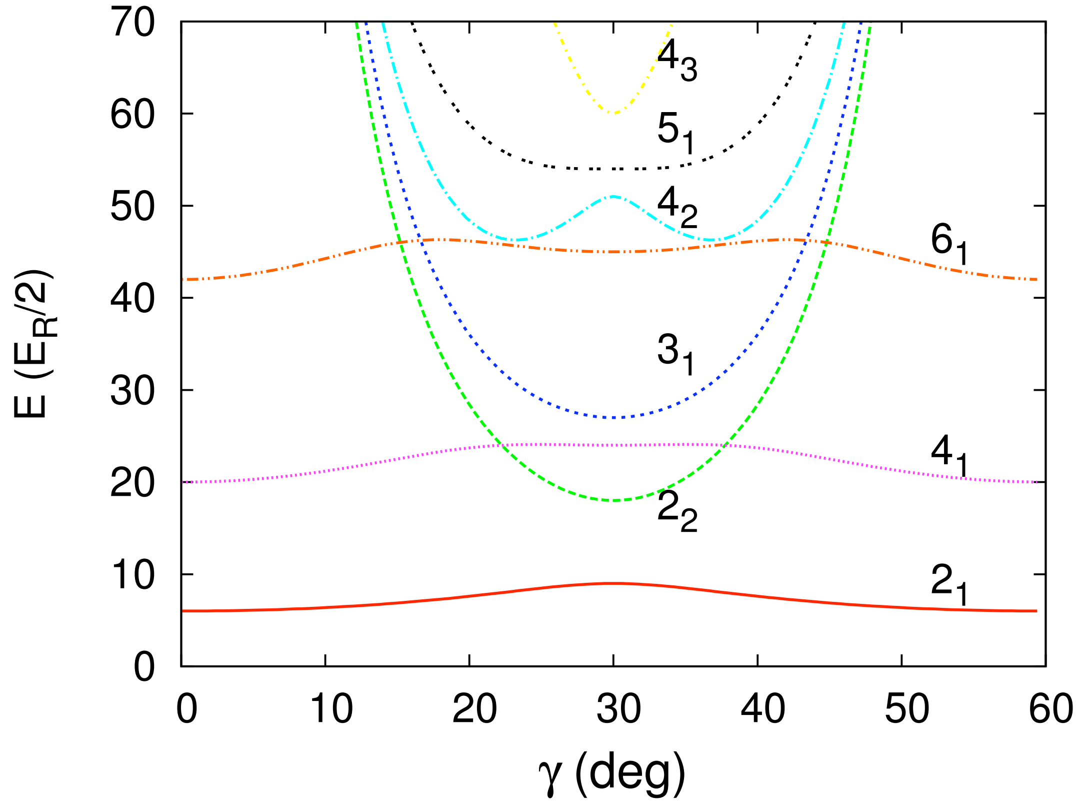
Three different parameters $A_1,\ A_2,\ A_3$

K is mixed (diagonalize H)

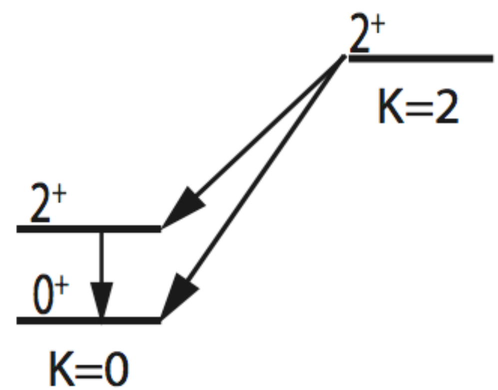
Spectral relations

$$E(2_1) + E(2_2) \ = \ E(3_1)$$

$$4E(2_1) + E(2_2) = E(5_1)$$



Mixed Transitions



See also: J. M. Allmond, et.al.
Phys. Rev. C 78, 014302 (2008).

Examining Triaxiality Parameters

H_{rot} parameters $A_1,\ A_2,\ A_3$ instead we use:

1.) Overall energy scale

2.) K mixing angle for 2_1 and 2_2 states Γ $\tan 2\Gamma = \frac{\sqrt{3}(A_1 - A_2)}{A_1 + A_2 - 2A_3}$

3.) Energy ratio of $E(2_1)$ and $E(2_2)$ $\gamma^2_{\mathrm{DF}} \approx \frac{E(2_1)}{2E(2_2)}$

Shape parameters: β γ define $\mathcal{M}_{\lambda\mu}$

how to measure?

$$\tan^2(\gamma - \Gamma) = \frac{B(E2, 0 \to 2_2)}{B(E2, 0 \to 2_1)} \quad \tan^2(\gamma + 2\Gamma) = \frac{2B(E2, 2_1 \to 2_2)}{7Q^2(2_1)}$$

See also: J. M. Allmond, et.al. Phys. Rev. C 78, 014302 (2008).

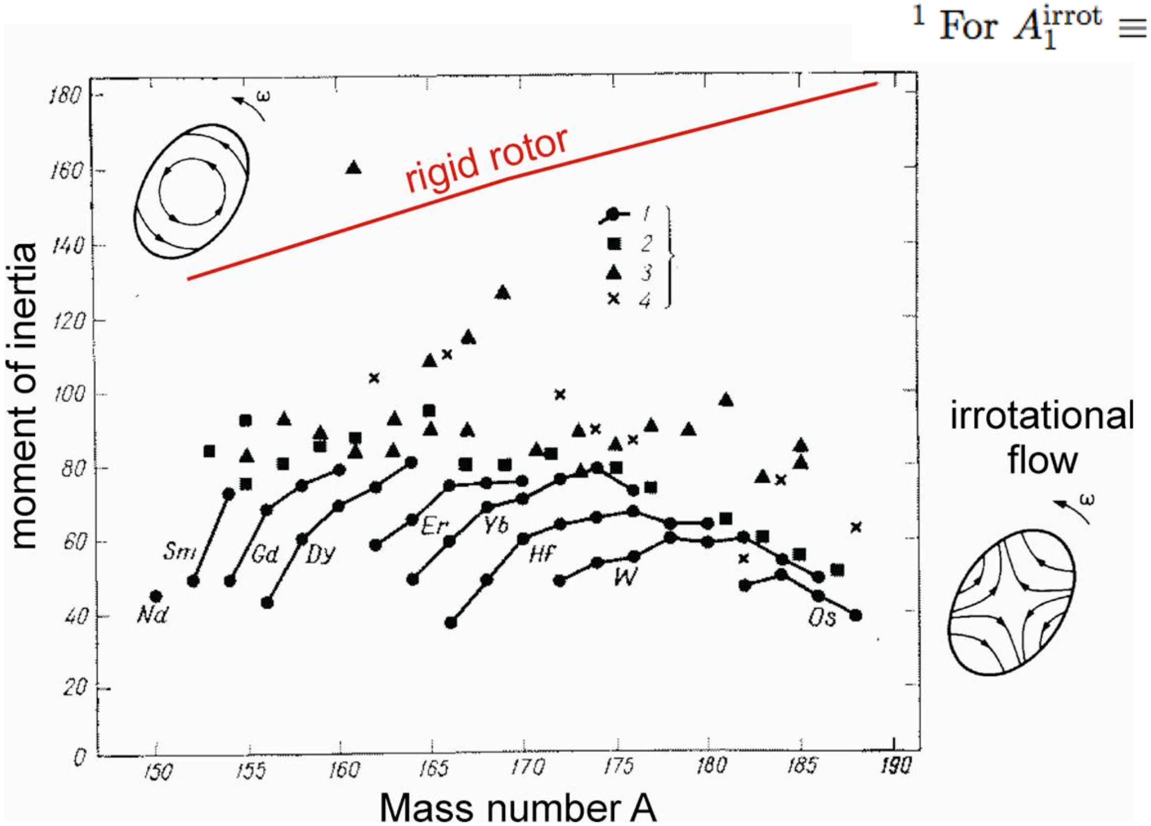
Models for moments of inertia

Relationship between H_{rot} and β γ is model-dependent.

$$\mathcal{L}^{\text{irr}} < \mathcal{L}^{\text{exp}} < \mathcal{L}^{\text{rid}}$$

Nucleus	A_1^{expt}	A_2^{expt}	A_3^{expt}	$\frac{A_2^{\text{expt}}}{A_2^{\text{irrot}}}$ [1]	$\frac{A_3^{\text{expt}}}{A_3^{\text{irrot}}}$ [1]
^{186}Os	17.5	28.1	181	0.918	0.614
^{188}Os	20.2	31.4	145	0.768	0.633
^{166}Er	11.4	15.4	189	0.885	0.610
^{172}Yb	11.9	14.3	360	0.841	0.798
^{184}W	16.4	20.6	217	0.821	0.485

[1] For $A_1^{\text{irrot}} \equiv A_1^{\text{expt}}$



From: J. M. Allmond,Ph.D thesis,. Georgia Institute of Technology, 2007

Evidence for nuclear superfluidity

Surface vibrations

$$R(\theta,\phi) = R_0\left(1+\sum_{\lambda=0}^{\infty}\sum_{\mu=-\lambda}^{\lambda}\alpha^*_{\lambda\mu}Y_{\lambda\mu}(\theta,\phi)\right)$$

Collective Hamiltonian

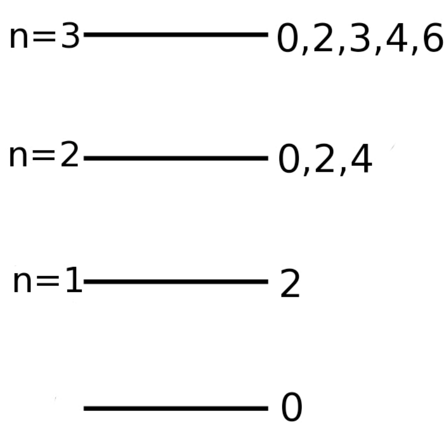
$$H_{\text{vib}} = \frac{1}{2}\sum_{\lambda\mu}\left(B_\lambda|\dot{\alpha}_{\lambda\mu}|^2 + C_\lambda|\alpha_{\lambda\mu}|^2\right)$$

Quantized Hamiltonian

$$H_{\text{vib}} = \sum_{\lambda\mu}\hbar\omega_\lambda\left(b^\dagger_{\lambda\mu}b_{\lambda\mu}+\frac{1}{2}\right)$$

$$\alpha_{\lambda\mu} = \sqrt{\frac{\hbar}{2B_\lambda\omega_\lambda}}\left(b^\dagger_{\lambda\mu}+(-1)^\mu b_{\lambda-\mu}\right)$$

spectrum



Transitions

$$\mathcal{M}_{\lambda\mu} \sim \alpha_{\lambda\mu} \qquad B(E\lambda) \sim \frac{1}{\omega_\lambda}$$

systematics

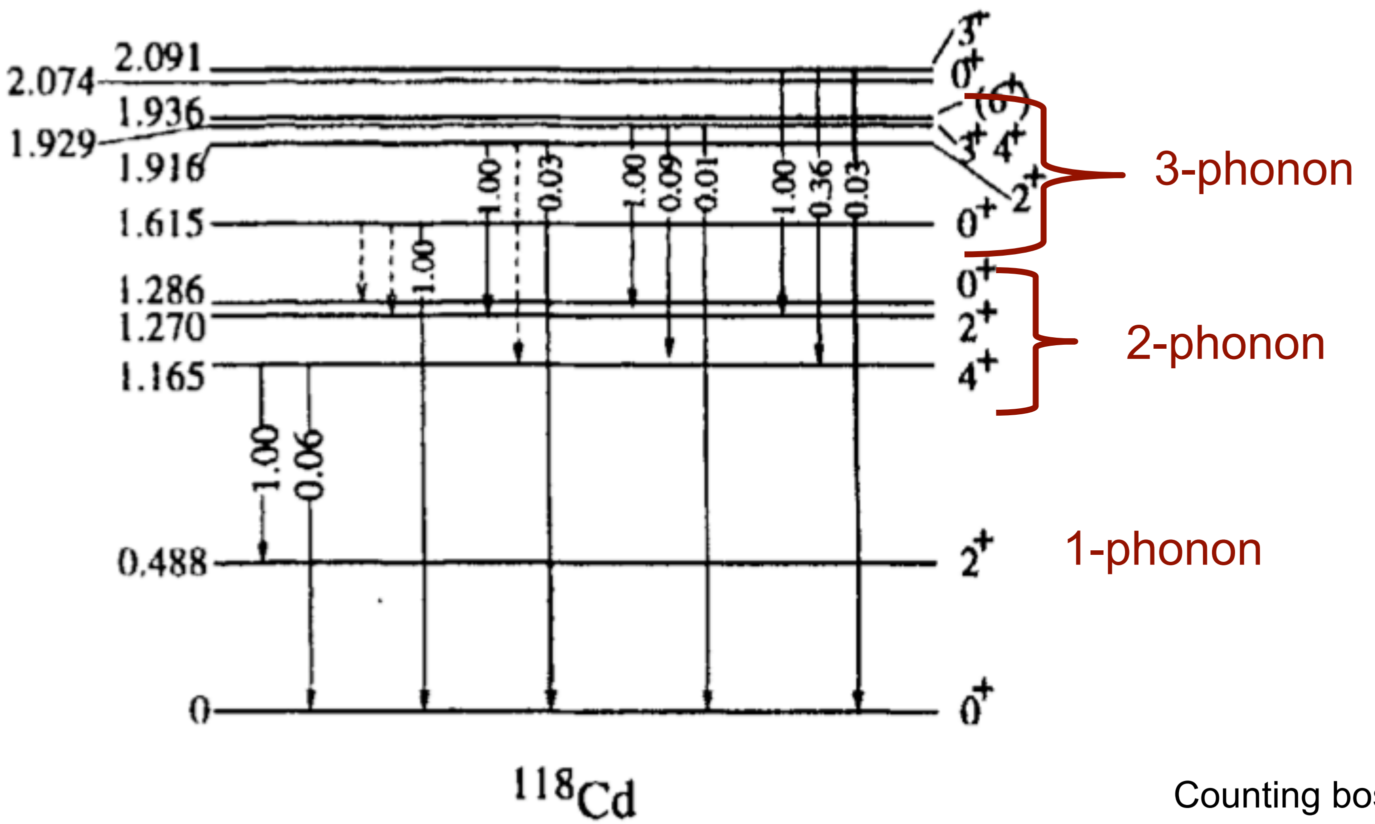
$$E_{2_1^+}B(E2, 2_1^+ \to 0^+) \approx 25\frac{Z^2}{A}(\text{MeV}e^2\text{fm}^4)$$

Bosonic enhancement

$$\sum_{J_f} B(E2, J_i \to J_f) = nB(E2, 2_1 \to 0_{gs})$$

Note: Giant resonances

Quadrupole Vibrations ^{118}Cd



Counting boson configurations:
http://www.volya.net/spins

From S. Wong,
Introductory nuclear physics

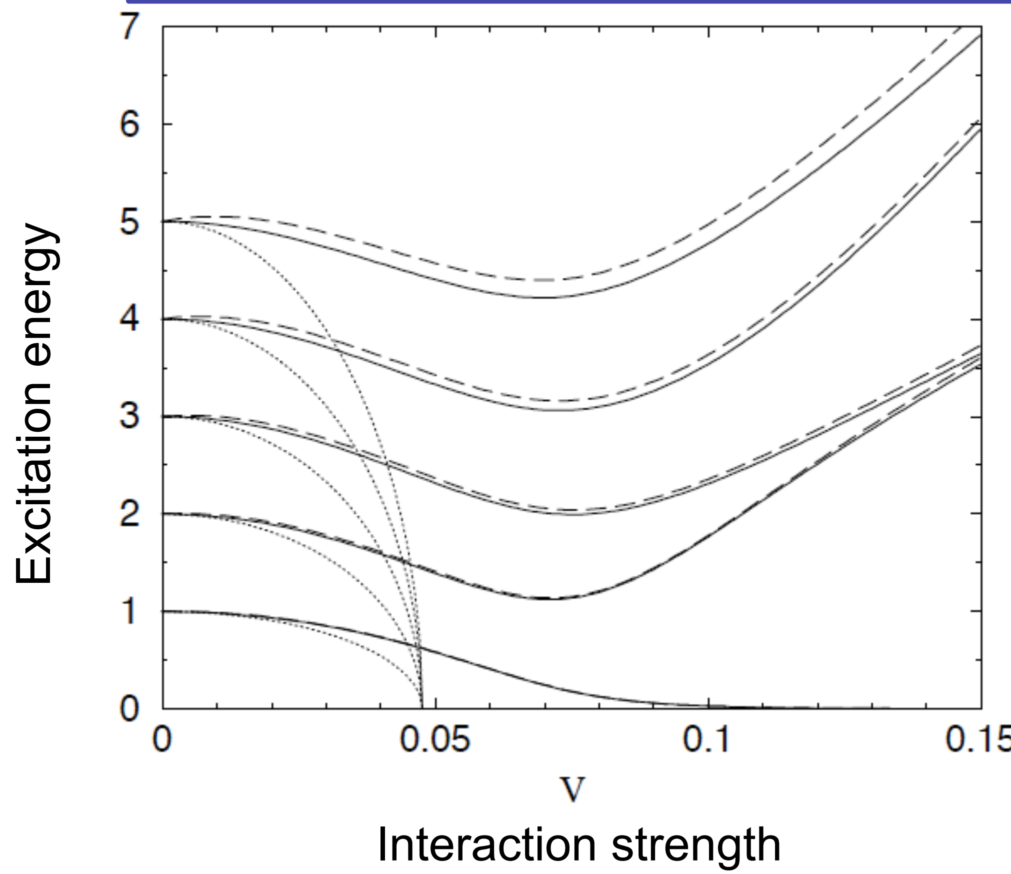
Transition to deformation, soft mode

$$H = \frac{\Lambda^{(0\,2)}\pi^2}{2} + \frac{\Lambda^{(2\,0)}}{2}\alpha^2 + \frac{\Lambda^{(3\,0)}}{3}\alpha^3 + \frac{\Lambda^{(1\,2)}}{4}[\alpha, \pi^2]_+ +$$

$$\frac{\Lambda^{(4\,0)}}{4}\alpha^4 + \frac{\Lambda^{(0\,4)}}{4}\pi^4 + \frac{\Lambda^{(2\,2)}}{8}[\alpha^2, \pi^2]_+ +$$

Two-level model with 20 particles
RPA, anharmonic solution, exact solution

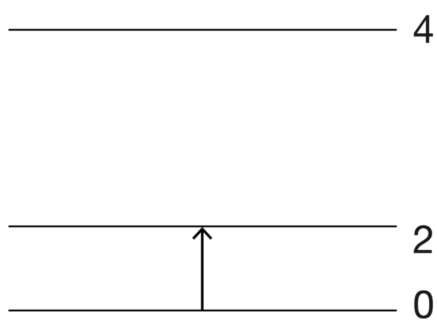
Lowering



Low-lying Collective modes

$$b = \frac{B(E2, 0 \to 2_1)}{\sum_i B(E2, 0 \to 2_i)} \qquad q = \frac{Q(2_1)}{Q_{\text{rot}}(2_1)} \qquad R_{42} = \frac{E(4_1)}{E(2_1)} \qquad B_{42} = \frac{B(E2, 4_1 \to 2_1)}{B(E2, 2_1 \to 0_{gs})}$$

Rotations



$$b = 1$$

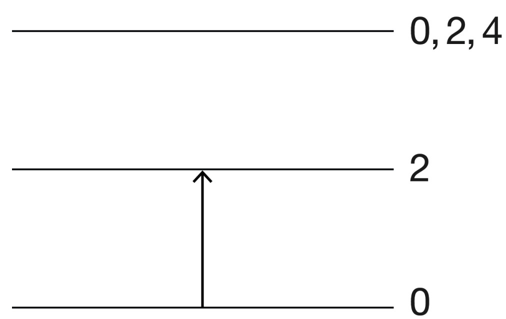
$$q = +1 \ (\text{prolate})$$

$$q = -1 \ (\text{oblate})$$

$$R_{42} = 10/3 \approx 3.33$$

$$B_{42} = 10/7 \approx 1.41$$

Vibrations



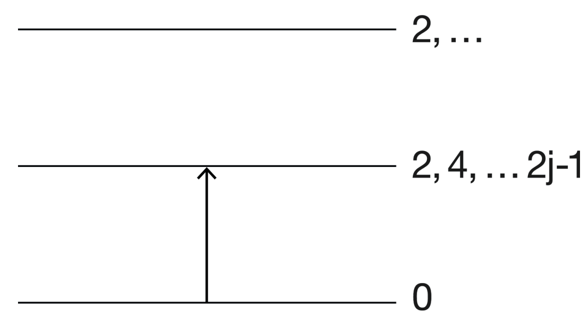
$$b = 1$$

$$q = 0$$

$$R_{42} = 2$$

$$B_{42} = 2$$

Pairing



$$b = 1$$

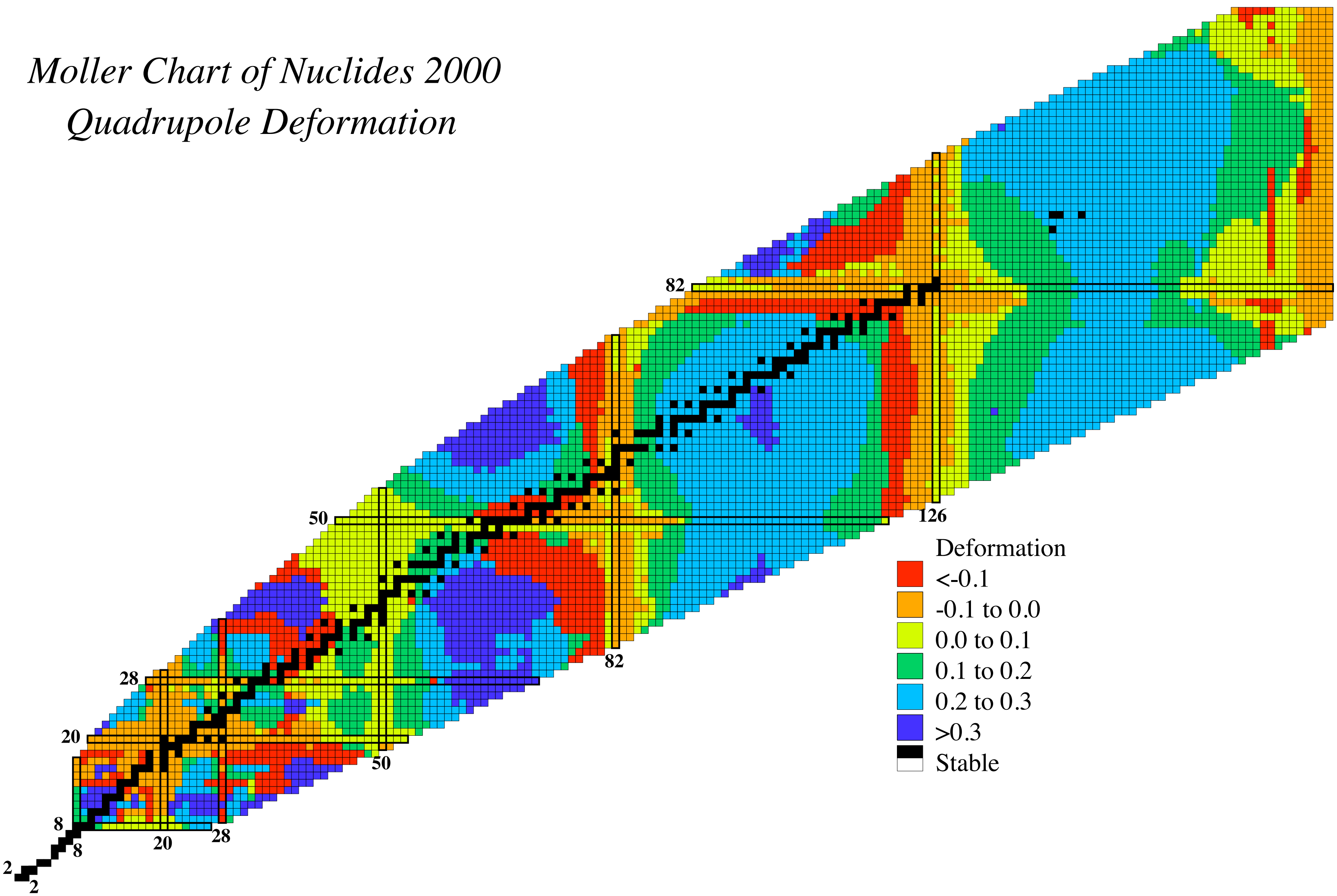
$$q \approx 0$$

$$R_{42} = 1$$

$$B_{42} \approx 0$$

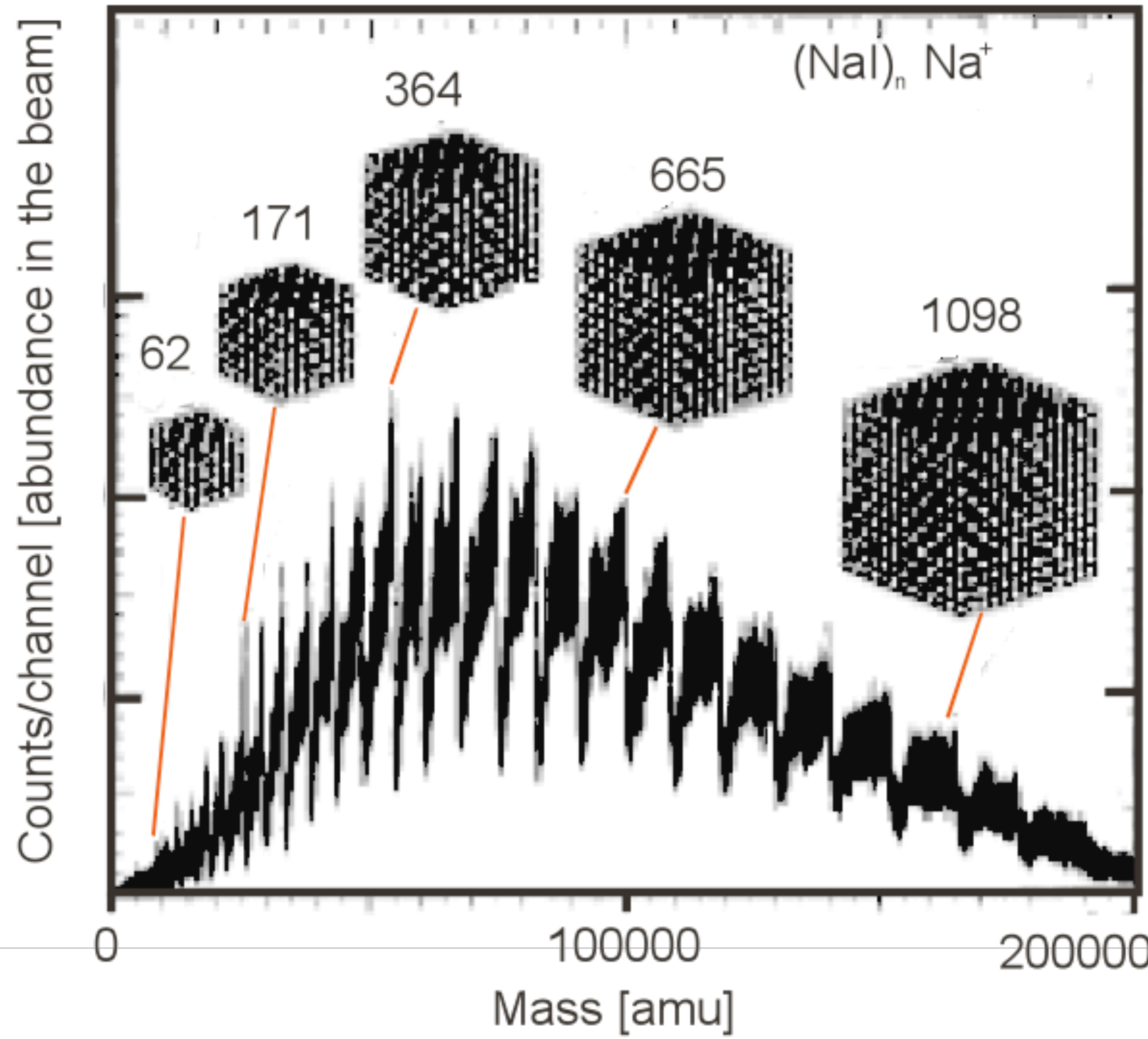
Single-Particle Motion

- Evidence of shell structure
- Single-particle modes and magic numbers
- Shells and supershells
- Classical periodic orbits
- Shells, nuclear surface and deformation


Moller Chart of Nuclides 2000
Quadrupole Deformation
Deformation
<-0.1
-0.1 to 0.0
0.0 to 0.1
0.1 to 0.2
0.2 to 0.3
>0.3
Stable
2
2
8
8
20
20
28
28
50
50
82
82
126

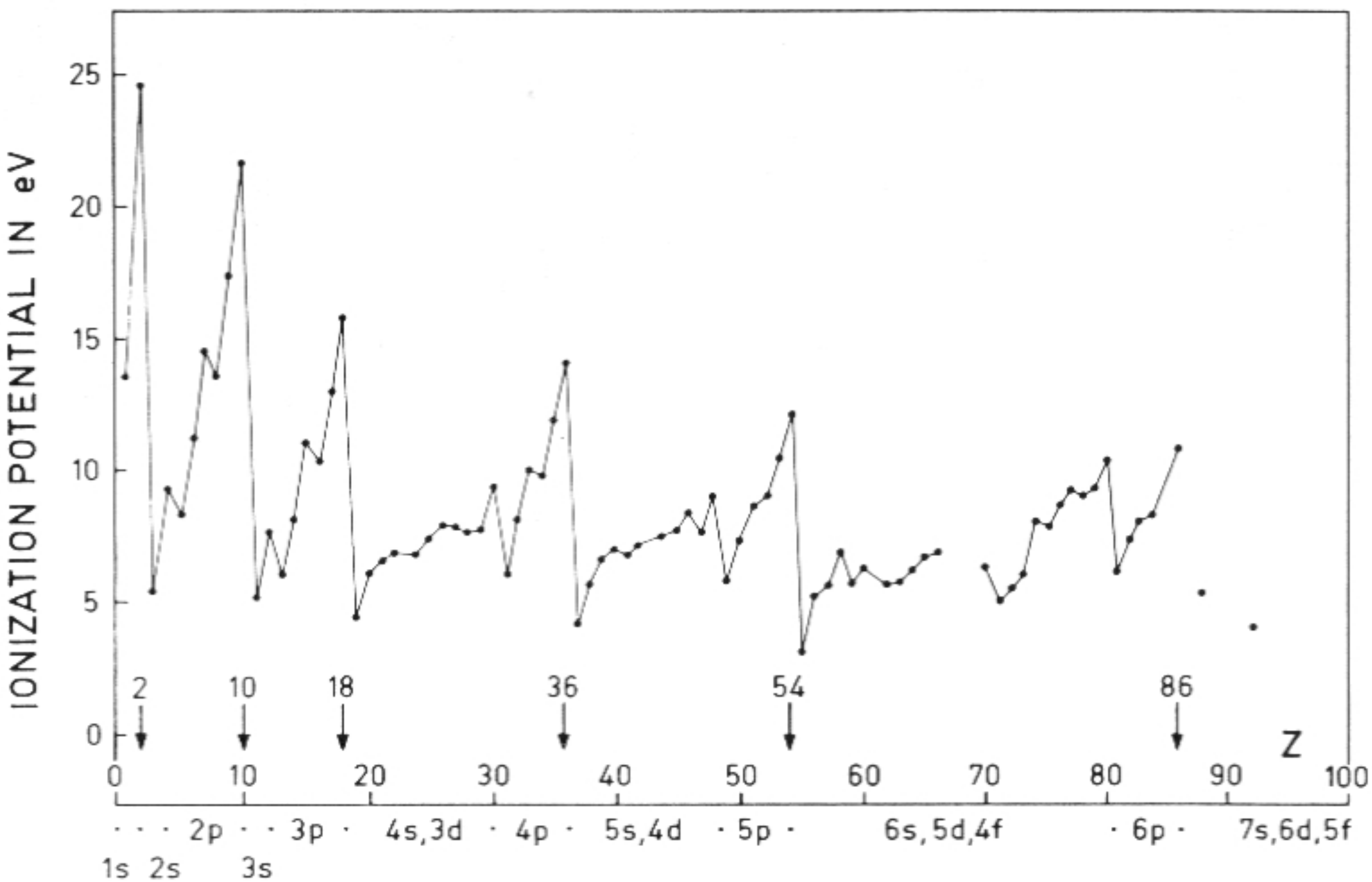
Salt Clusters, transition from small to bulk

- Symmetry
- Surface
- “Shells”



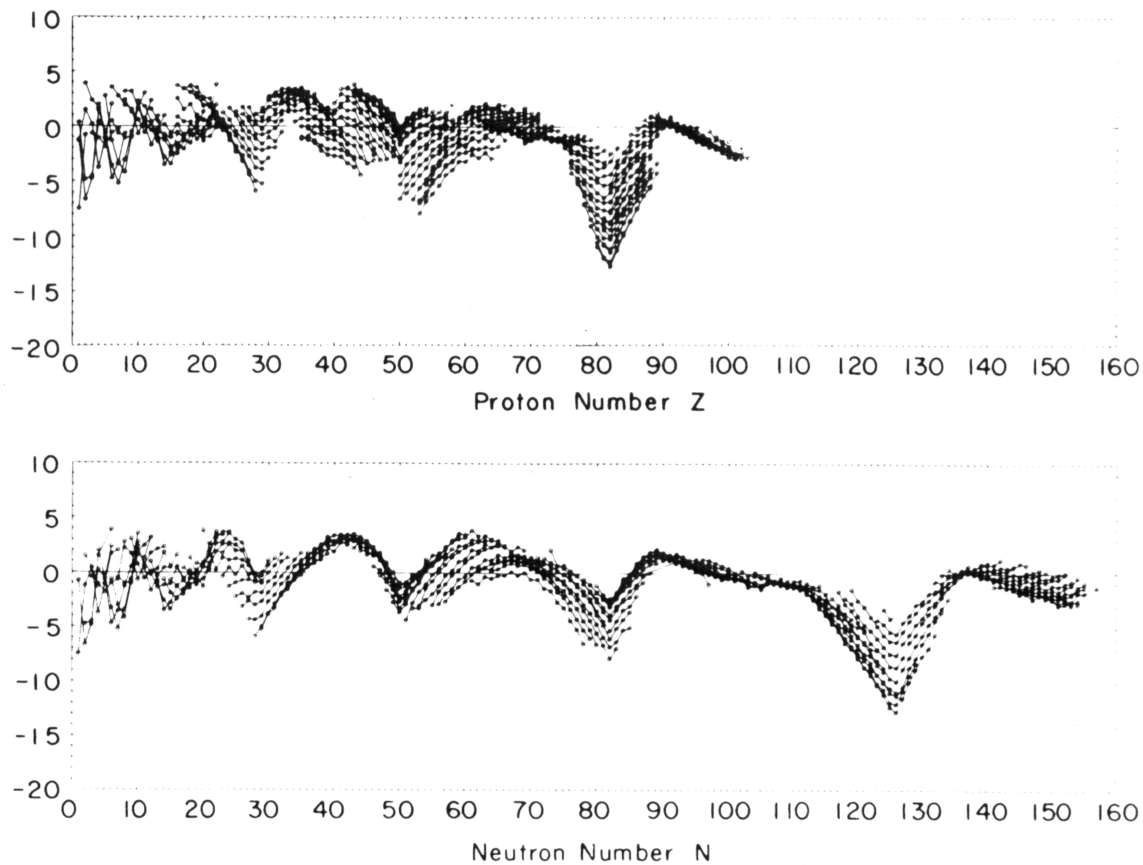
T. P.Martin Physics Reports **273** (1966) 199-241

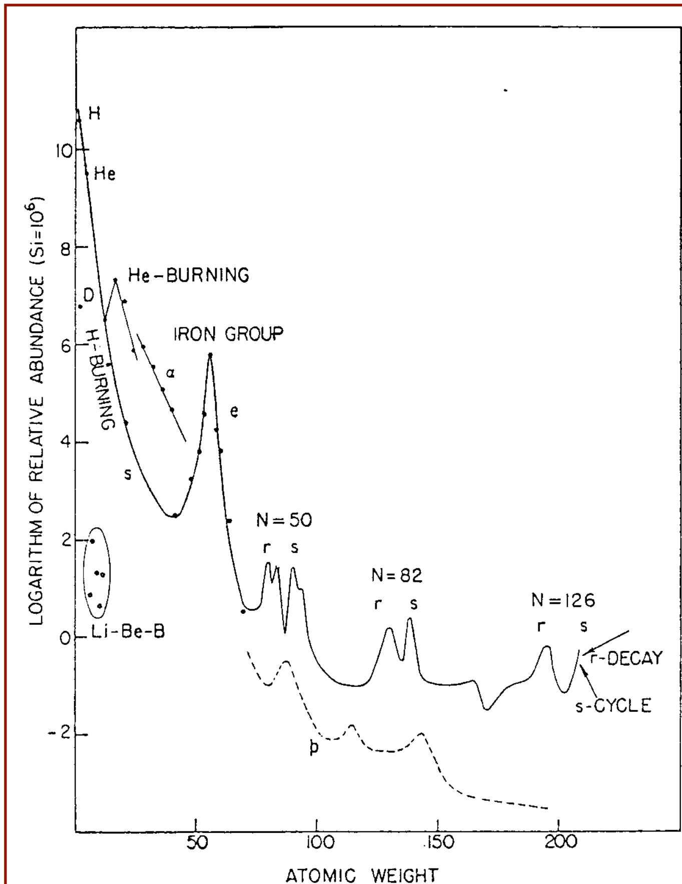
Shell Structure in atoms



From A. Bohr and B.R.Mottleson, *Nuclear Structure*, vol. 1, p. 191 Benjamin, 1969, New York

Nuclear Magic Numbers, nucleon packaging, stability, abundance of elements

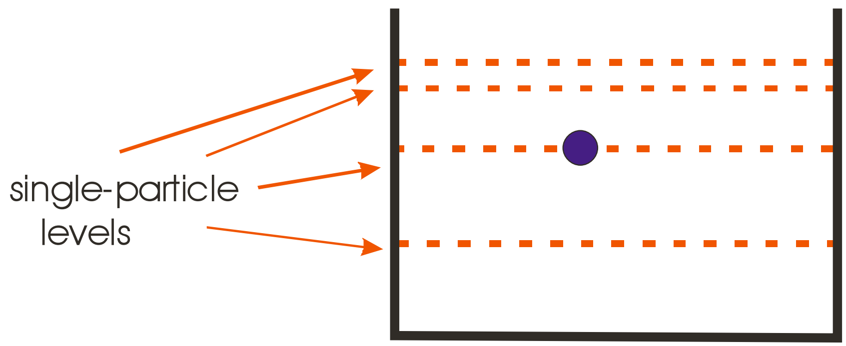




From W.D. Myers and W.J. Swiatecki, Nucl. Phys. **81**, 1 (1966)

Mean field

Nucleon in a box

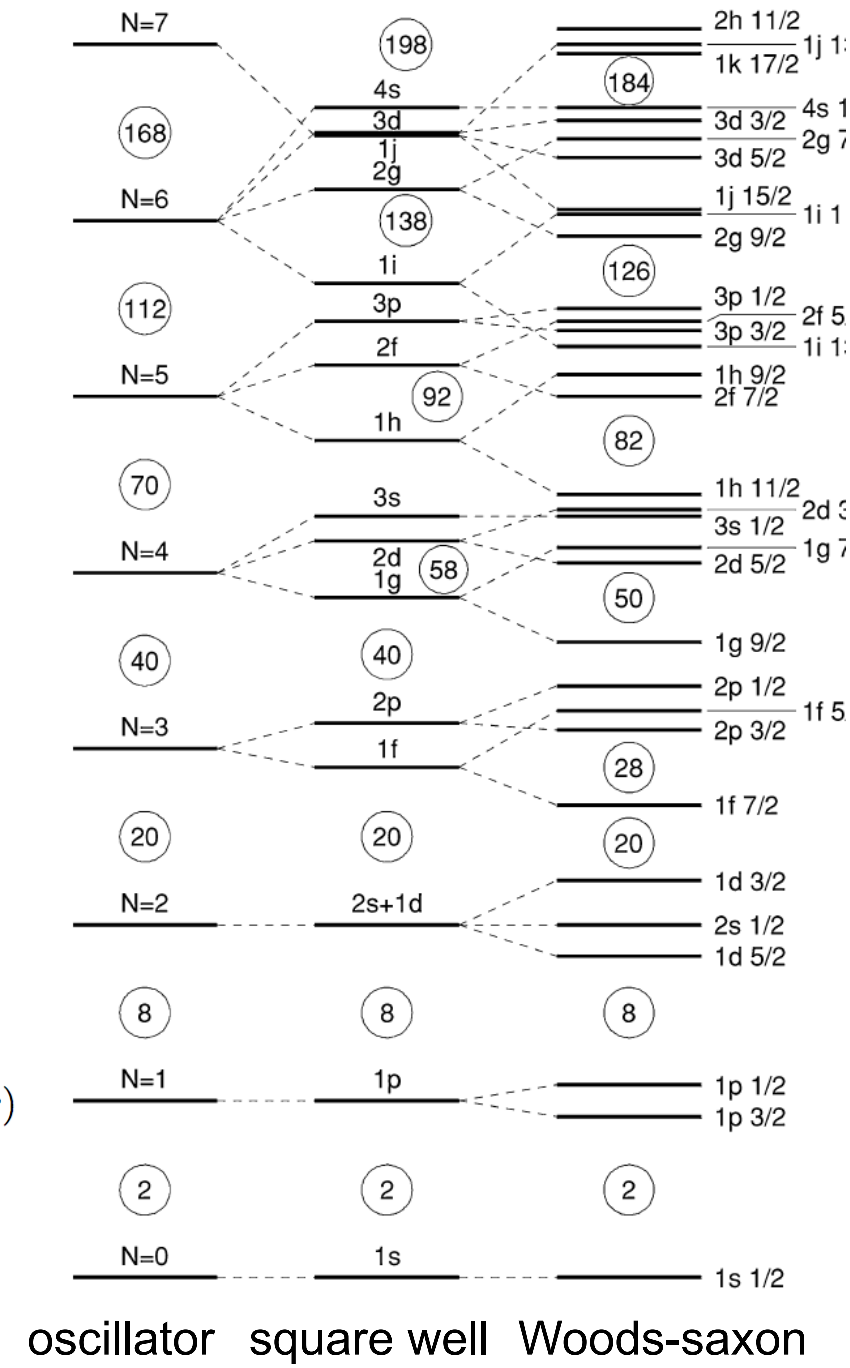


Shell gaps N=2,8,20,

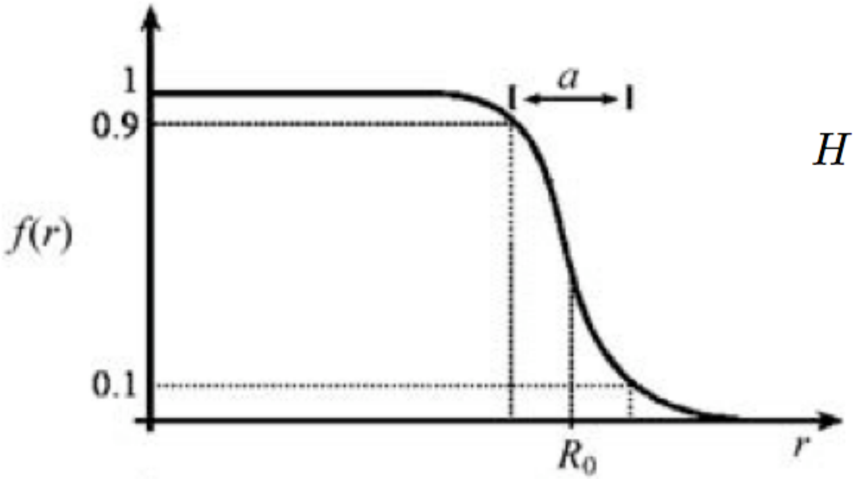
Radial equation to solve

$$\left\{-\frac{d^2}{dr^2}+\frac{l(l+1)}{r^2}+2\mu[v(r)+\alpha\frac{Zz}{r}]\right\}u_l(r)=k^2u_l(r)$$

$$Y_{lm}\frac{u_l(r)}{r}$$



Woods-Saxon potential



$$H = \frac{\mathbf{p}^2}{2\mu} + V(r) + V_c(r) + \frac{1}{2\mu^2 r}\left(\frac{\partial}{\partial r}\tilde{V}(r)\right)\ \mathbf{l}\cdot\mathbf{s}.$$

Central potential

$$V(r) = -V f(r, R, a)$$

Coulomb potential

(uniform charged sphere)

$$f(r, R, a) = \left[1 + \exp\left(\frac{r - R}{a}\right)\right]^{-1}$$

$$V_c(r) = Z' e^2 \begin{cases} (3R^2 - r^2)/(2R^3), & r \le R, \\ 1/r, & r > R, \end{cases}$$

Spin-orbit potential $\tilde{V}(r) = \tilde{V} f(r, R_{SO}, a_{SO})$

Origin of spin-orbit term is non-relativistic reduction of Dirac equation

$$H = \frac{\mathbf{p}^2}{2\mu} - \frac{\mathbf{p}^4}{8\mu^3} + V_c(\mathbf{r}) + \frac{1}{4\mu^2}\sigma\cdot[\nabla V_c(\mathbf{r}) \times \mathbf{p}] + \frac{1}{8\mu^2}\nabla^2 V_c(\mathbf{r})$$

Parameterization:

$$R = R_C = R_0 A^{1/3} \qquad R_{SO} = R_{0,SO} A^{1/3}$$

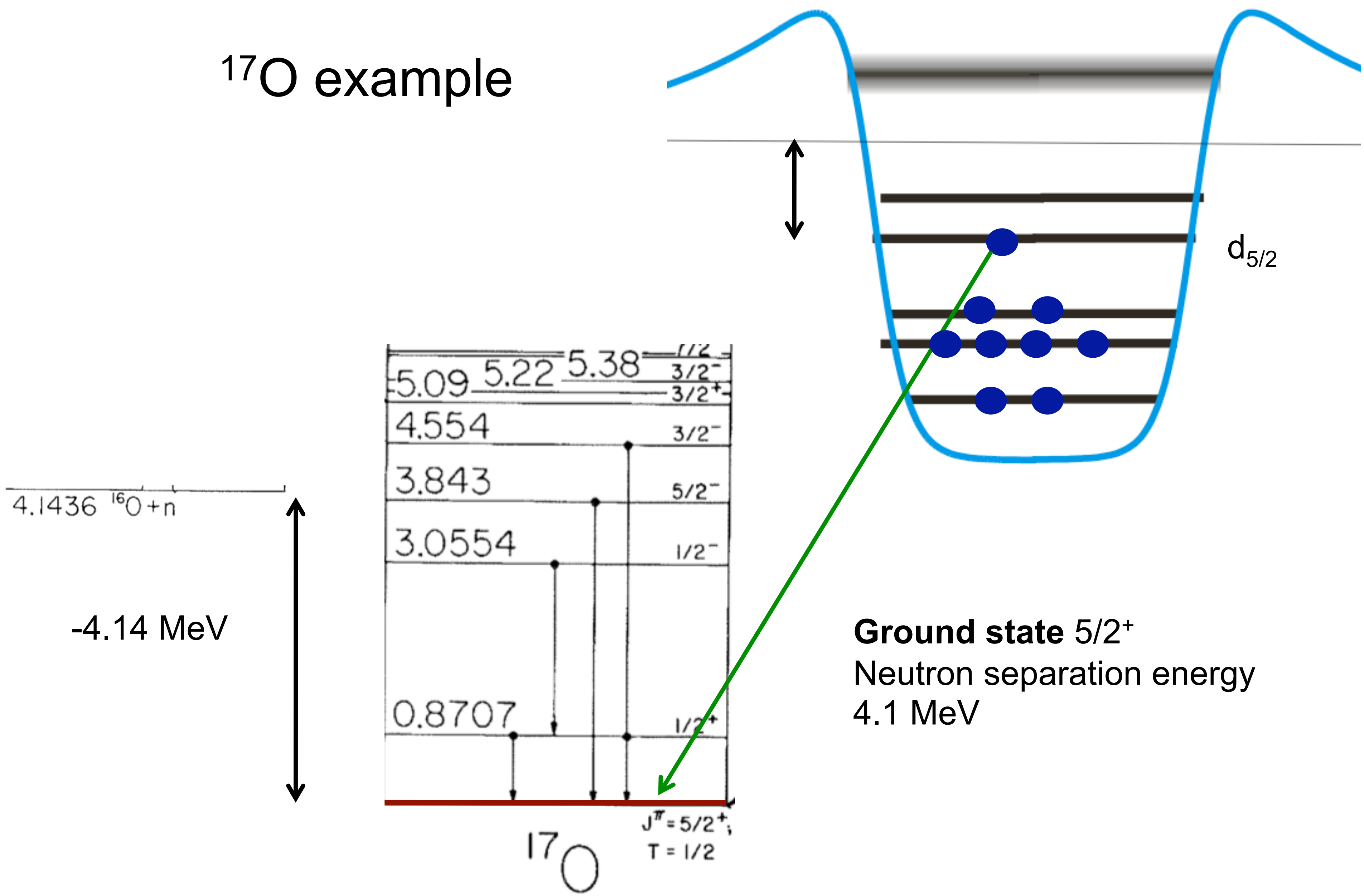
$$V = V_0\left(1 \pm \kappa \frac{(N - Z)}{A}\right)$$

$$a = a_{SO} = const$$

$$\tilde{V} = \lambda V_0$$

Single-particle states in potential model

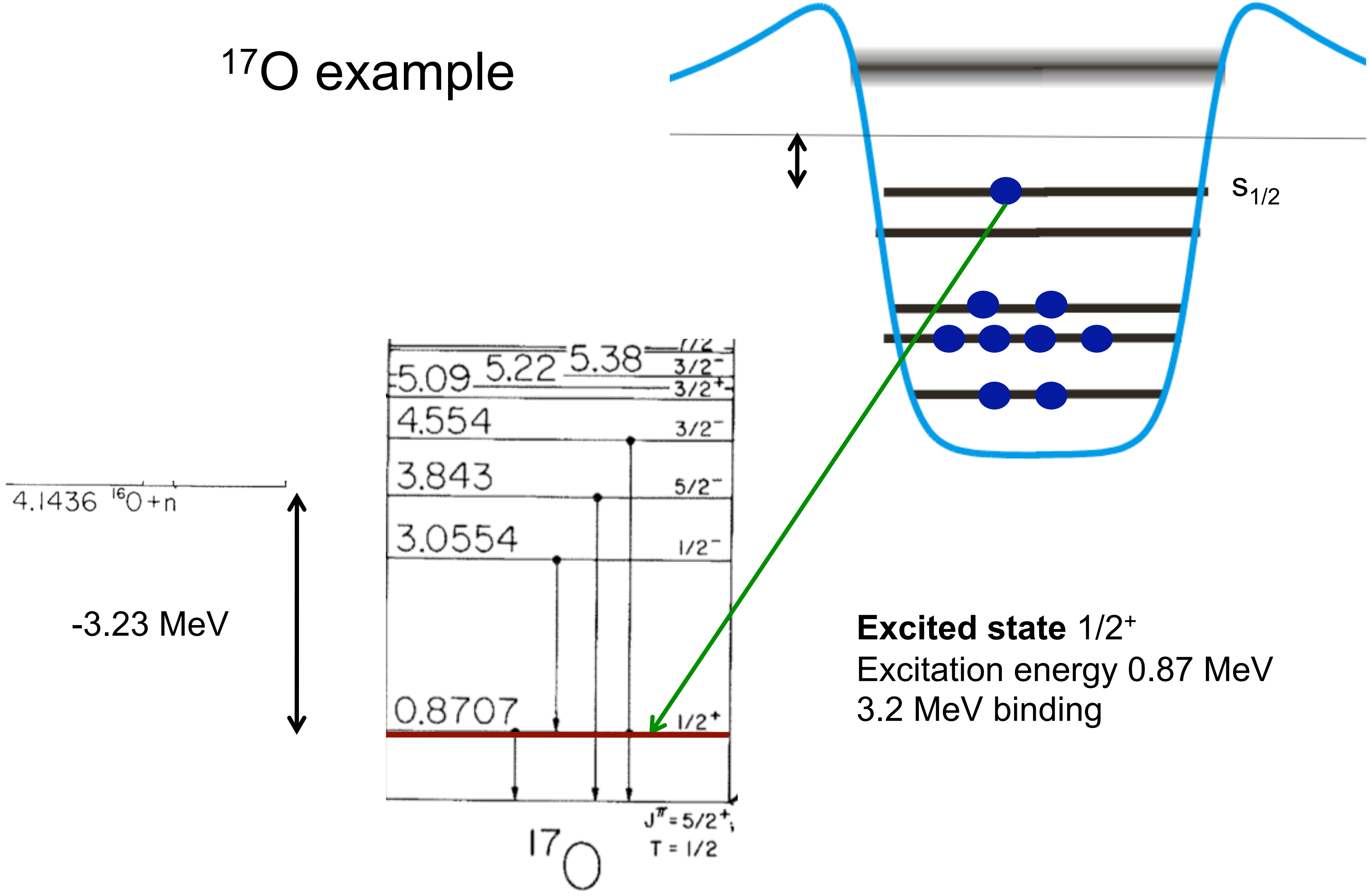
^{17}O example



Ground state $5/2^+$
Neutron separation energy
4.1 MeV

Single-particle states in potential model

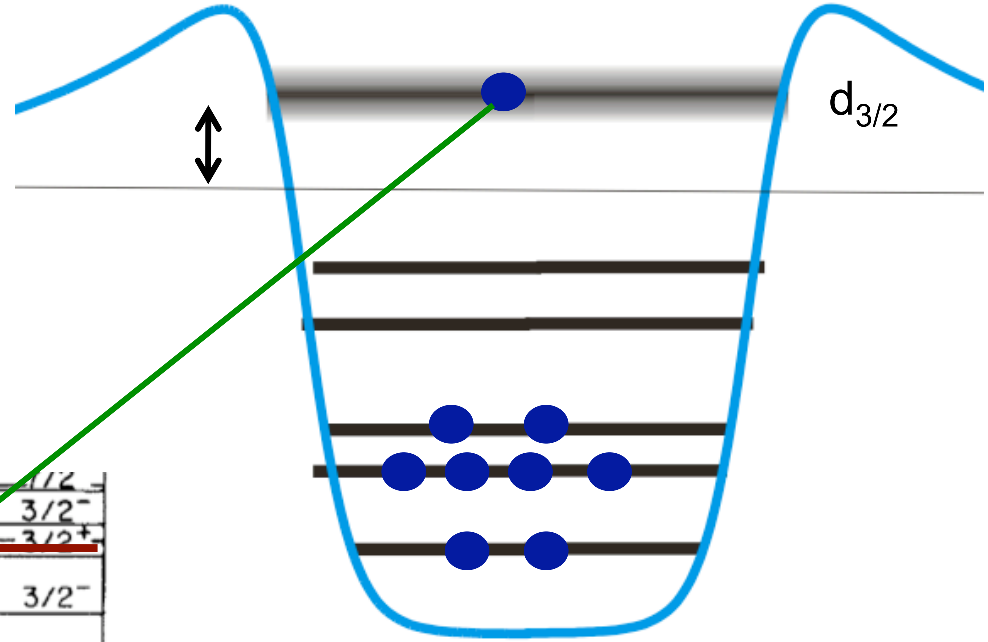
^{17}O example

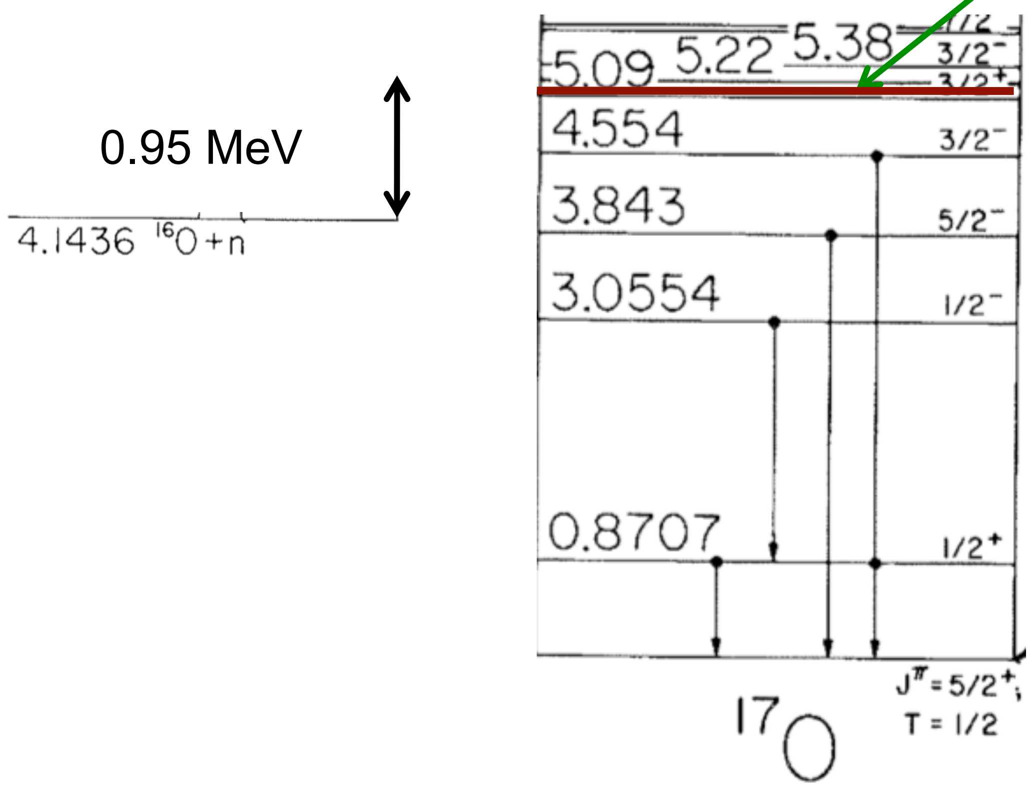


Excited state $1/2^+$
Excitation energy 0.87 MeV
3.2 MeV binding

Single-particle states in potential model

^{17}O example





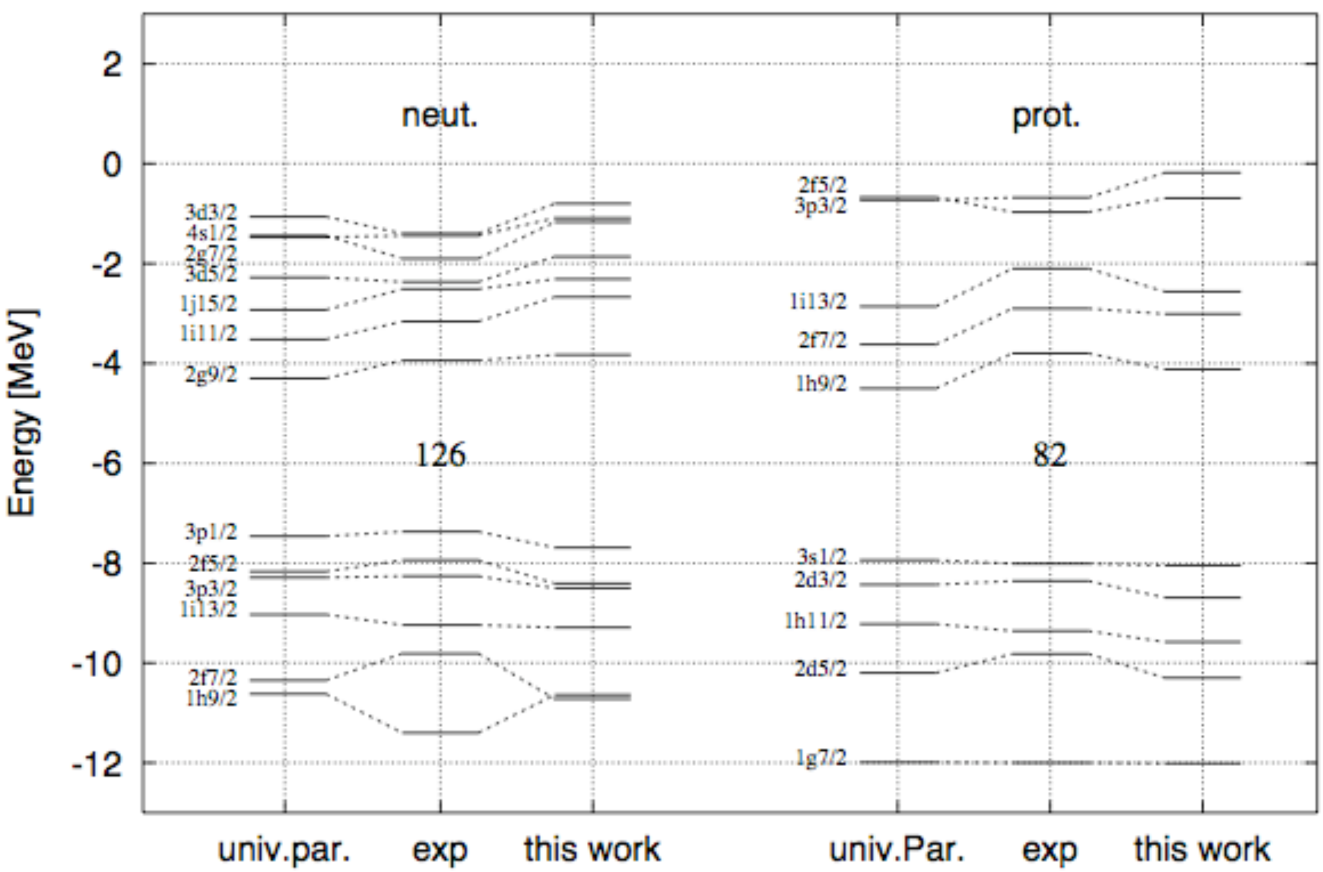
Unbound resonance state $3/2^{+}$
Excitation energy 5.09 MeV
unbound by 0.95 MeV

Woods-Saxon parameterization potential

V_0 [MeV]	κ	R_0 [fm]	$a = a_{SO}$ [fm]	λ	$R_{0,SO}$ [fm]
52.06	0.639	1.260	0.662	24.1	1.16

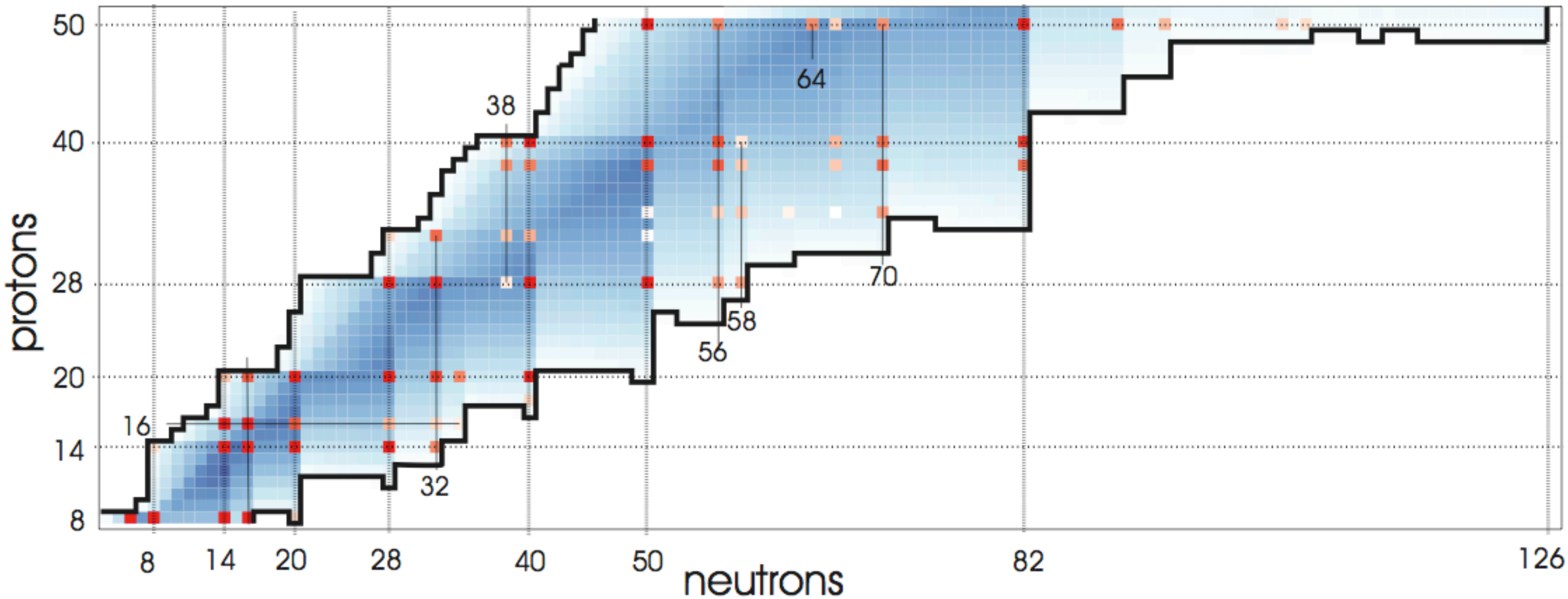
Nuclear Woods-Saxon solver
http://www.volya.net/ws/

^{208}Pb



Woods-Saxon potential

V_0 [MeV]	κ	R_0 [fm]	$a = a_{SO}$ [fm]	λ	$R_{0,SO}$ [fm]
52.06	0.639	1.260	0.662	24.1	1.16



N. Schwierz, I. Wiedenhover, A. Volya, *Parameterization of the Woods-Saxon Potential for Shell-Model Calculations,* *arXiv:0709.3525 [nucl-th]*

Investigation of Supershells

Nishioka et. al. Phys. Rev. B **42**, (1990) 9377
R.B. Balian, C. Block Ann. Phys. **69** (1971) 76

Consider WS potential

$$U(R) = \frac{V_0}{1+\exp[(R-R_0)/a_0]} ,$$

with parameter values

$$V_0 = -6.0 \text{ eV} ,$$

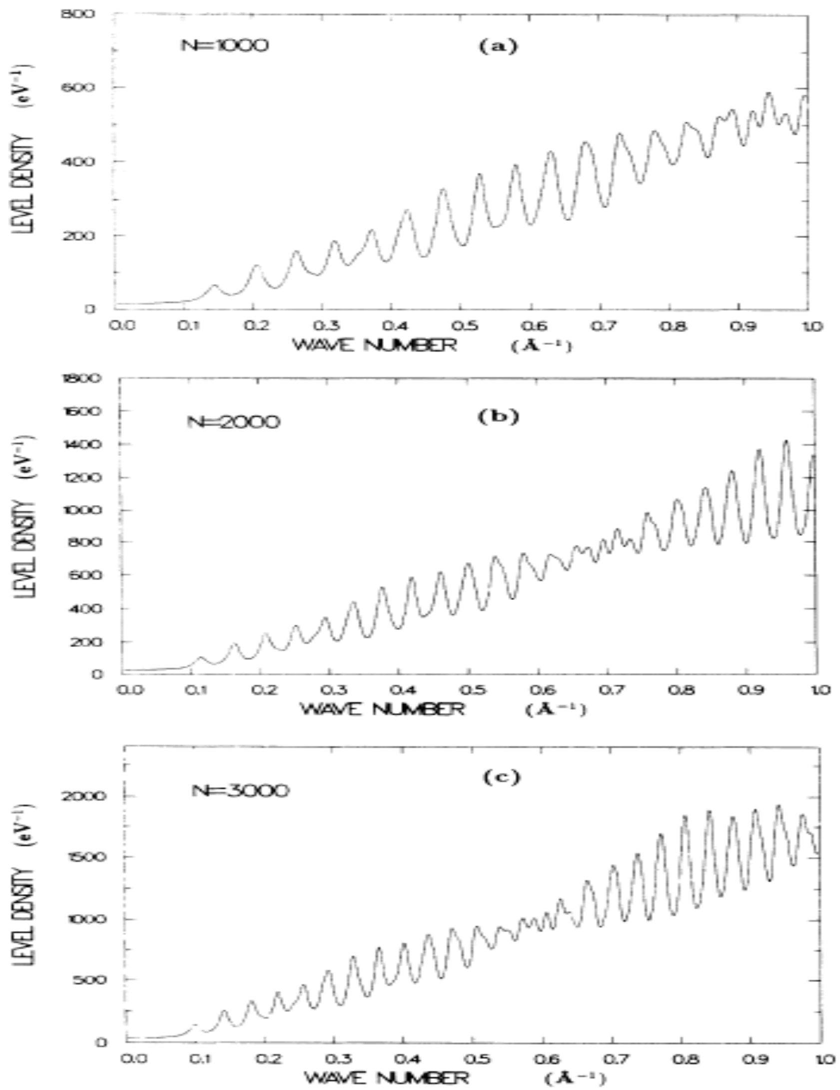
$$R_0 = r_0 N^{1/3}, \quad r_0 = 2.25 \text{ Å} ,$$

$$a_0 = 0.74 \text{ Å} .$$

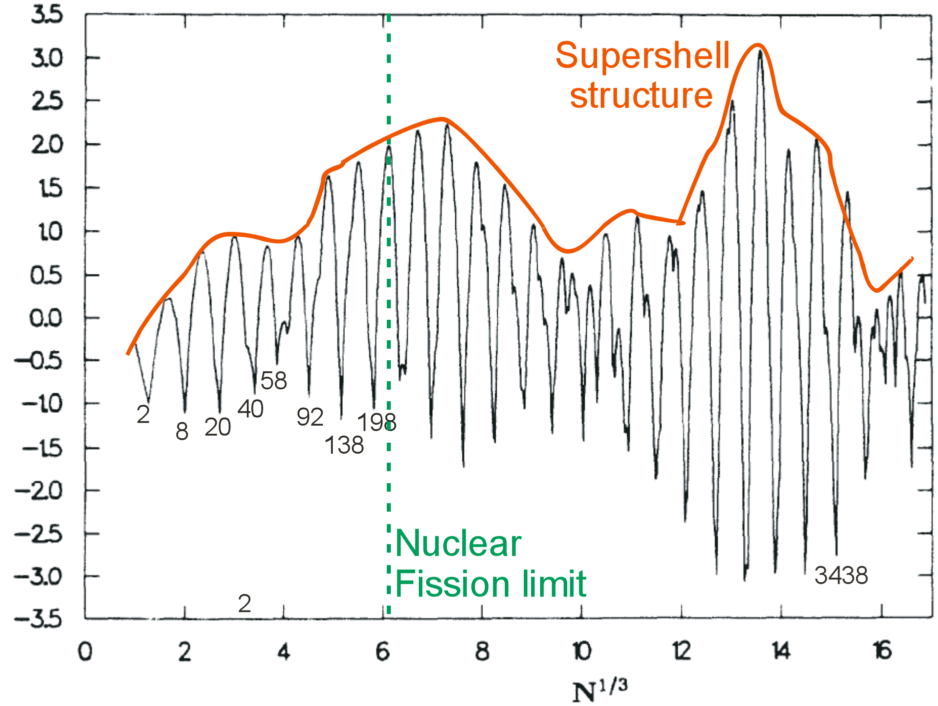
Assuming flat bottom use momentum

$$k = \frac{1}{\hbar}[2m(E-V_0)]^{1/2}$$

Supershells



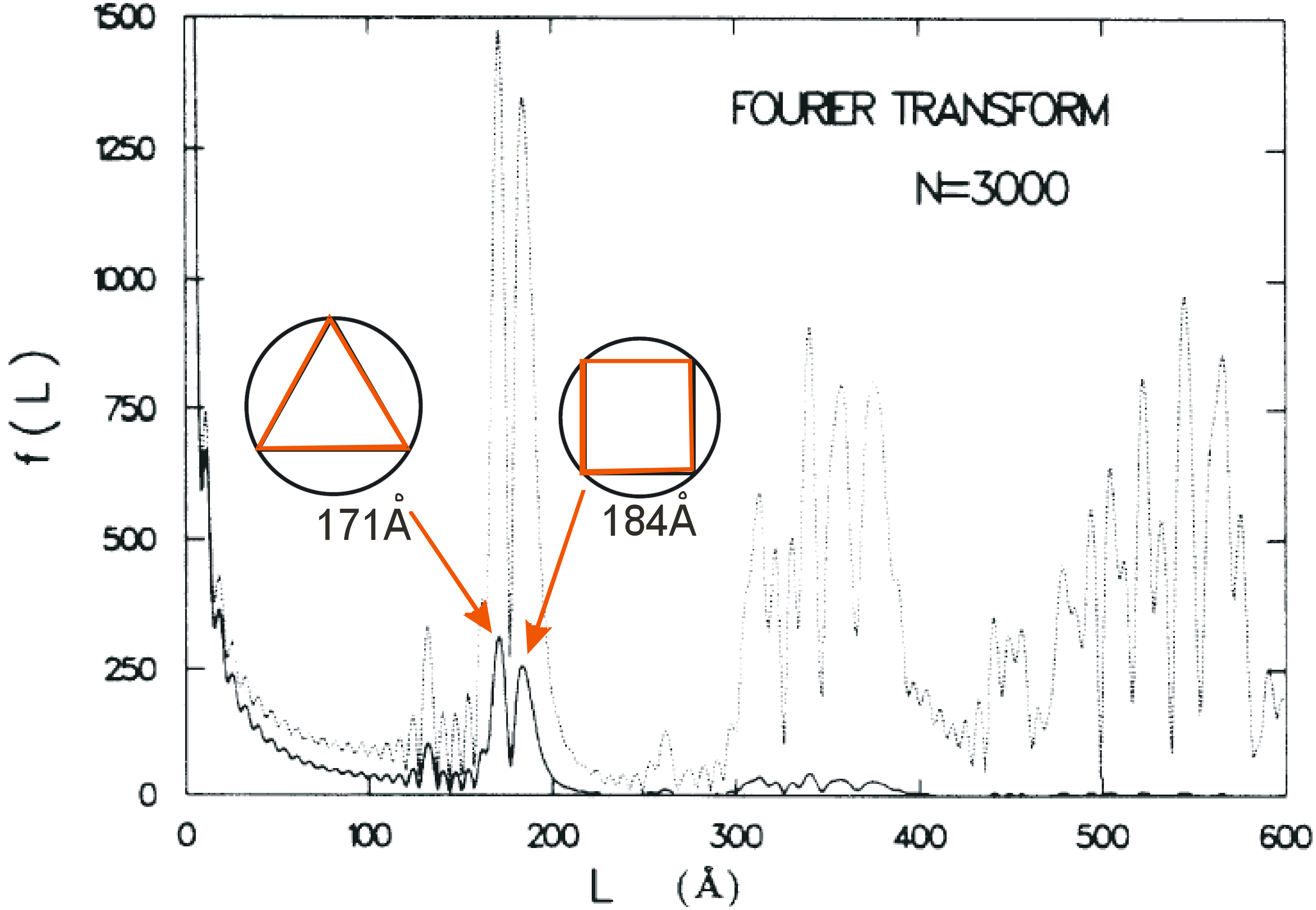
Level density in the Woods-Saxon Potential: N=1000, 2000, and 3000



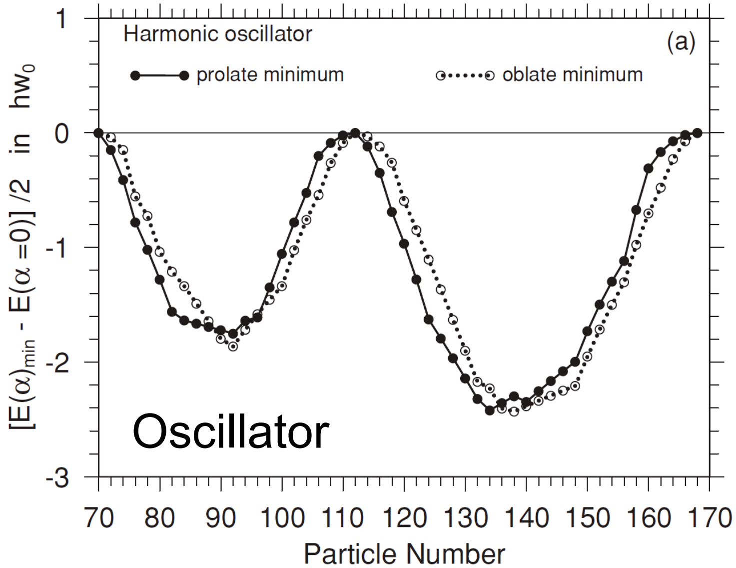
Binding energy, deviation from average

Nishioka et. al. Phys. Rev. B **42**, (1990) 9377
R.B. Balian, C. Block Ann. Phys. **69** (1971) 76

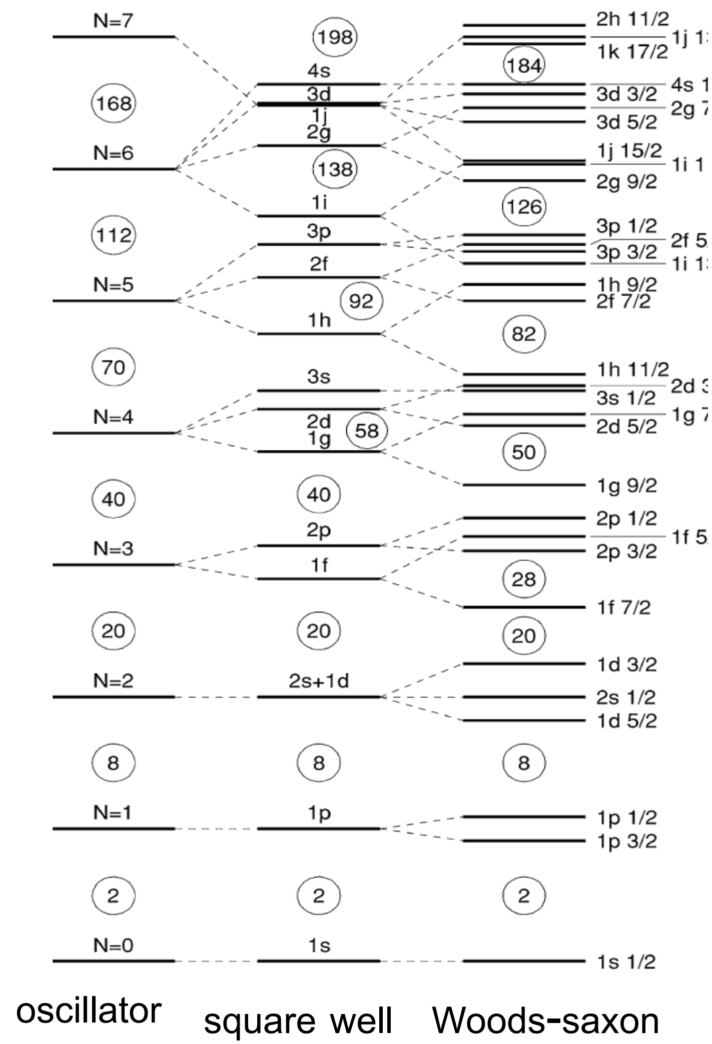
Supershells and classical periodic orbits

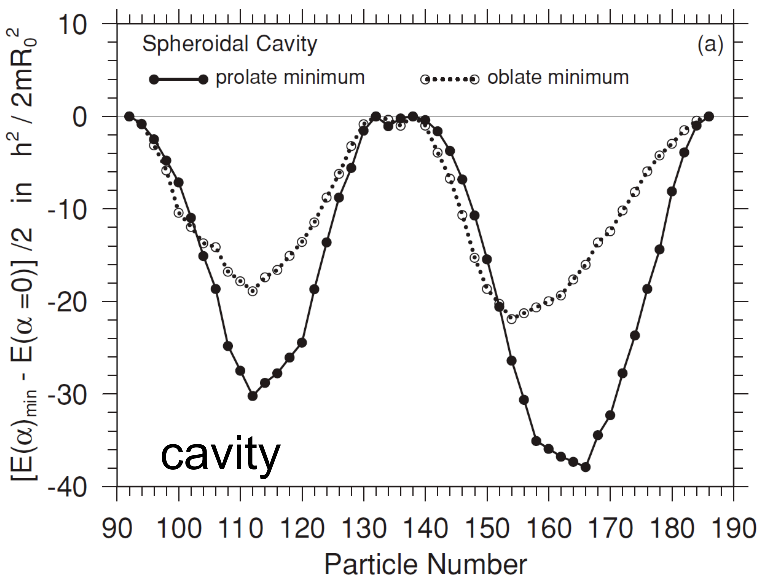


Prolate-shape dominance in nuclear deformation



The sharpness of the nuclear surface





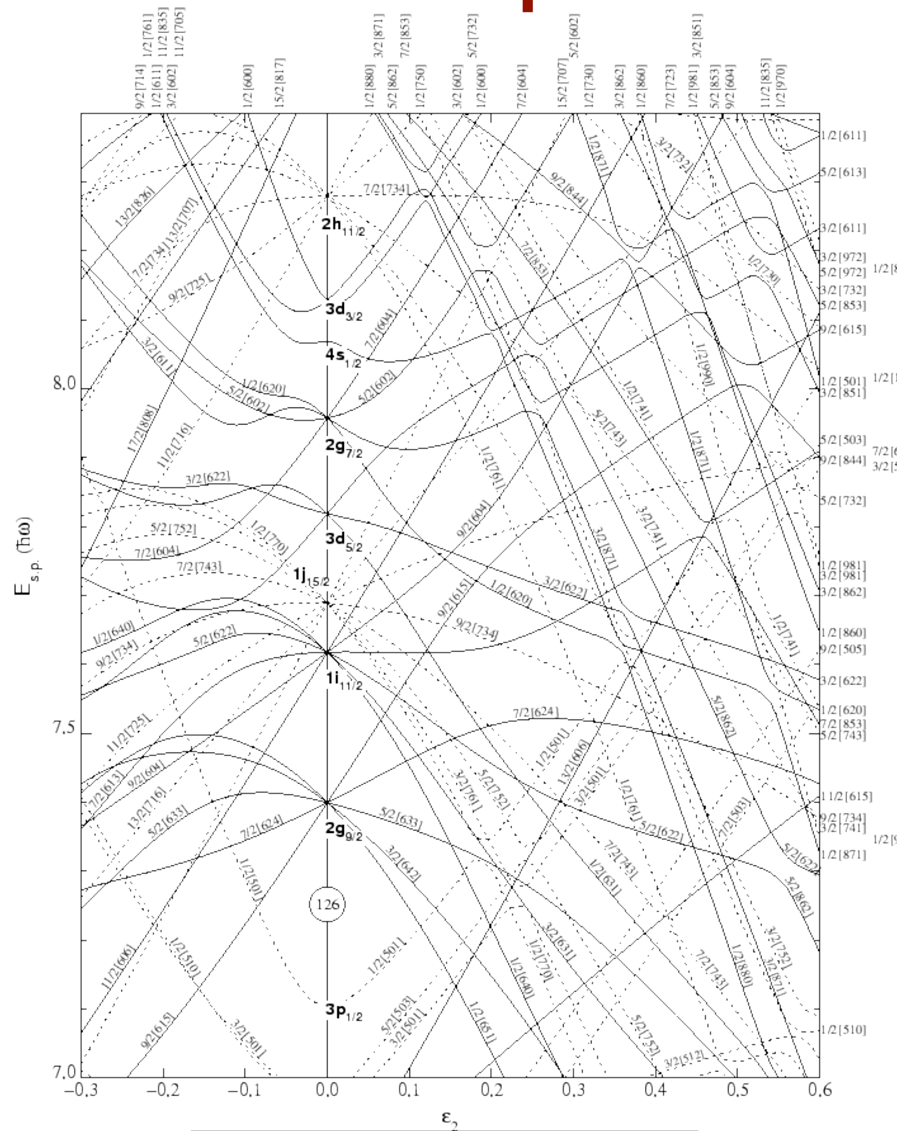
I. Hamamoto and B. R. Mottelson, Phys. Rev. C **79,** 034317 (2009)

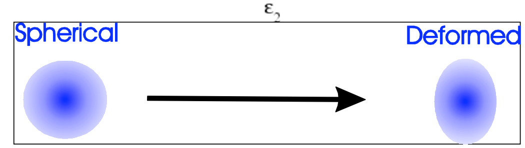
Evolution of shells

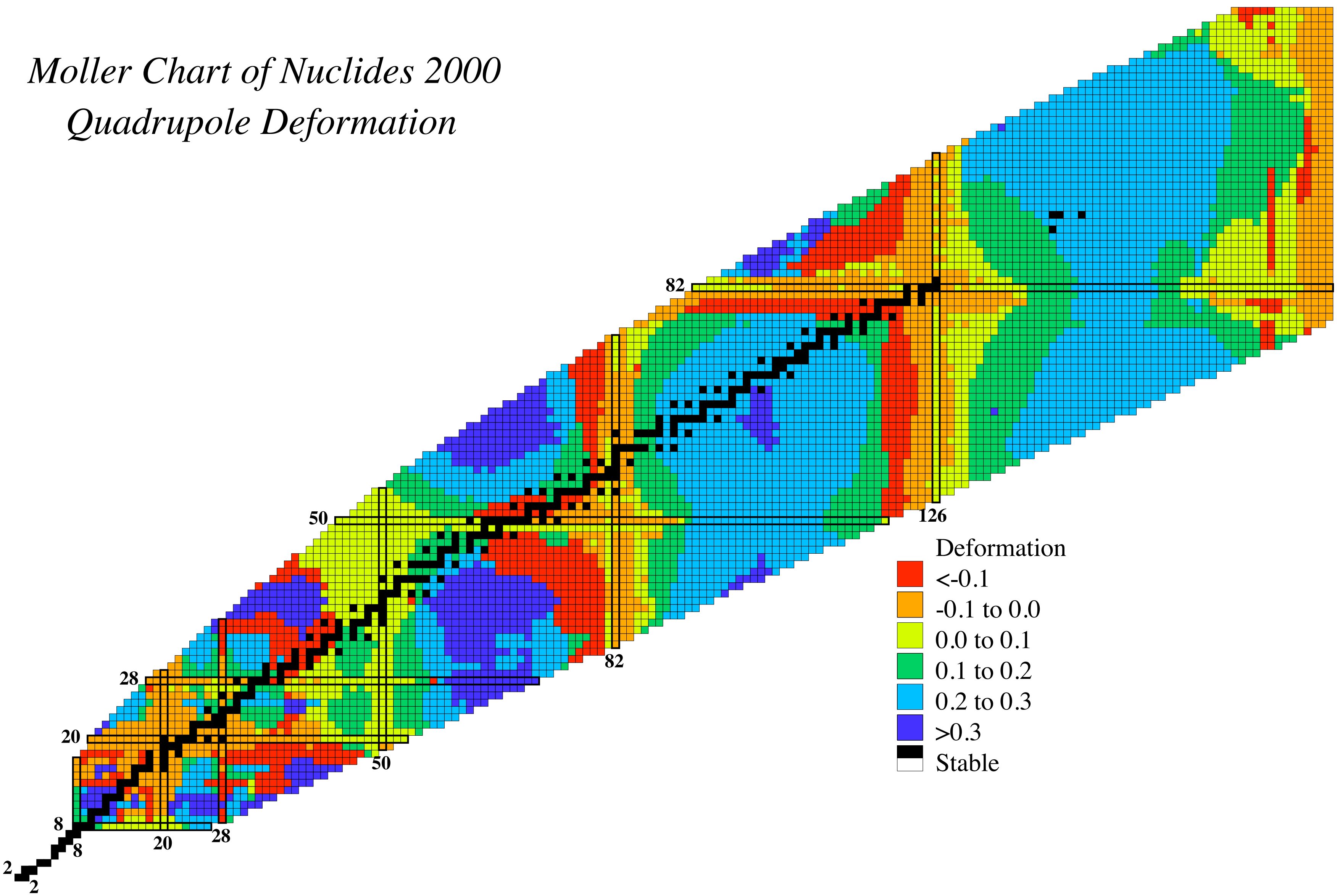
- Melting of shell structure
- Shells in deformed nuclei
- Shells in weakly bound nuclei
- Is the mean field concept valid?

Single-nucleon motion in deformed potential

Nilsson Hamiltonian: Anisotropic Harmonic oscillator Hamiltonian

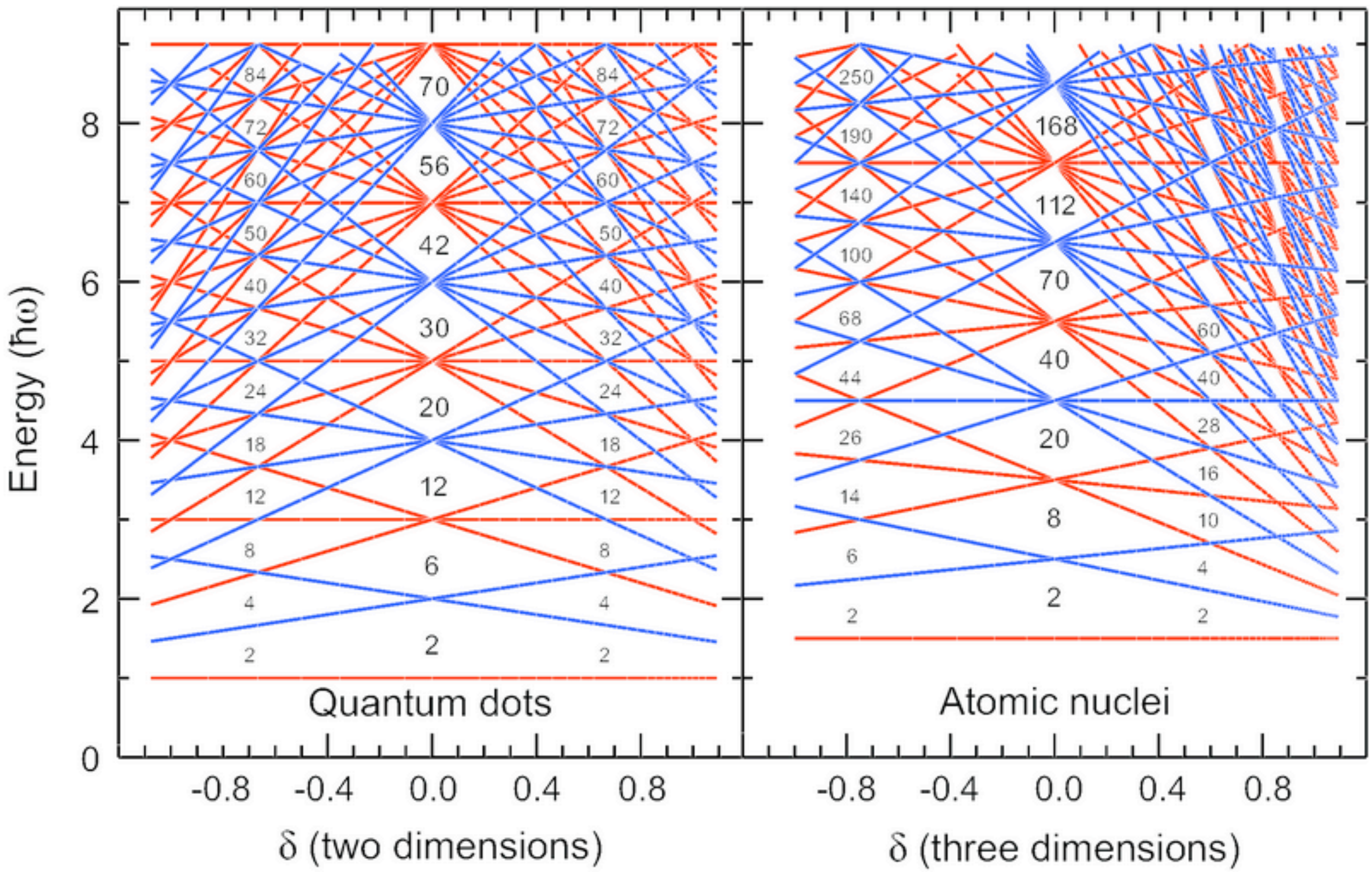






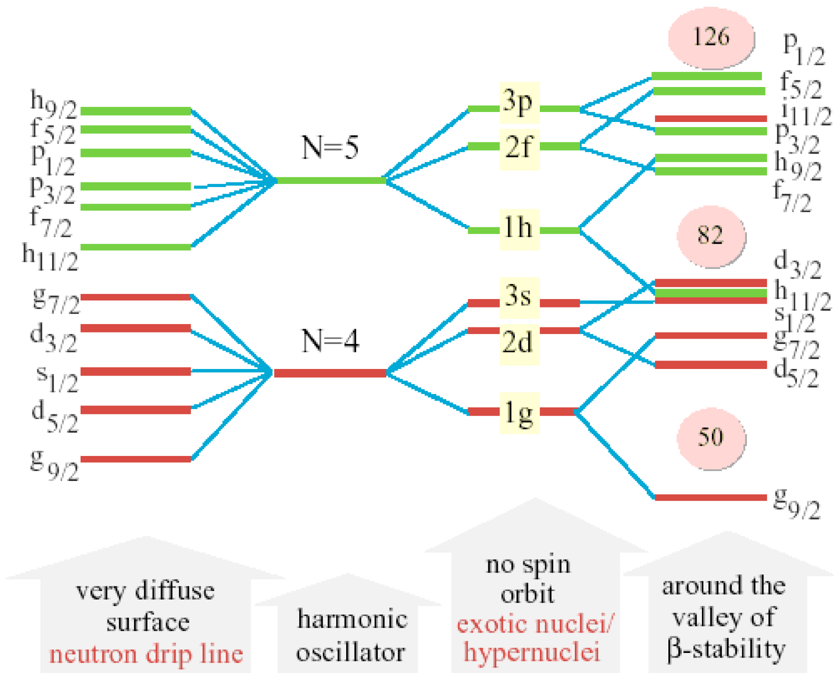
Moller Chart of Nuclides 2000
Quadrupole Deformation
Deformation
<-0.1
-0.1 to 0.0
0.0 to 0.1
0.1 to 0.2
0.2 to 0.3
>0.3
Stable
2
2
8
8
20
20
28
28
50
50
82
82
126

Deformation and shell gaps



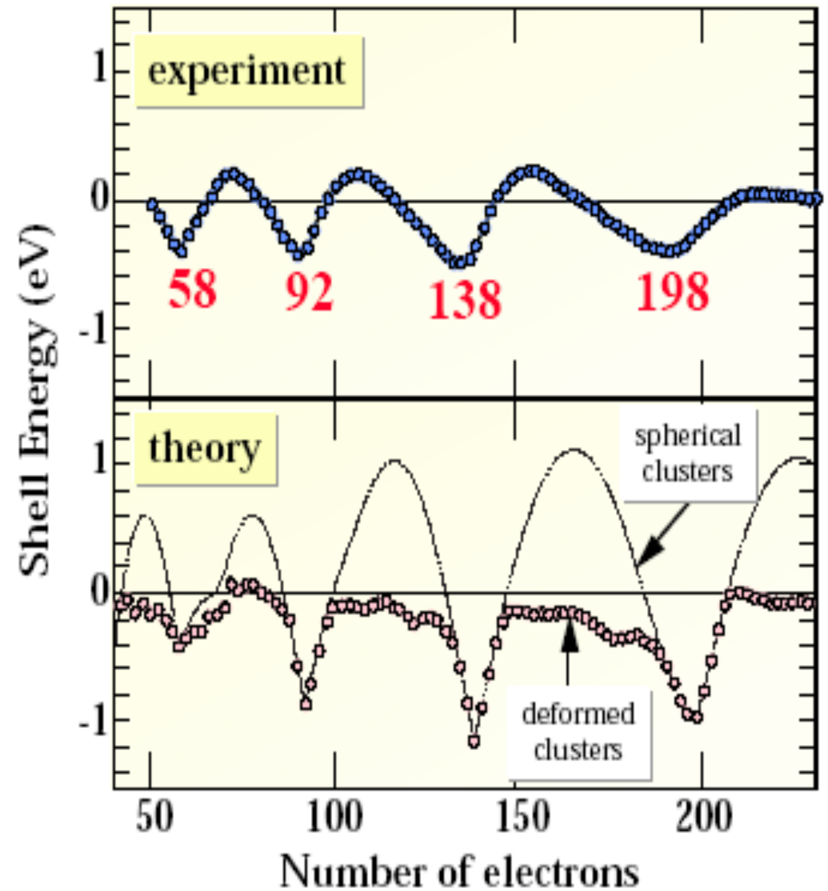
Shell structure in extreme limits

Shells in nuclei far from stability



J. Dobaczewski *et al.*, PRC53, 2809 (1996)

Melting of shell structure



T=0 and T=0.4 ev,
Frauendorf S, Pahskevich VV. *NATO ASI Ser. E: Appl. Sci.*, ed. TP Martin, 313:201. Kluwer (1996)

Single-particle decay

- Decay rate and width
- Potential size
- Single-particle structure and decay from deformed nucleus
- Decay and recoil.

Single-particle decay

Coordinate wave function $Y_{lm}\frac{u_l(r)}{r}$

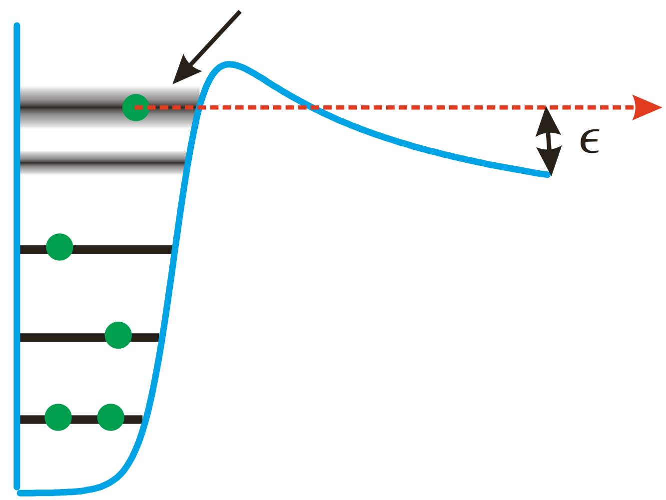
Radial equation to solve $\left\{-\frac{d^2}{dr^2}+\frac{l(l+1)}{r^2}+2\mu[v(r)+\alpha\frac{Zz}{r}]\right\}u_l(r)=k^2u_l(r)$

$$u_l(r)=\cos(\delta_l)F_l(r)+\sin(\delta_l)G_l(r) \qquad O_l^{\pm}(r)=G_l(r)\pm iF_l(r)$$

$$\lim_{r\to\infty}u(r)=\mathcal{N}O^{+}(kr)$$

Consider an “almost” stationary state

$$\gamma=\frac{k}{\mu}|\mathcal{N}|^2$$



Square well of size R gives

$$\gamma=\frac{2\hbar^2}{\mu R^2}kR\left|\frac{2l-1}{2l+1}\right|T_l$$

Barrier transmission probability $T_l\approx\left[\frac{kR^l}{(2l-1)!!}\right]^2$ coulomb $T=\exp(-2\pi\eta)$

One-body decay

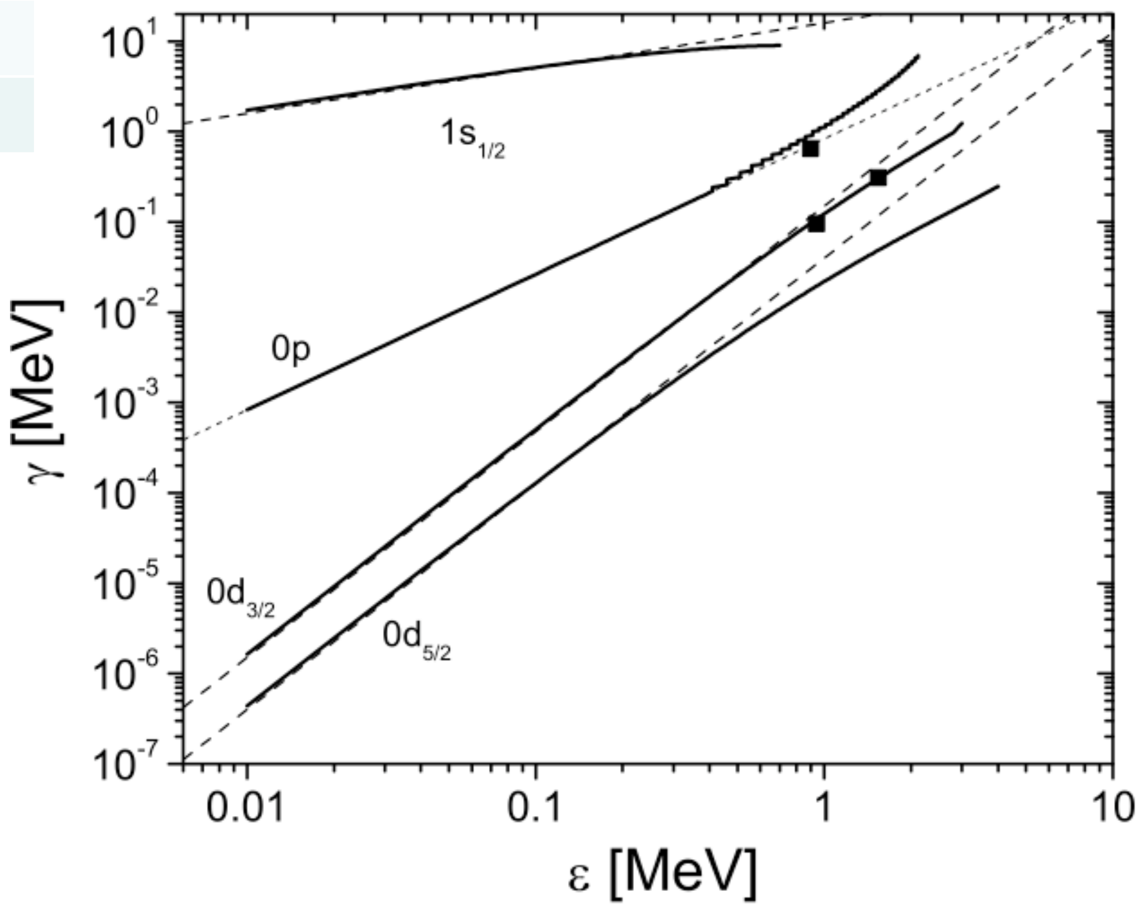
realistic one-body potential

$$\gamma(\epsilon) \sim \epsilon^{l+1/2}$$

	e(MeV)	γ(keV)	R(fm)
^{5}He	0.895	648	4.5*
^{17}O	0.941	98	3.8
^{19}O	1.540	310	3.9

*^{5}He is too broad R=4.5 fm gives max width of 0.6 MeV

Decay width calculation using Woods-Saxon



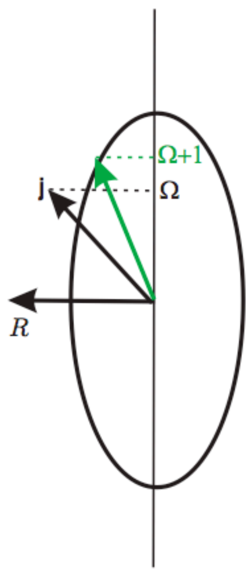
Single-particle decay and rotating mean field

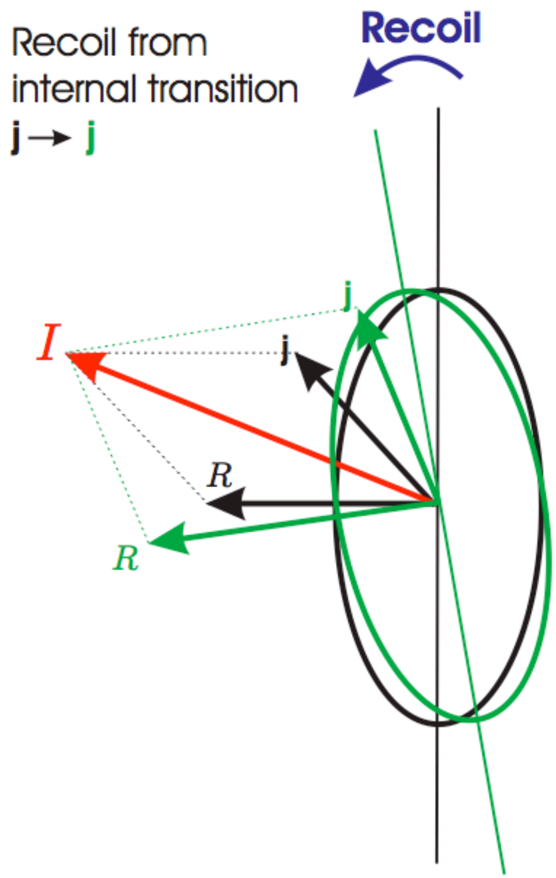
Odd-nucleon systems with deformation: particle-rotor model:

Total angular momentum $\mathbf{I} = \mathbf{R} + \mathbf{j}$ $\frac{\mathbf{R}^2}{2\mathcal{L}} = \frac{(\mathbf{I}-\mathbf{j})^2}{2\mathcal{L}}$ Recoil term "Coriolis"

Rotor part $H = \mathbf{I}^2/(2\mathcal{L}) + H'$ $H' = \frac{1}{2\mathcal{L}}(\mathbf{j}^2 - 2j_3^2) - \frac{1}{2\mathcal{L}}(j_+I_- + j_-I_+) + H_{\mathrm{intr}}$

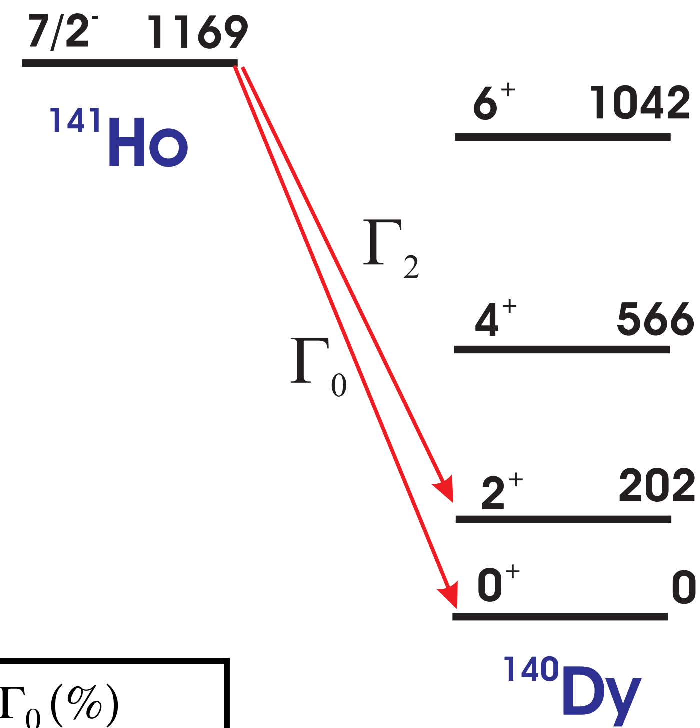
Adiabatic treatment





Particle decay from deformed proton emitters

•single particle motion
•deformed mean field
•recoil of mean field
•Coriolis attenuation



	$\Gamma_0\ (10^{-20}$ MeV)		$\Gamma_2/\Gamma_0\ (\%)$	
	Rotor	RHFB	P+Rotor	RHFB
Experiment	10.9		0.71	
Adiabatic	15.0		0.73	
Coriolis	1.4	5.9	1.8	1.2
Coriolis+pairing	1.7	7.0	1.7	0.3

Nuclear Structure Theory II

The Nuclear Many-body Problem

Alexander Volya

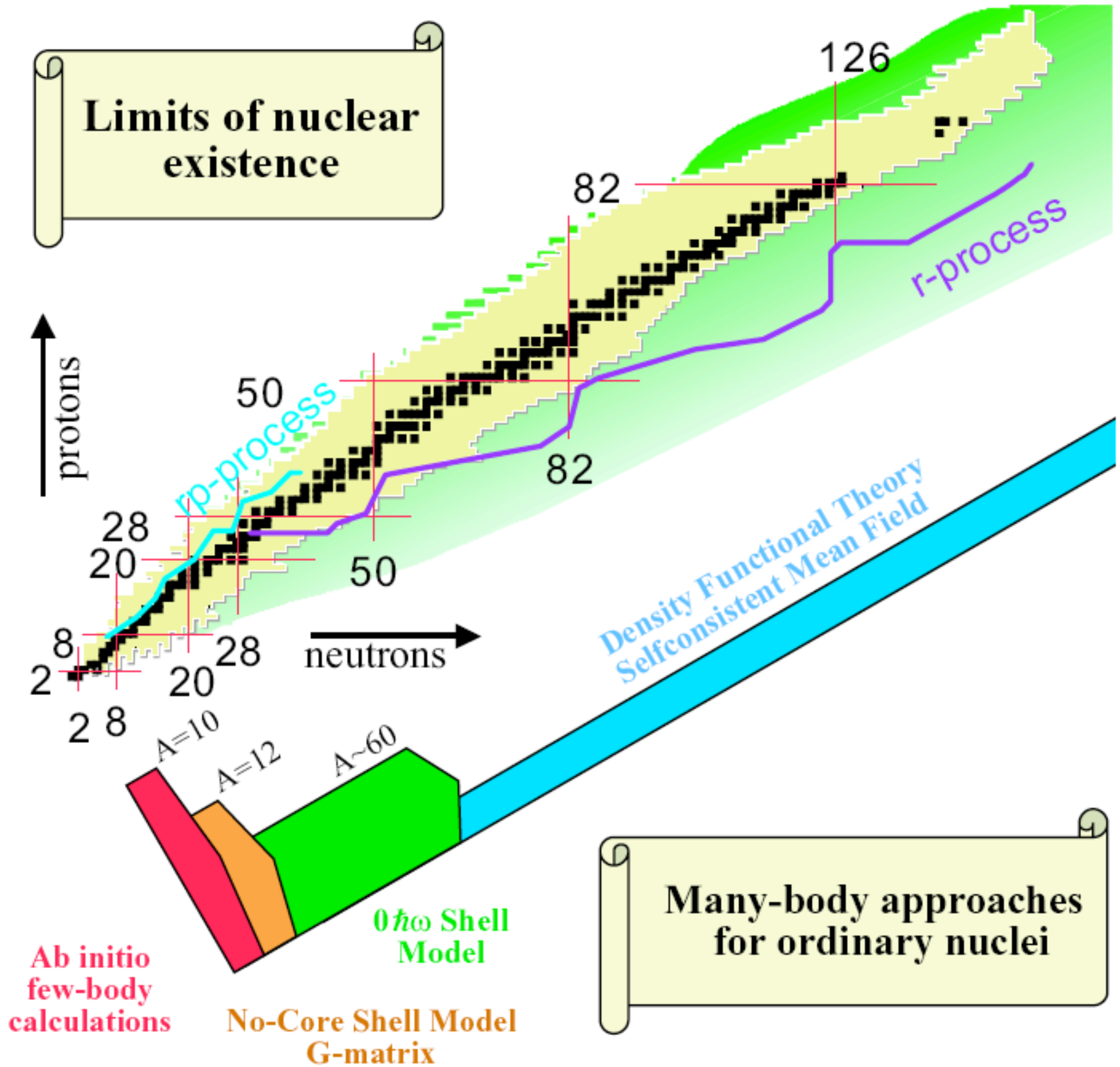
Florida State University

10th Exotic Beam Summer School - EBSS2011
East Lansing, Michigan. 25-30 July, 2011

Nuclear many-body problem configuration interaction, the shell model

- Applicability and limitations
- Many-body configurations and Hamiltonian
- Example study
- Binding energy, shell evolution and monopole
- Pairing interaction
- Multipole-multipole interaction, emergence of deformation and rotations
- Statistical approach and random matrix theory

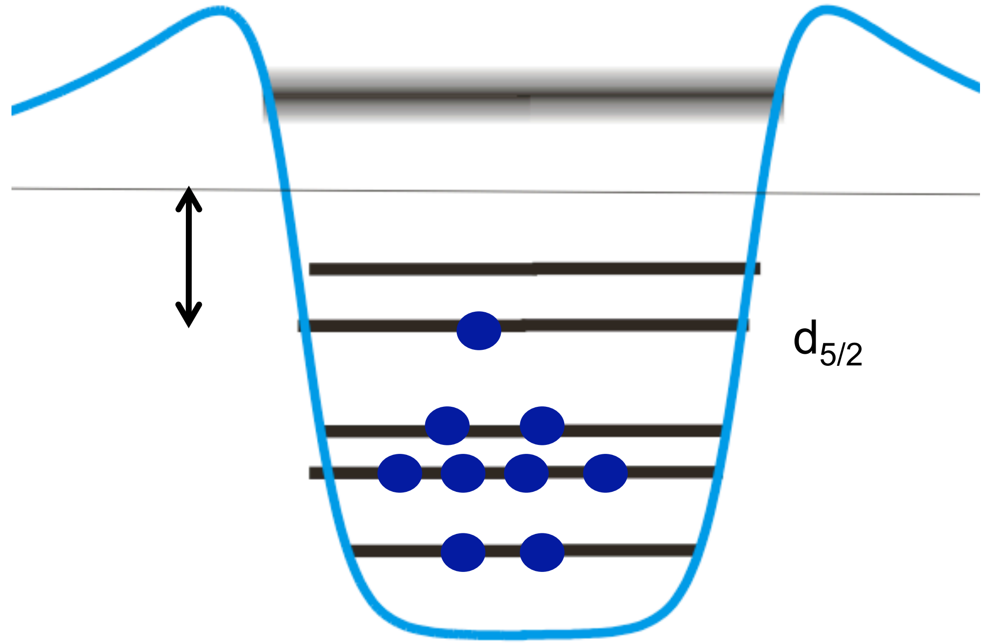
Chart of Isotopes



The Nuclear Shell Model

Many-body Hamiltonian

$$H = \sum_a \frac{\vec{p}_a^{\,2}}{2m} + \sum_{a>b} V_{NN}(r_a - r_b)$$



Mean field and residual interactions

$$H = \sum_a \left[\frac{\vec{p}_a^{\,2}}{2m} + U(r_a) \right] + \underbrace{\sum_{a>b} V_{NN}(r_a - r_b) - \sum_a U(r_a)}_{\text{Residual Interaction}}$$

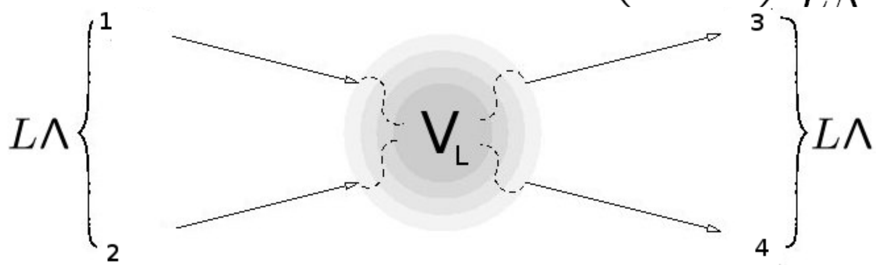
Residual interactions

- Residual, depends on mean-fied
- Depends on truncation of space and basis
- Not exactly two-body

Shell model interactions

Two-body Hamiltonian in the particle-particle channel:

$$H = \sum_L V_L \sum_\Lambda P^\dagger_{L\Lambda} P_{L\Lambda} \quad \text{where} \quad P^\dagger_{L\Lambda} \propto \left(a^\dagger_1 a^\dagger_2\right)_{L\Lambda}$$



No-core shell model
•bare NN interaction
•Renormalized interactions improve convergence
Traditional shell model
•Simple potential interactions
•Renormalized bare interactions to include core and core excitations
•Phenomenological interactions determined from fits.

Typical shell model study

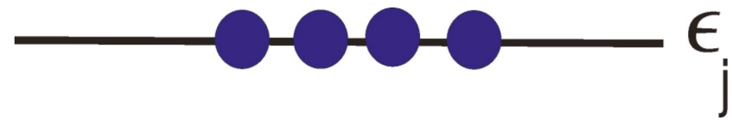
Shell model codes:

NuShell: http://knollhouse.eu/NuShellX.aspx
Antoine: http://sbgat194.in2p3.fr/~theory/antoine/menu.html
Redstick: http://www.phys.lsu.edu/faculty/cjohnson/redstick.html
CoSMo: http://www.volya.net (click CoSMo)
For demonstration we use CoSMo code.

Anatomy of a shell model study

1. Identify system, valence space, limitations on many-body states to study (cosmoxml)
2. Create a list of many-body states, typically fixed J_z projection T_z, and parity (Xsysmbs)
3. Create many-body Hamiltonian (XHH+JJ)
4. Diagonalize many-body Hamiltonian using exact, lanczos, davidson or other method (texactev, davidson_file).
5. Database eigenstates states and determine their spins (XSHLJT)
6. Define other operators and compute various properties
   - Overlaps and spectroscopic factors (SHLSF)
   - Electromagnetic transition rates (SHLEMB)

The simple model

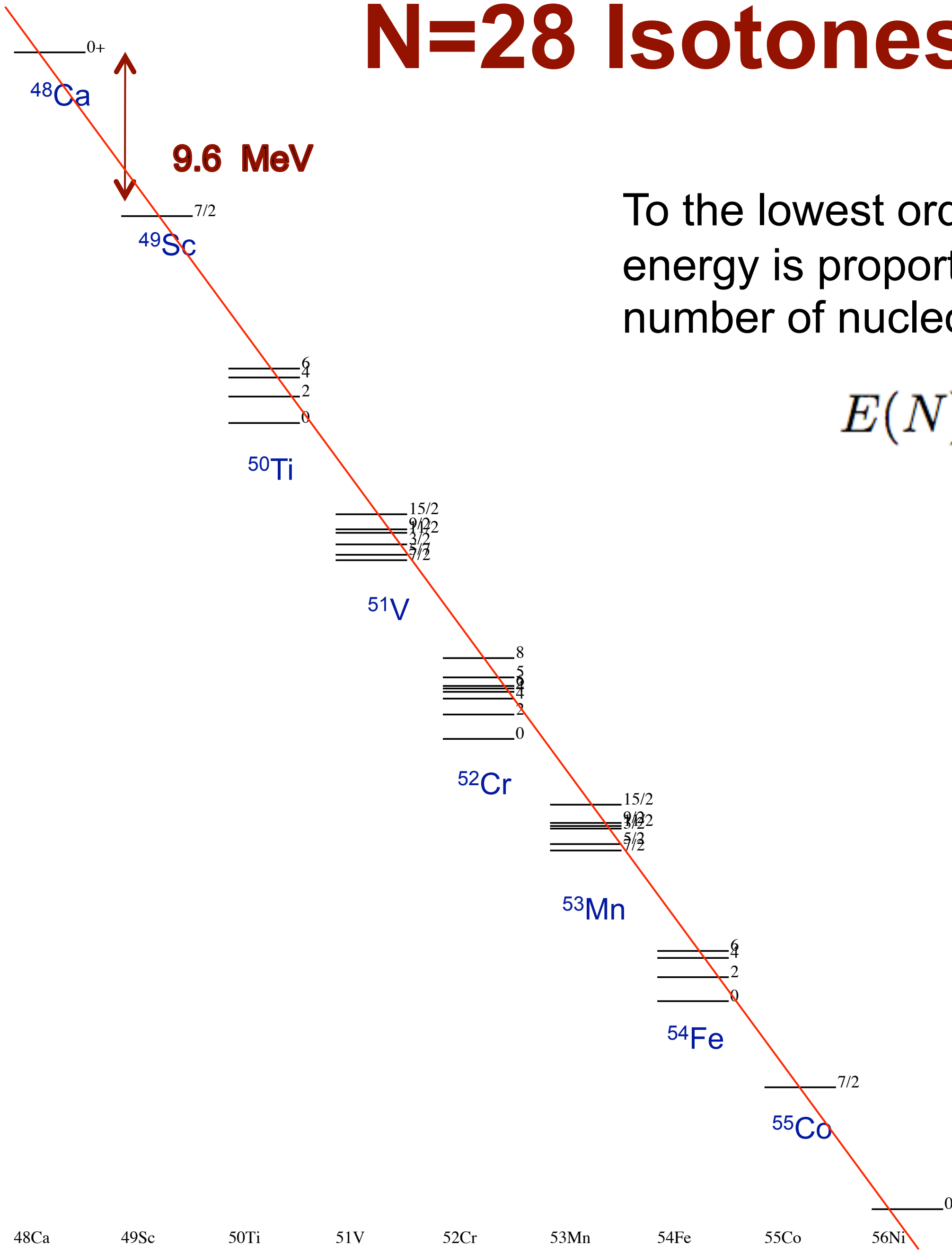


- Single-j level
- $\Omega=2j+1$ single-particle orbitals: $m=-j, j-1, \dots j$
- Number of nucleons N: $0 \le N \le \Omega$
- Number of many-body states: $\Omega!/((N!(\Omega-N)!)$
- Many-body states classified by rotational symmetry: (J,M)

Dynamics

- Rotational invariance and two-body interactions
  particle-particle pair operator $P_{L\,M}=(a\ a)_{L\,M}$
  particle-hole pair operator $M_{K\,\kappa}=(a\ a^\dagger)_{K\,\kappa}$
- Hamiltonian $H = \sum_L V_L \sum_M P^\dagger_{LM} P_{LM}$
- Dynamics is fully determined by j+1/2 parameters V_L

N=28 Isotones, data



To the lowest order binding energy is proportional to the number of nucleons.

$$E(N) \sim \epsilon N$$

Spin	ν	Name	Binding
0	0	^{48}Ca	0
7/2	1	^{49}Sc	9.626
0	0	^{50}Ti	21.787
2	2	1.554	20.233
4	2	2.675	19.112
6	2	3.199	18.588
7/2	1	^{51}V	29.851
5/2	3	0.320	29.531
3/2	3	0.929	28.922
11/2	3	1.609	28.241
9/2	3	1.813	28.037
15/2	3	2.700	27.151
0	0	^{52}Cr	40.355
2_1	2*	1.434	38.921
4_1	4*	2.370	37.986
4_2	2*	2.768	37.587
2_2	4*	2.965	37.390
6	2	3.114	37.241
5	4	3.616	36.739
8	4	4.750	35.605
7/2	1	^{53}Mn	46.915
5/2	3	0.378	46.537
3/2	3	1.290	45.625
11/2	3	1.441	45.474
9/2	3	1.620	45.295
15/2	3	2.693	44.222
0	0	^{54}Fe	55.769
2	2	1.408	54.360
4	2	2.538	53.230
6	2	2.949	52.819
7/2	2	^{55}Co	60.833
0	0	^{56}Ni	67.998

N=28 Isotones, data

1-particle J=j=7/2
2 particles J=0,1,2,..7
but Pauli principle J=0,2,4,6
3 particles

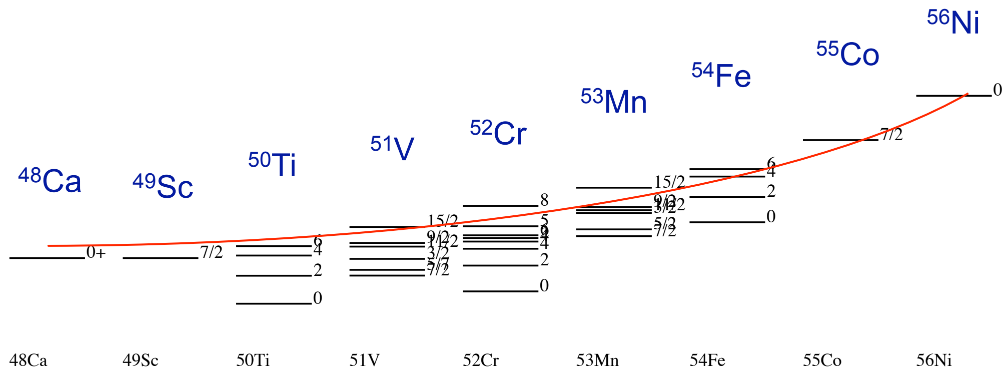
Total states 56=8!/(5!3!)

J=15/2,11/2,9/2,7/2,5/2,3/2

3-particles on j=7/2, 28 m>0 states

m_1	m_2	m_3	M	
7/2	5/2	3/2	15/2	1
7/2	5/2	1/2	13/2	1
7/2	5/2	-1/2	11/2	
7/2	3/2	1/2	11/2	2
7/2	5/2	-3/2	9/2	
7/2	3/2	-1/2	9/2	
5/2	3/2	1/2	9/2	3
7/2	5/2	-5/2	7/2	
7/2	3/2	-3/2	7/2	
7/2	1/2	-1/2	7/2	
5/2	3/2	-1/2	7/2	4
7/2	5/2	-7/2	5/2	
7/2	3/2	-5/2	5/2	
7/2	1/2	-3/2	5/2	
5/2	3/2	-3/2	5/2	
5/2	1/2	-1/2	5/2	5
7/2	3/2	-7/2	3/2	
7/2	1/2	-5/2	3/2	
7/2	-1/2	-3/2	3/2	
5/2	3/2	-5/2	3/2	
5/2	1/2	-3/2	3/2	
3/2	1/2	-1/2	3/2	6
7/2	1/2	-7/2	1/2	
5/2	3/2	-7/2	1/2	
5/2	1/2	-5/2	1/2	
5/2	-1/2	-3/2	1/2	
5/2	-3/2	-1/2	1/2	
3/2	1/2	-3/2	1/2	6

Monopole term



$$H = \epsilon N + \sum_{L=0,2,4,6} V_L \sum_{M=-L}^{L} P^\dagger_{LM} P_{LM}$$

Consider a constant component "shift" $\quad V_L \to V_L - \tilde{V}_0$

"Shift term" counts number of pairs $\quad H \to H - \frac{N(N-1)}{2}\tilde{V}_0$

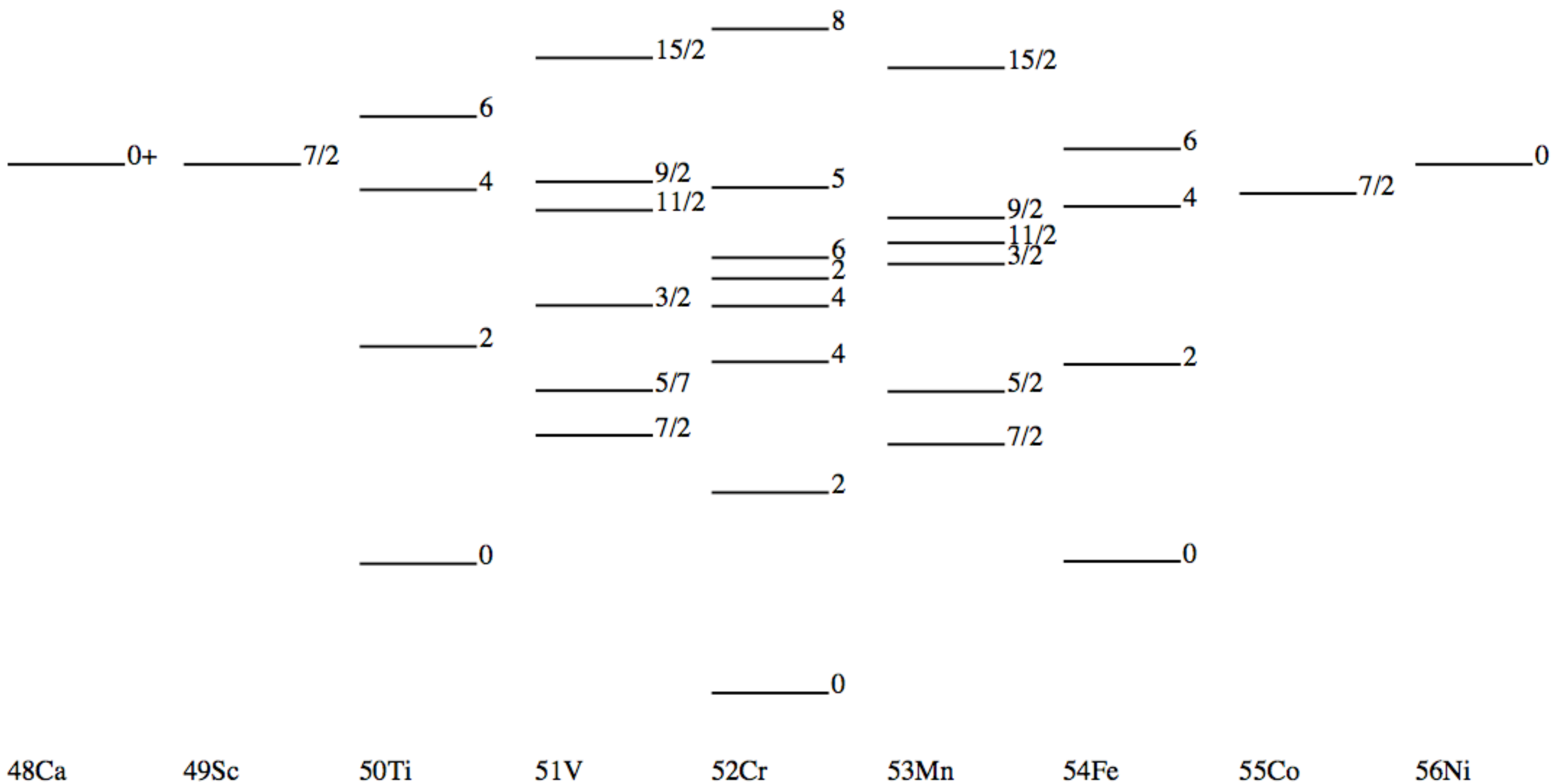
What is needed to fix closed shell ^{56}Ni ? $\quad N = \Omega = 2j+1$

Fully occupied shell ^{56}Ni $\quad \langle \Omega | P^\dagger_{LM} P_{LM} | \Omega \rangle = 1 \quad E_{^{56}\mathrm{Ni}} = 8\epsilon + \sum_L V_L(2L+1)$

Monopole term $\quad \tilde{V}_0 = \frac{\sum_L (2L+1) V_L}{\sum_L (2L+1)} \quad \sum_L (2L+1) = \frac{\Omega(\Omega-1)}{2}$

Matching binding across the shell gives $\quad \tilde{V}_0 = 0.3217\,\mathrm{MeV}$

N=28 isotones Monopole term



$$E - \epsilon N - N(N-1)\tilde{V}_0/2$$

$$\epsilon = -9.626\,\mathrm{MeV} \quad \tilde{V}_0 = 0.3217\,\mathrm{MeV}$$

Prediction S_p=7.374 MeV in ^{56}Ni

The single-particle energy changes from 9.6 MeV to 7.3 MeV

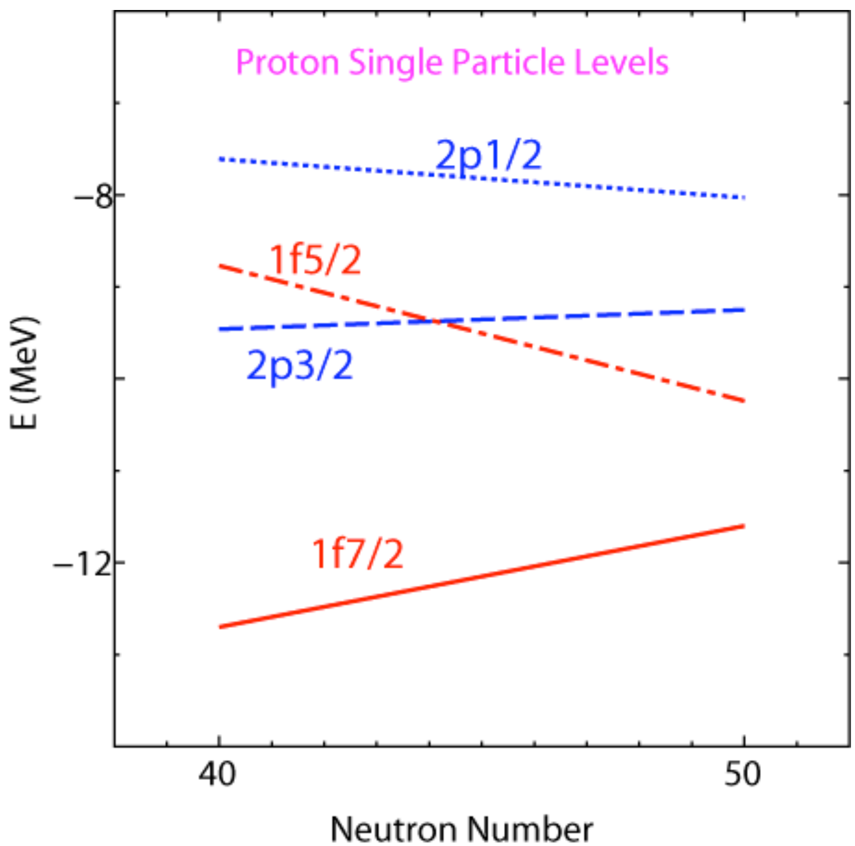
Woods-Saxon prediction 9.9 MeV to 7.2 MeV

Shell evolutions and monopole term

Energy: $E(N) \sim \epsilon N + N(N-1)\tilde{V}_0/2$

$$\varepsilon = \partial E/\partial N \sim \epsilon + \tilde{V}_0 N$$

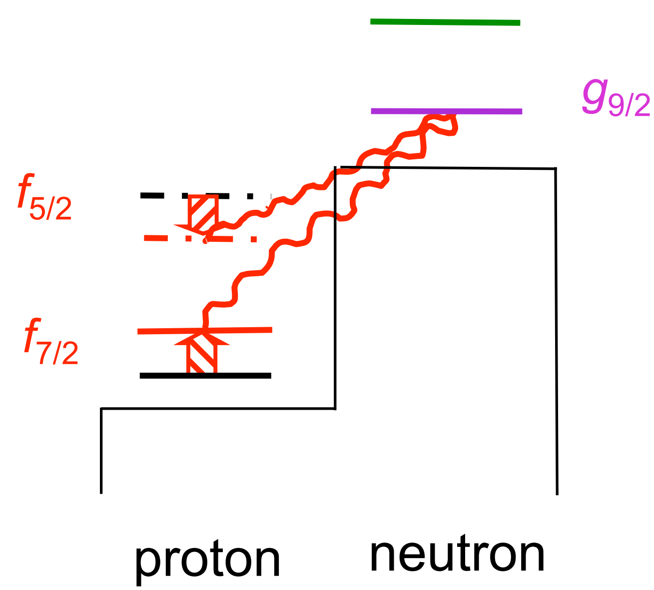
Effective single-particle energies



Example from From Otsuka, GXPF1 interaction

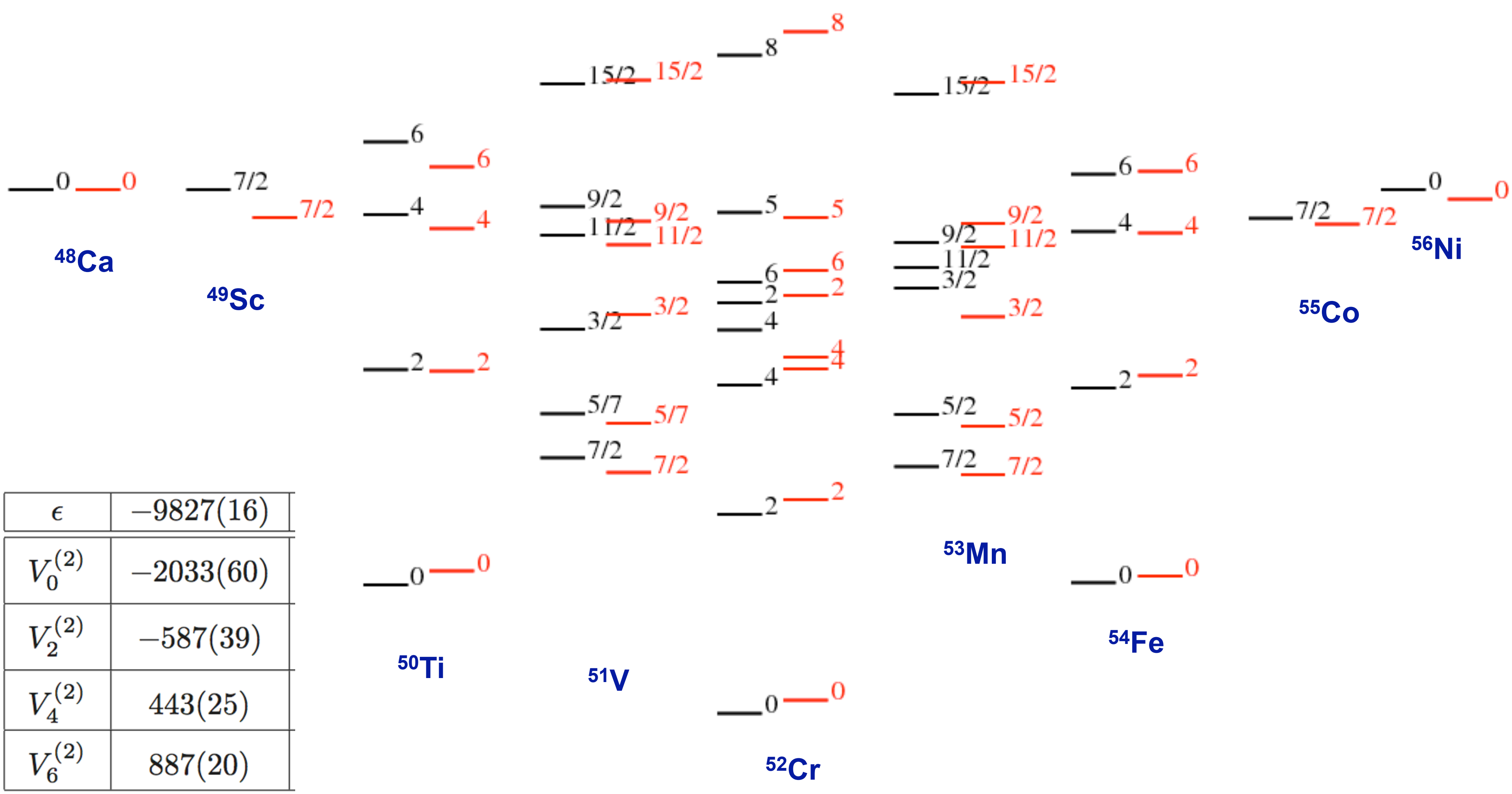
Tensor nucleon-nucleon interaction
off-diagonal monopole term

$$\varepsilon_i = \epsilon_i + \sum \tilde{V}_0^{(i,j)} N_j$$



N=28 Best fit

Overall spectrum, ordering is well reproduced 31 state only 5 parameters
There are discrepancies, p-h symmetry, seniority



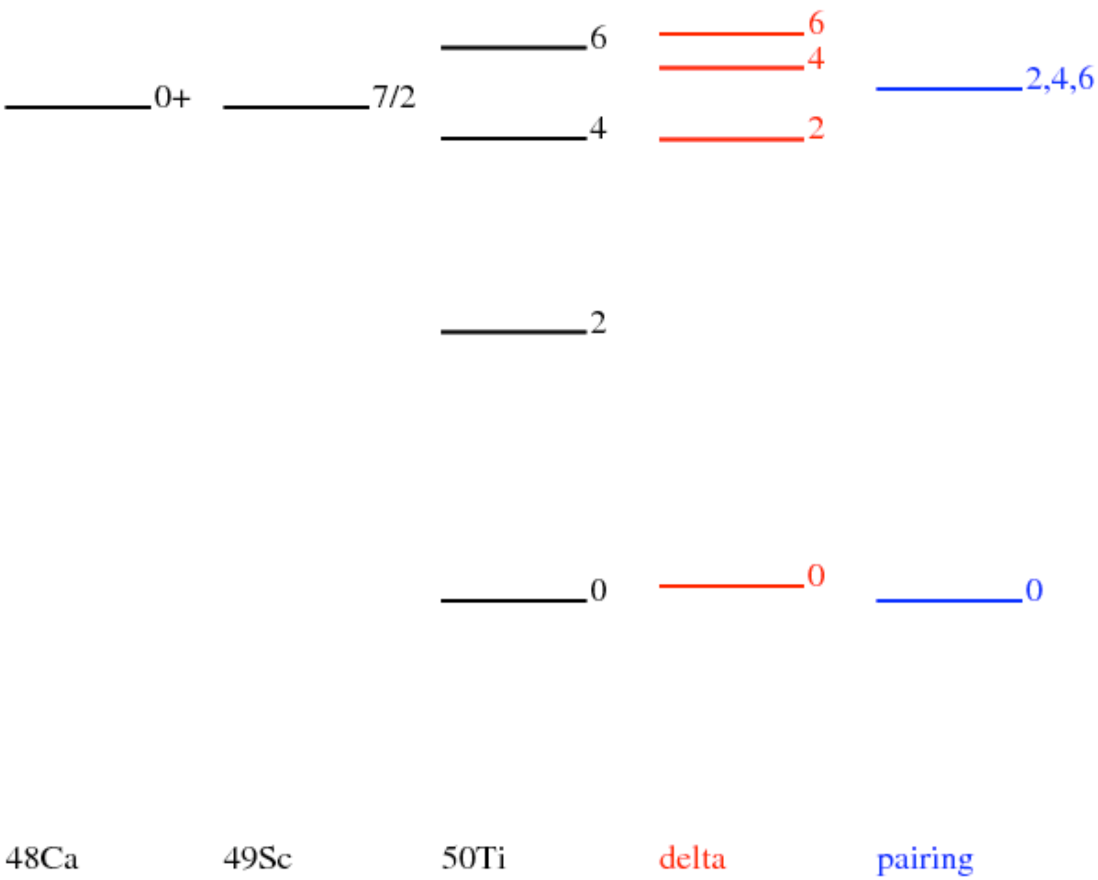
ϵ	$-9827(16)$
$V_0^{(2)}$	$-2033(60)$
$V_2^{(2)}$	$-587(39)$
$V_4^{(2)}$	$443(25)$
$V_6^{(2)}$	$887(20)$

Two-body interaction pairing and potential model

Phenomenological residual interaction

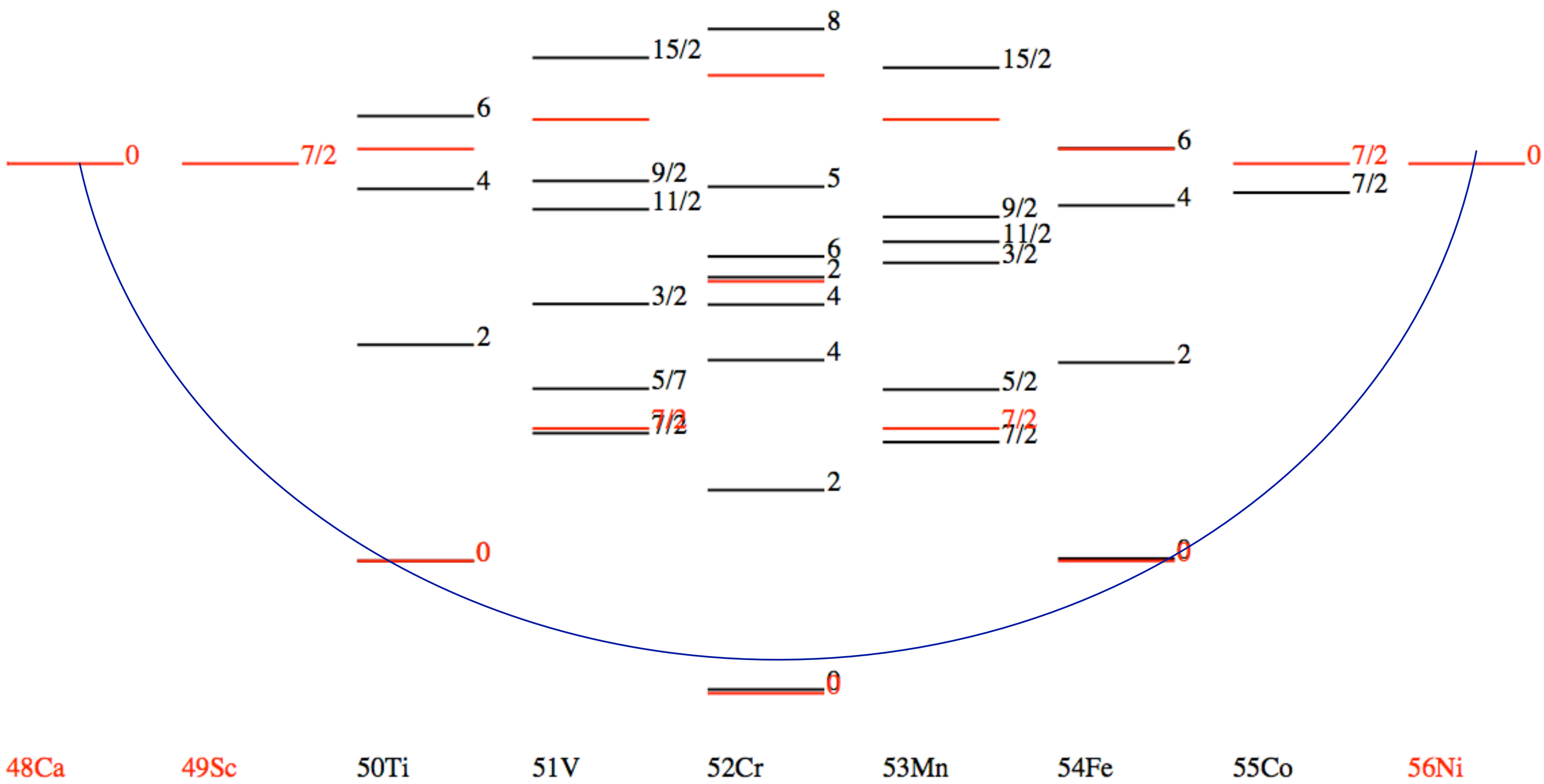
$$V(\mathbf{r}_1 - \mathbf{r}_2) \sim \delta^3(\mathbf{r}_1 - \mathbf{r}_2)$$

$$V_L \sim \frac{1}{2L+1} \left| C^{L\,0}_{j\,1/2, j\,-1/2} \right|^2$$

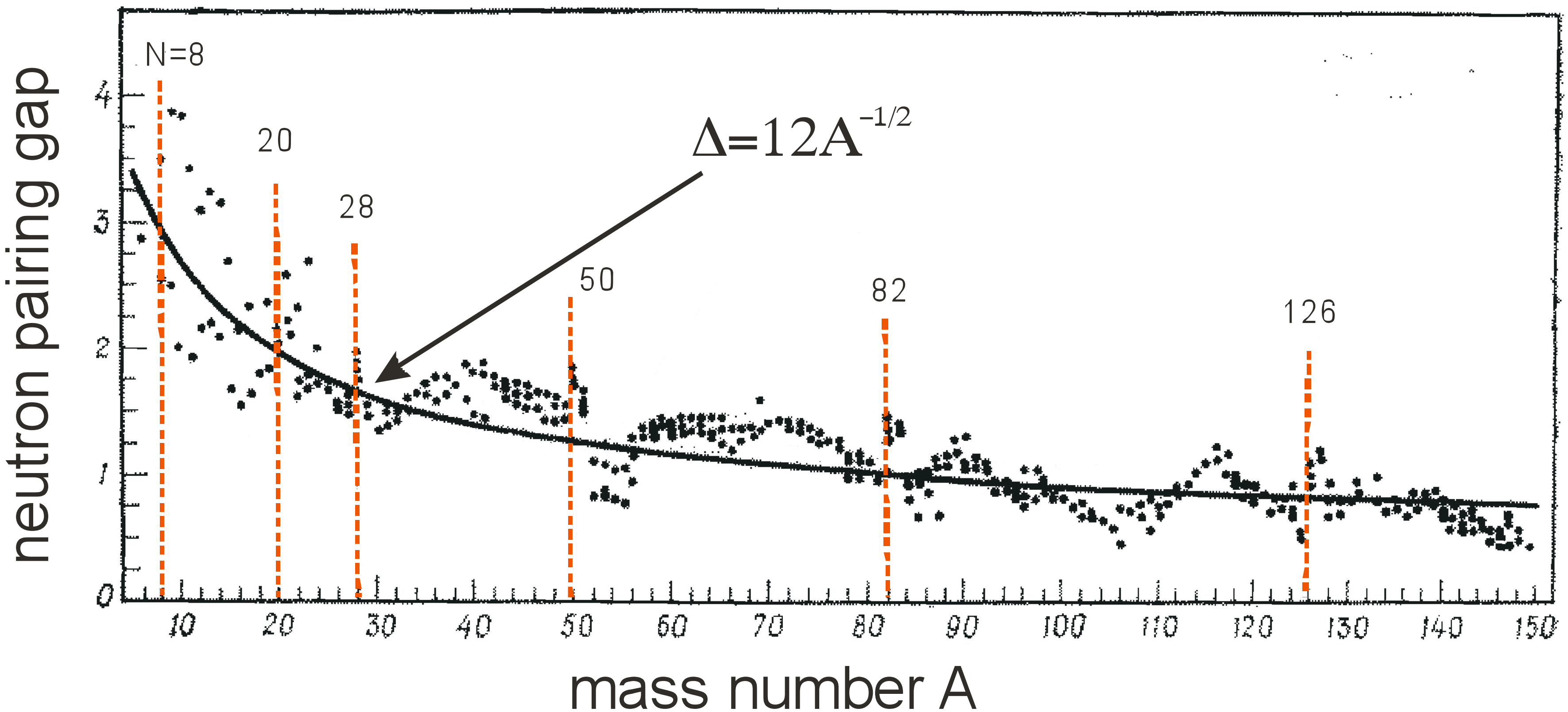


Short range interaction contributes most to pairing matrix element V_0

Pairing interaction in f7/2 shell nuclei



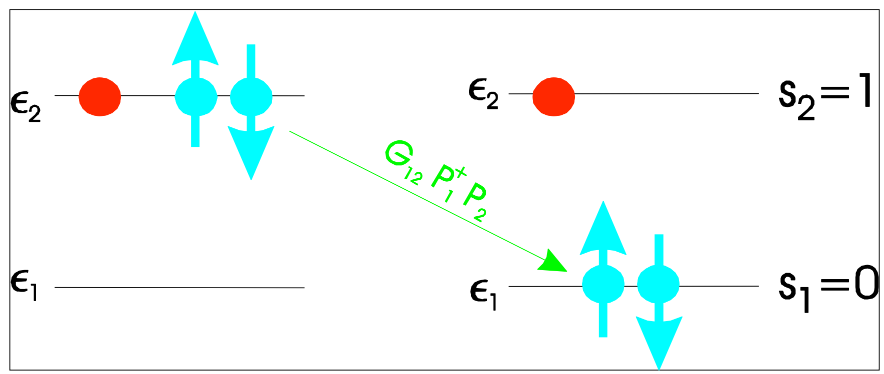
Pairing interaction in nuclei



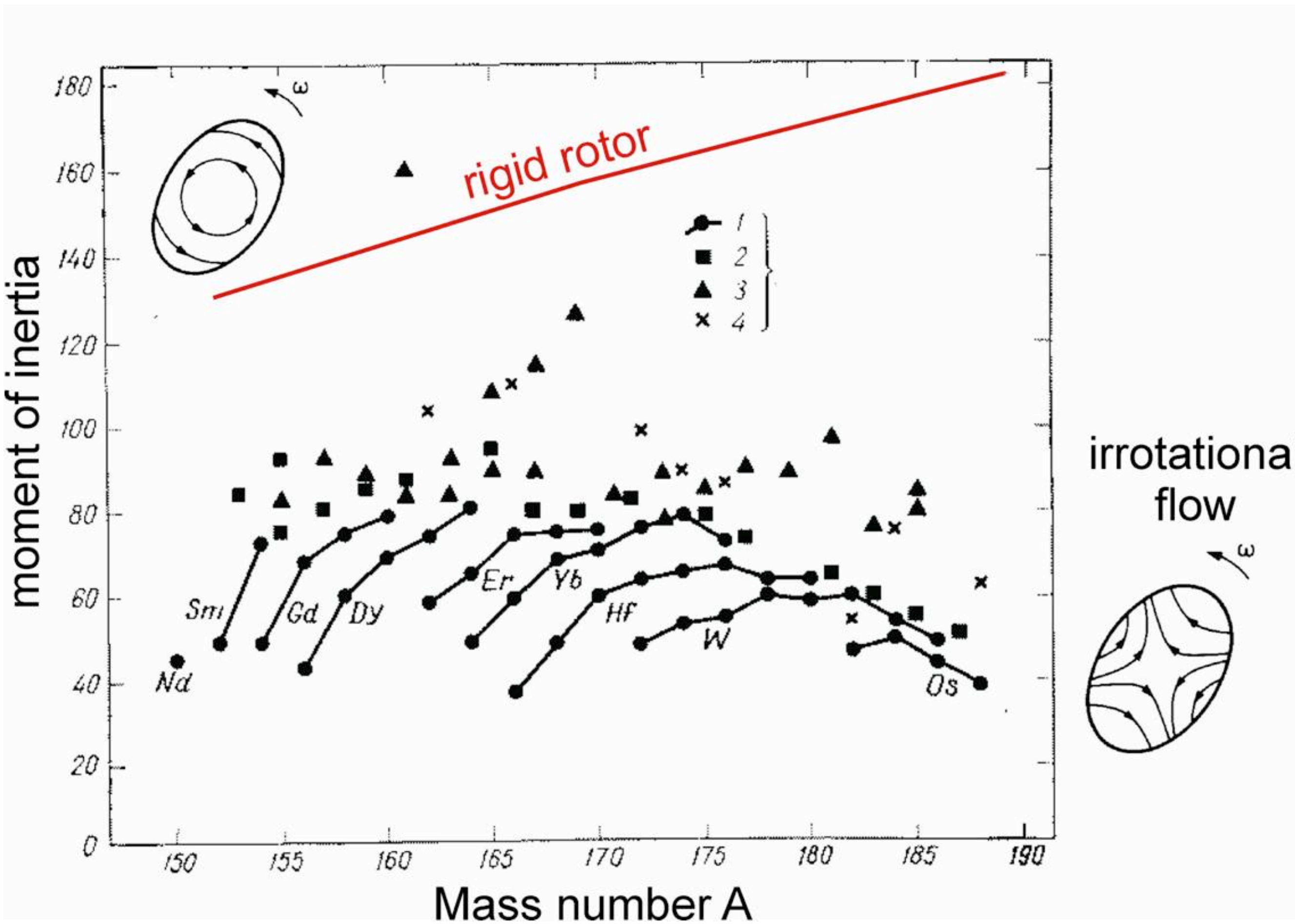
Pairing Hamiltonian

- Pairing on degenerate time-conjugate orbitals
  $|1\rangle \leftrightarrow |\tilde{1}\rangle \qquad |\widetilde{jm}\rangle = (-1)^{j-m}|j-m\rangle$
- Pair operators P =$(a_1 a_1)_{J=0}$ (J=0, T=1)
- Number of unpaired fermions is seniority v
- Unpaired fermions are untouched by H

$$H = \sum_1 \epsilon_1 N_1 - \sum_{12} G_{12} P_1^\dagger P_2$$



Evidence of nuclear superfluidity



Approaching the solution of pairing problem

- Approximate
  - BCS theory
    - HFB+correlations+RPA
  - Iterative techniques
- Exact solution
  - Richardson solution
  - Algebraic methods
  - Direct diagonalization + quasispin symmetry[1]

[1] A. Volya, B. A. Brown, and V. Zelevinsky, Phys. Lett. B 509, 37 (2001).

Quasispin and exact solution of pairing problem

The pair operators from SU(2) algebra "quasispin"

Algebra of pair operators

$$[P,\, P^\dagger] = 1 - \frac{2N}{\Omega}$$

$$[N,\, P^\dagger] = 2P^\dagger$$

Algebra of spin operators

$$[\mathcal{L}^+,\, \mathcal{L}^-] = -2\mathcal{L}_z$$

$$[\mathcal{L}_z,\, \mathcal{L}^+] = \mathcal{L}_z$$

$$\mathcal{L}^+ = \sqrt{\frac{\Omega}{2}}P^\dagger \qquad \mathcal{L}^- = \sqrt{\frac{\Omega}{2}}P$$

$$\mathcal{L}_z = \frac{N}{2} - \frac{\Omega}{4}$$

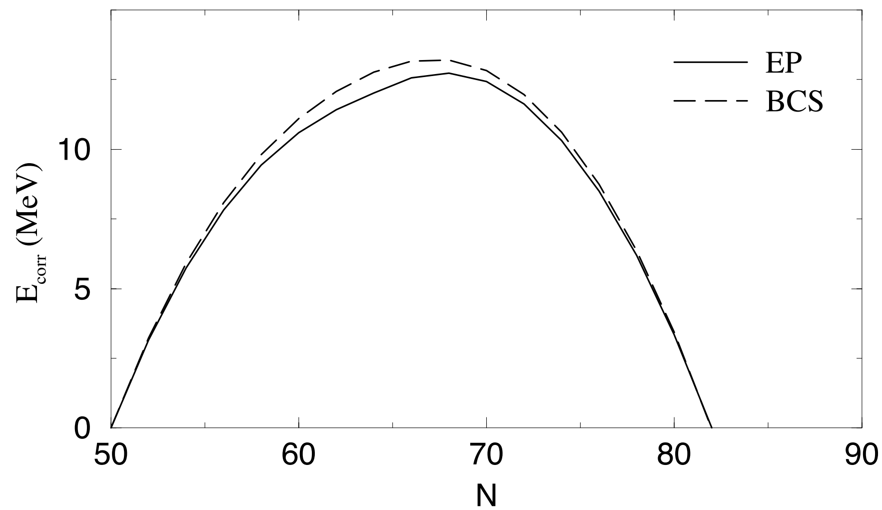
Magnitude of the quasispin, define seniority

$$\mathcal{L} = \frac{\Omega}{4} - \frac{\nu}{2}$$

Hamiltonian

$$H = \epsilon N + V_0 P^\dagger P = \epsilon N + V_0 \frac{2}{\Omega}\mathcal{L}^+\mathcal{L}^-$$

$$E(N,\nu) = \epsilon N + V_0 \frac{N-\nu}{2\Omega}(\Omega - N - \nu + 2)$$



Quasispin and single-particle operators

Single-particle operators are quasispin 1/2

$a^\dagger,\ a \quad \mathcal{L}=1/2, \mathcal{L}_z=+1/2,\ -1/2$

Example: Decay and spectroscopic factors

Initial state with N even $\quad \nu=0,\ N\rightarrow \mathcal{L}=\frac{\Omega}{4}\,\mathcal{L}_z=\frac{N}{2}-\frac{\Omega}{4}$

Final odd-N state $\quad \nu=1,\ N-1\rightarrow \mathcal{L}'=\frac{\Omega}{4}-\frac{1}{2}\,\mathcal{L}'_z=\frac{N}{2}-\frac{1}{2}-\frac{\Omega}{4}$

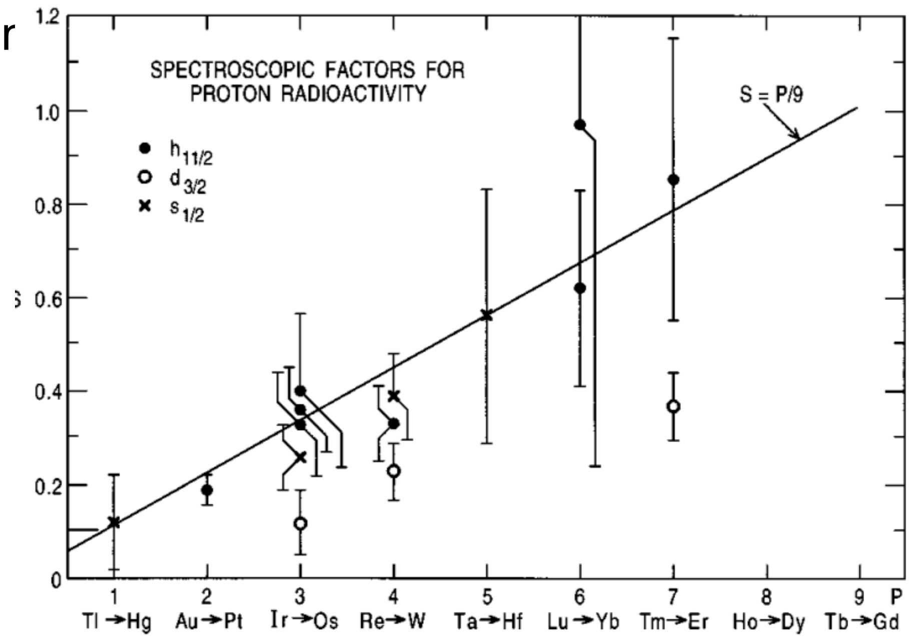
Decay width of a many-particle state $\quad \Gamma=S\gamma$

N-dependence and Wigner-Eckard theorem

Spectroscopic factor

$$S\sim|\langle I'|a|I\rangle|^2\sim\left|C^{\mathcal{L}-1/2,\mathcal{L}_z-1/2}_{1/2\,-1/2,\,\mathcal{L}\,\mathcal{L}_z}\right|^2\sim N$$

Chances to decay are proportional to the number of particles



From. Phys. Rev. C 55, 2255–2266 (1997)

Quasispin and two-particle operators

Even multopoles (quadrupole momentL=2) are quasivectors

$$P^{\dagger}_{LM} \sim \{a^{\dagger}a^{\dagger}\}_{LM}, \ \mathcal{M}_{LM} \sim (a^{\dagger}a)_{LM}, \ P_{LM} \sim (aa)_{LM}$$

$$\mathcal{L} = 1, \ \mathcal{L}_z = -1, 0, 1, \quad L = 0, \ 2, \ldots$$

Odd-multipoles (magnetic moment L=1

$$\mathcal{M}_{LM} \sim (a^{\dagger}a)_{LM}$$

$$\mathcal{L} = 0, \ \mathcal{L}_z = 0, \quad L = 1, \ 3, \ldots$$

Example:
Reduced electromagnetic decay rates

$$B(E2) \sim \left| C^{\mathcal{L},\mathcal{L}_z}_{1\,0,\mathcal{L}-1\,\mathcal{L}_z} \right|^2 \sim N(\Omega - N)$$

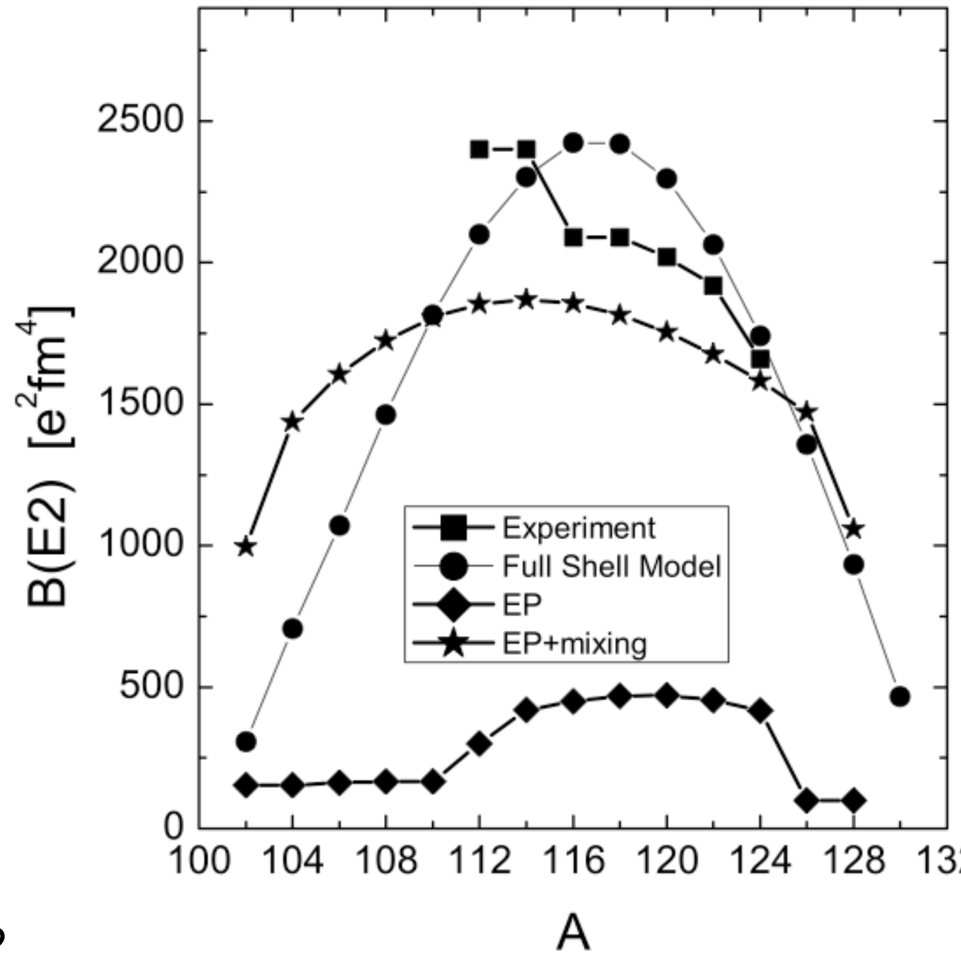


Figure. Models and data for
B(E2) rates across shell A=100-132

Quasispin and exact solution of pairing problem

For many levels each level is associated with a spin

- Operators $P^\dagger_j$, P_j and N_j form a SU(2) group
  $P^\dagger_j \sim L^+_j$, $P_j \sim L_j$, and $N_j \sim L_j^z$
- Quasispin L^2_j is a constant of motion,
  seniority $s_j=(2j+1) -2L_j$
- States can be classified with set
  $(L_j\ L_j^z)$, (s_j, N_j)
- Each s_j is conserved but N_j is not
- Extra conserved quantity simplifies solution.
  Example: ^{116}Sn: 601,080,390 m-scheme states
  272,828 J=0 states
  110 s=0 states
  Linear algebra with sparse matrices is fast. Deformed basis Nmax~50-60
- Generalization to isovector pairing, R_5 group

BCS theory

Trial wave-function

$$|0) = \prod_{\nu} \left(u_\nu - v_\nu a^\dagger_\nu a^\dagger_{\tilde{\nu}} \right) |0\rangle \,, \quad \text{where} \quad \underbrace{|u_\nu|^2}_{\text{empty}} + \underbrace{|v_\nu|^2}_{\text{occupied}} = 1$$

Minimization of energy determines

$$|v_\nu|^2 = \frac{1}{2}\left(1 - \frac{\epsilon_\nu - \mu}{e_\nu}\right) \,, \quad |u_\nu|^2 = \frac{1}{2}\left(1 + \frac{\epsilon_\nu - \mu}{e_\nu}\right)$$

Gap equation

$$\Delta_\nu = \frac{1}{4} \sum_{\nu'} G_{\nu\,\nu'} \frac{\Delta_{\nu'}}{e_{\nu'}} \,, \quad \text{where} \quad e_\nu = \sqrt{(\epsilon_\nu - \mu)^2 + \Delta_\nu^2}$$

Shortcomings of BCS

- Particle number non-conservation

$$|\mathrm{BCS}\rangle = \prod_{\nu(\mathrm{doublets})} \left\{ u_\nu - v_\nu P_\nu^\dagger \right\} |0\rangle$$

- Phase transition and weak pairing problem

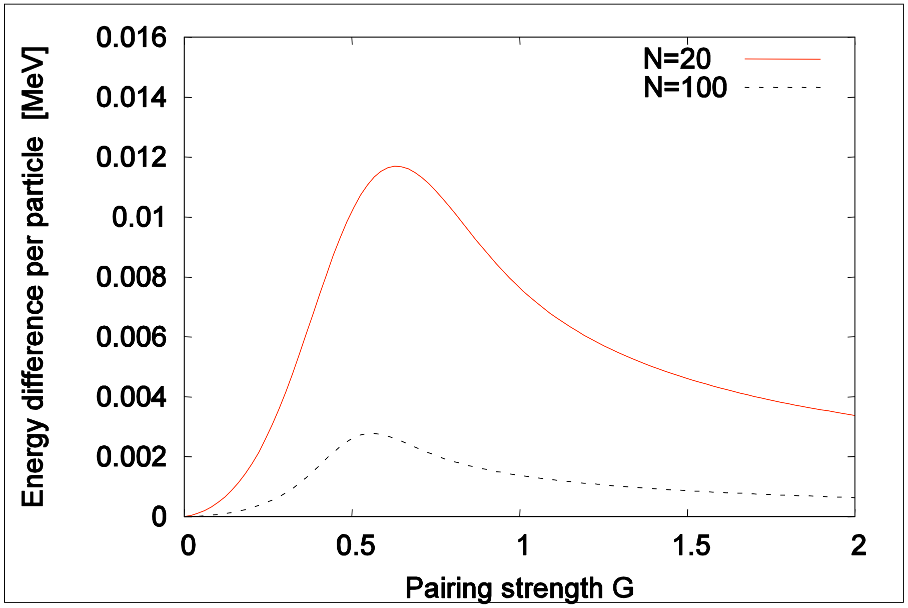
Example $\quad G = G_{\nu\nu'}\,, \quad \text{gap eq. } 1 = G \sum_\nu \frac{1}{2E_\nu}$

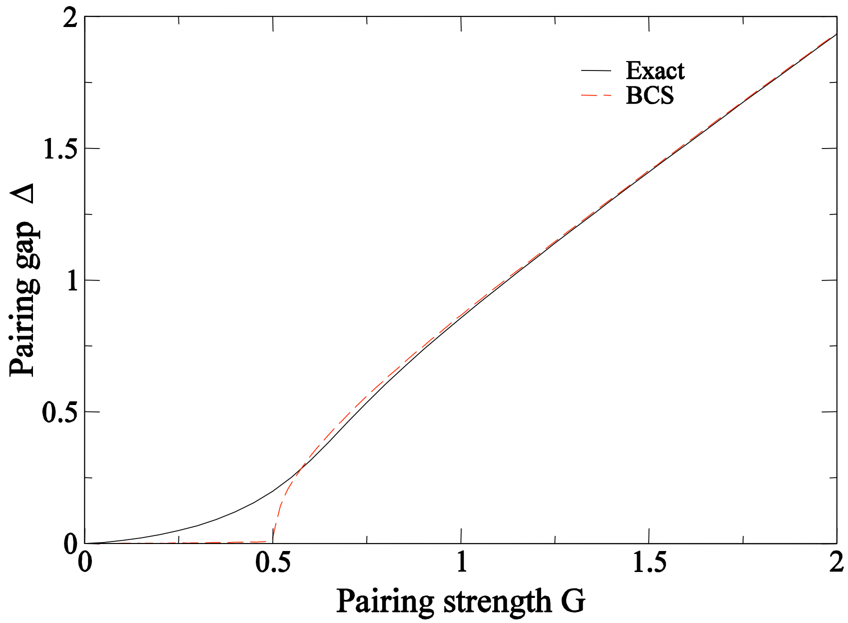
$$G < G_c \quad \Delta = 0\,, \quad \text{where } 1 = G_c \sum_\nu \frac{1}{2\epsilon'_\nu}$$

- Excited states, pair vibrations

Cooper Instability in mesoscopic system

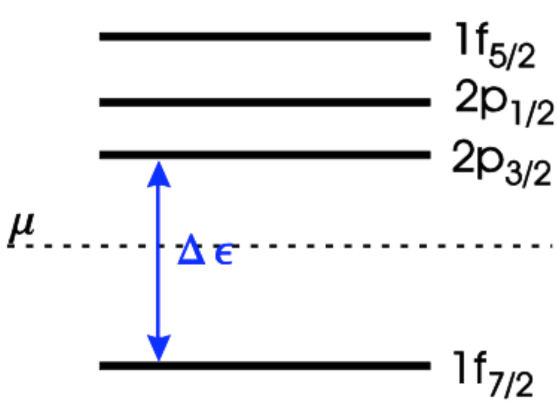
BCS versus Exact solution



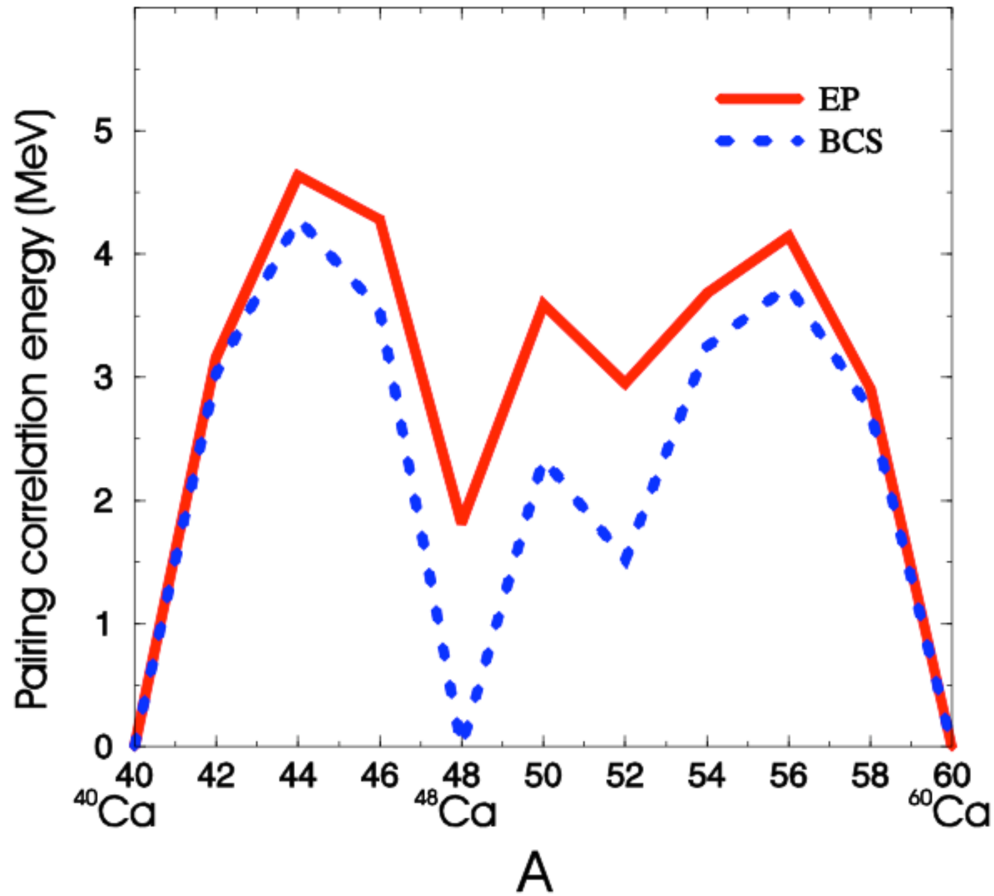


Pairing in Ca isotopes

BCS fails to describe pairing correlations in ^{48}Ca



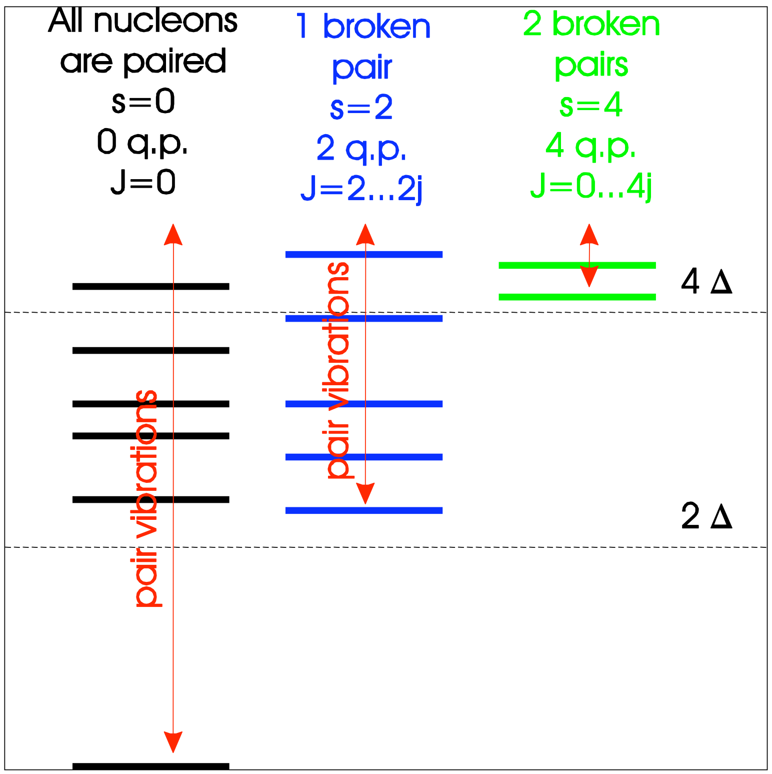
Pairing becomes weak if $G/\Delta\varepsilon \sim 1$ at Fermi surface



Low-lying states in paired systems

- Exact treatment
  - No phase transition and $G_{critical}$
  - Different seniorities do not mix
  - Diagonalize for pair vibrations
- BCS treatment

	$G<G_{critical}$	$G>G_{critical}$
Ground state	Hartree-Fock	BCS
Elementary excitations	single-particle excitations $E_{s=2}=2\ \varepsilon$	quasiparticle excitation $E_{s=2}=2\ e$
Collective excitations	HF+RPA	HFB+RPA



Multipole-multipole interaction

Hamiltonian operator can be written in multipole-multipole form

$$P^\dagger_{LM}P_{LM} \sim (a_1^\dagger a_2^\dagger)(a_3 a_4) \sim \delta_{23} a_1^\dagger a_4 - \underbrace{(a_1^\dagger a_3)(a_2^\dagger a_4)}_{\mathcal{M}^\dagger_{K\kappa}\mathcal{M}_{K\kappa}}$$

$$H = \epsilon N + \sum_{L=0,2,4,6} V_L \sum_{M=-L}^{L} P^\dagger_{LM}P_{LM} \qquad H = \epsilon' N + \sum_K \tilde{V}_K \sum_\kappa \mathcal{M}^\dagger_{K\kappa}\mathcal{M}_{K\kappa}$$

Consider lowest terms K=0,1,2…

$\mathcal{M}_{00} \sim N$ Monopole $\tilde{V}_0$

$\mathcal{M}_{1\mu} \sim J_\mu$ (angular momentum) Moment of inertia $\tilde{V}_1$

$\mathcal{M}_{2\mu}$ Quadrupole-quadrupole interaction, lowest non-trivial term

$$H_{QQ} = \sum_\mu \mathcal{M}^\dagger_{2\mu}\mathcal{M}_{2\mu}$$

QQ-Interaction
- Creates deformation
- Leads to rotational featurs

5 operators $\mathcal{M}_{2\mu}$ 3 operators J_μ can be considered forming SU(3)

For a Harmonic oscillator shell the algebra is exact

Elliot's mode

SU(3) group g.s. representation

Consider a Cartesian distribution of particles in 3D HO $(n_x n_y n_z)$

Configurations (representations) are labeled $\lambda = n_z - n_x \; \mu = n_x - n_y$

Energy of the QQ hamiltonian

$$E_{SU(3)} = 4[\lambda^2 + \mu^2 + \lambda\mu + 3(\lambda + \mu)] + 3L(L+1)$$

$\bar{\lambda} = \max(\lambda, \mu)$ $\bar{\mu} = \min(\lambda, \mu)$

$K' \geq 0, K' = \bar{\mu}, \bar{\mu} - 2 \ldots,$

$$L = \begin{cases} K', K'+1 \ldots K' + \bar{\lambda} & \text{for } K' > 0 \\ \bar{\lambda}, \overline{\lambda - 2} \ldots & \text{for } K' = 0, \end{cases}$$

SU(3) spectrum and transitions are very close to rotational

(0,0,2)	**(1,0,1)**	(2,0,1)
	(0,1,1)	(1,1,0)
		(0,2,0)

Example ^{24}Mg, 4protons 4 neutrons

For each nucleon **(0,0,2)+(1,0,1)=(1,0,3)**

Total number of quanta (4,0,12) λ=8, μ=4

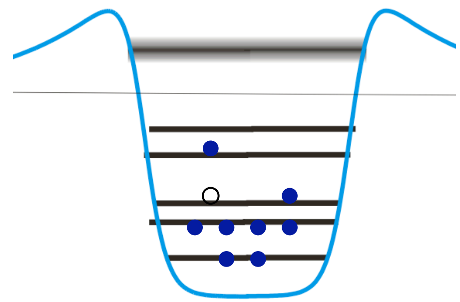
Three mixed rotational bands K=4,2,0, for
K=0 L=0,2,…8
K=2, L=2,3,…10
K=4, L=4,5,…12 (terminating spin for sd space)

Homework

1. Conduct a shell model study of ^{24}Mg using “sd”-valence space and “usd” interaction
2. Calculate the B(E2) transition rate between ground state and first excited 2^+ state. Calculate the lab quadrupole moment of the 2^+ state. You can use harmonic oscillator wave functions and oscillator length units.
3. Compare your results with the rotor prediction (see Alaga rules, lecture 1)

$$\frac{Q^2(2_1^+)}{B(E2, 0^+ \rightarrow 2^+)} = \frac{16\pi}{5}\left(\frac{2}{7}\right)^2 \approx 0.82$$

Giant resonances

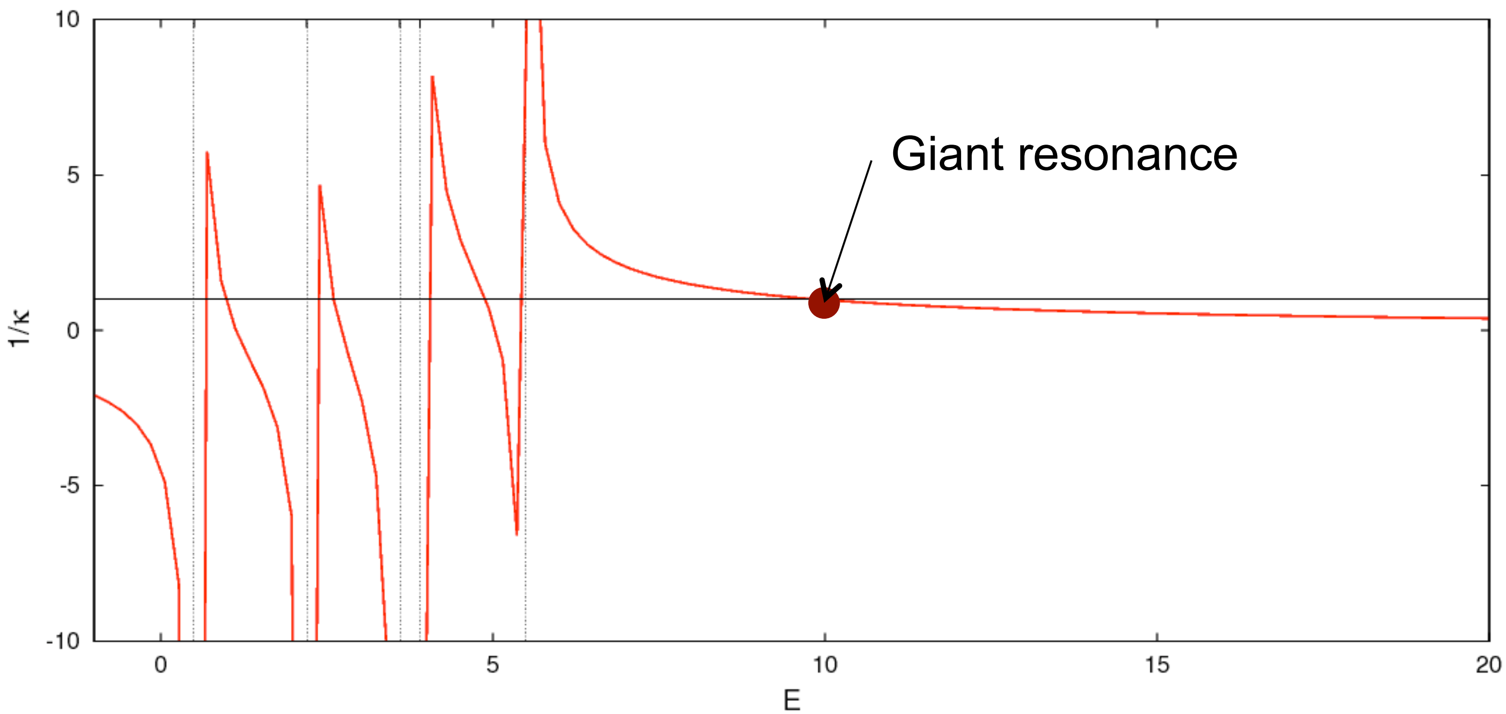


Consider interacting particle-hole excitations

$$\varepsilon_{k=\{1,2\}} = \epsilon_1 - \epsilon_2 \qquad H_{kk'} = \delta_{kk'}\varepsilon_k + \kappa\, d_k d_{k'}$$

$$\sum_{k'} H_{kk'}\, x_{k'} = E\, x_k \qquad D \equiv \sum_{k'} d_{k'} x_{k'}$$

$$x_k = \kappa \frac{D\, d_k}{E - \varepsilon_k} \qquad \varepsilon_k x_k + \kappa d_k \sum_{k'} d_{k'} x_{k'} = E\, x_k$$



Strength function

$$F_d(E) = \sum_\alpha |\langle d|x^\alpha\rangle|^2\, \delta(E - E^\alpha)$$

Dipole collectivity

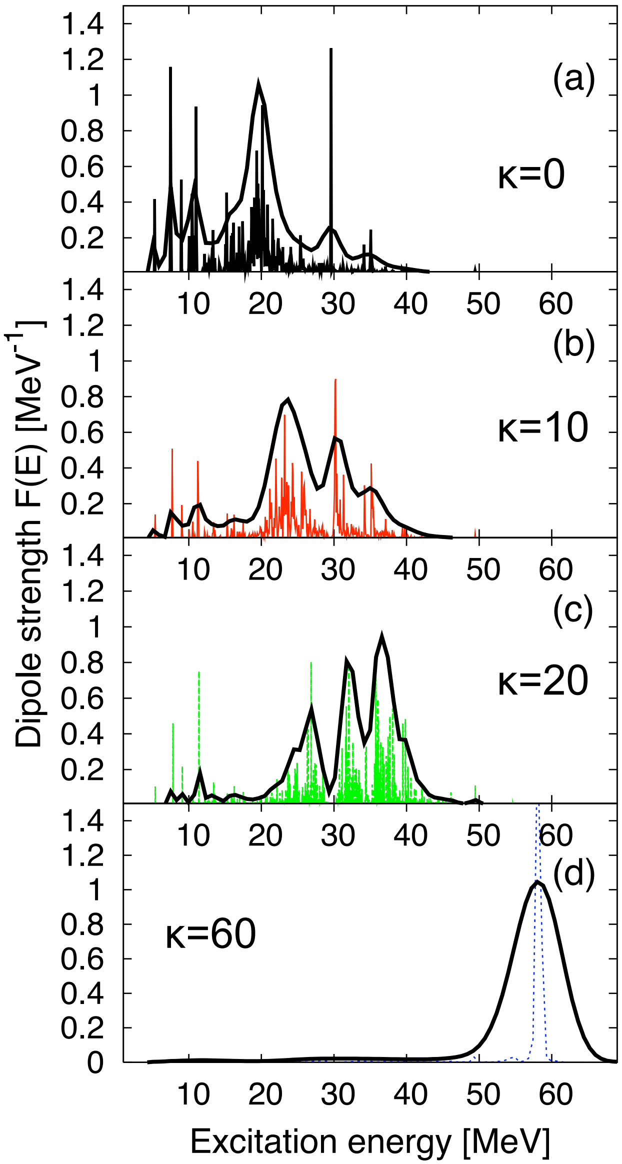


Figure: Strength function of the isovector dipole operator in ^{22}O. WBP SM Hamiltonian plus interaction term:

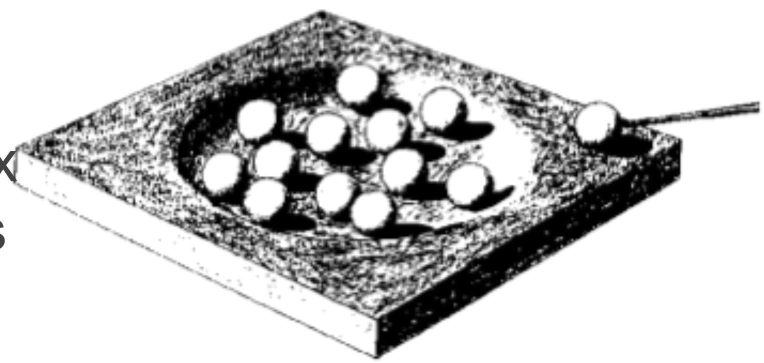
$$V = \kappa|D\rangle\langle D|$$

$$|D\rangle = D|0^+_{\mathrm{g.s.}}\rangle$$

$$\kappa = 10,\ 20,\ \mathrm{and}\ 60$$

Statistical approach, quantum chaos

- Nuclei are strongly interacting many-body systems.
- Many-body quantum systems have complex dynamics, similarly to the classical systems such as gases. Statistical approach, quantum chaos.

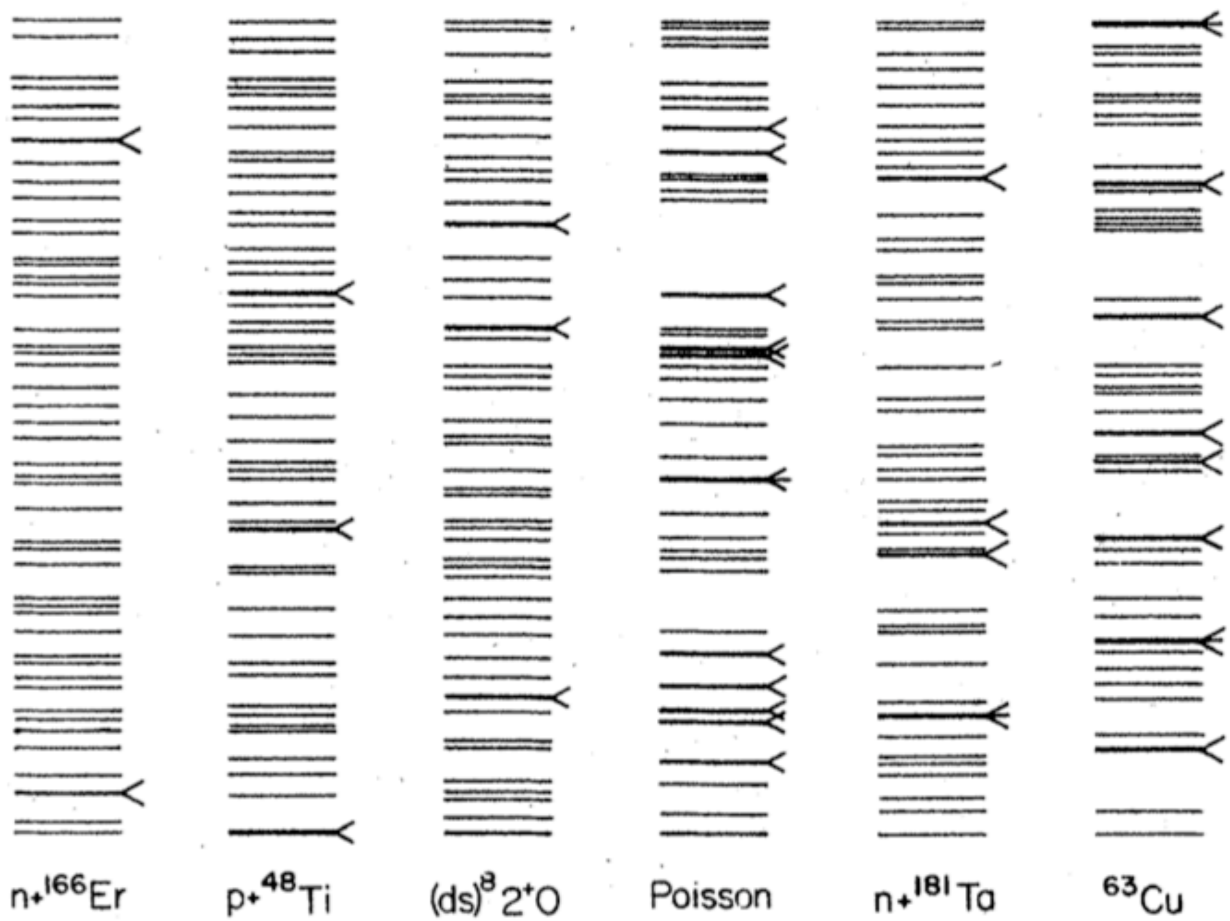


From N. Bohr, Nature 137, 344 (1936)

Why is this interesting?

- Detect missing states.
- Test of fundamental symmetries and their violations
- Help with exact solutions to the many-body problem

Level spacing distribution



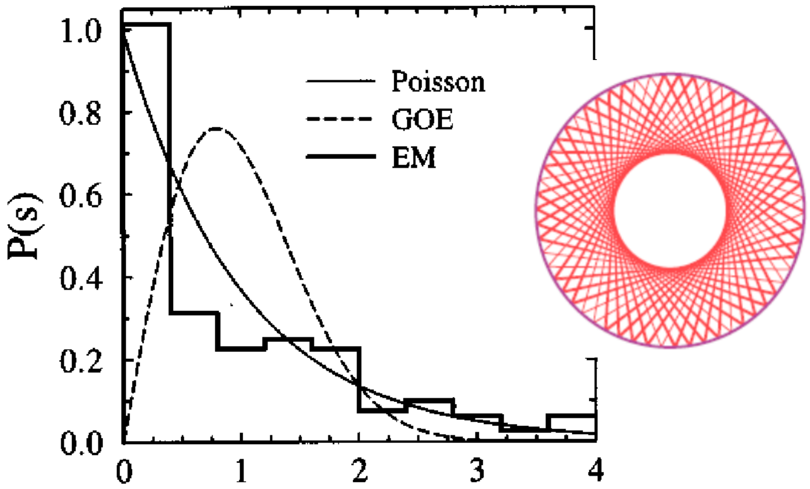
From Brody, Rev. Mod. Phys. 53 p 385

Arrows show
closely located states

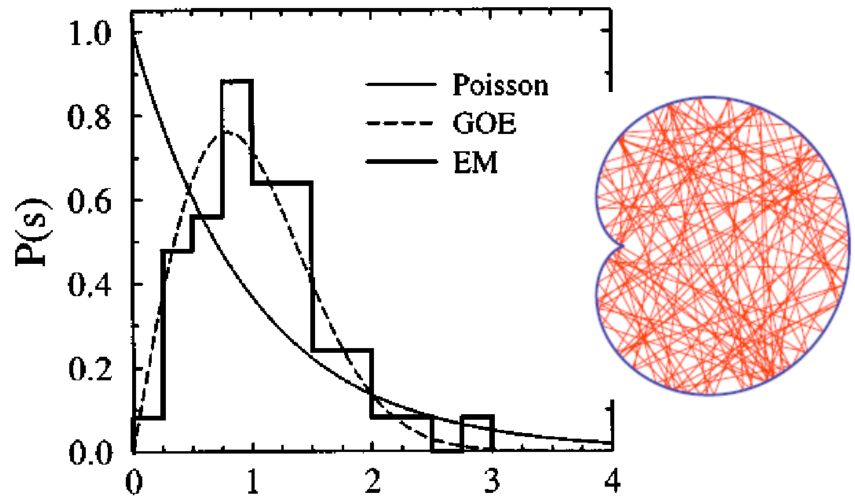
Quantum chaos

Distribution of energy spacing between neighboring states

Circular billiard



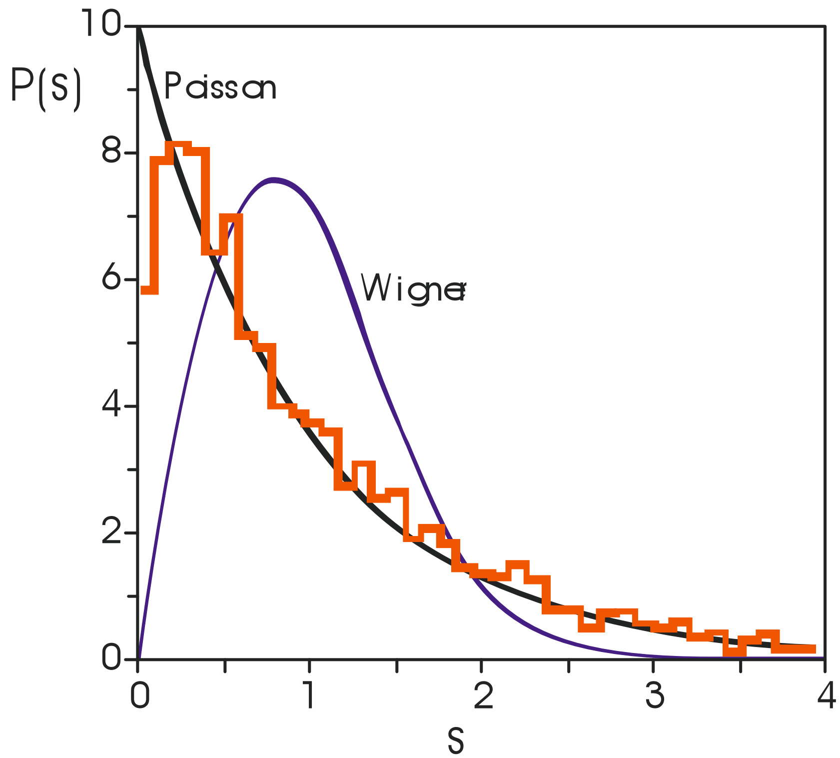
Irregular billiard



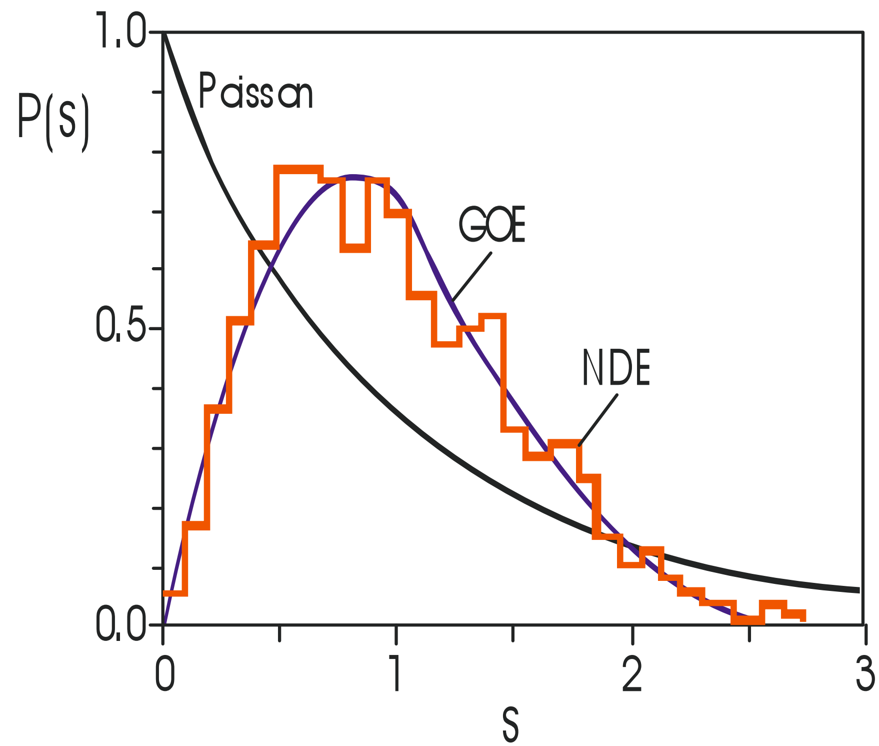
- Regular motion
  - Analog to integrable systems
  - No level repulsion
  - Poisson distribution $P(s)=\exp(-s)$
- Chaotic motion
  - Classically chaotic
  - Level repulsion
  - GOE (Random Matrix)

    $P(s)=s \exp(-p\, s^2/4)$

Chaotic motion in nuclei



"Cold" (low excitation)
rare-earth nuclei



High-Energy region,
Nuclear Data Ensemble
Slow neutron resonant date

Haq. et.al. PRL 48, 1086 (1982)

Many-body complexity and distribution of spectroscopic factors

$|c\rangle$ Channel-vector (normalized)

$|I\rangle$ Eigenstate

Reduced width (spectroscopic factor)

$$\gamma_I^c = |\langle I|c\rangle|^2$$

What is the distribution of the reduced width?

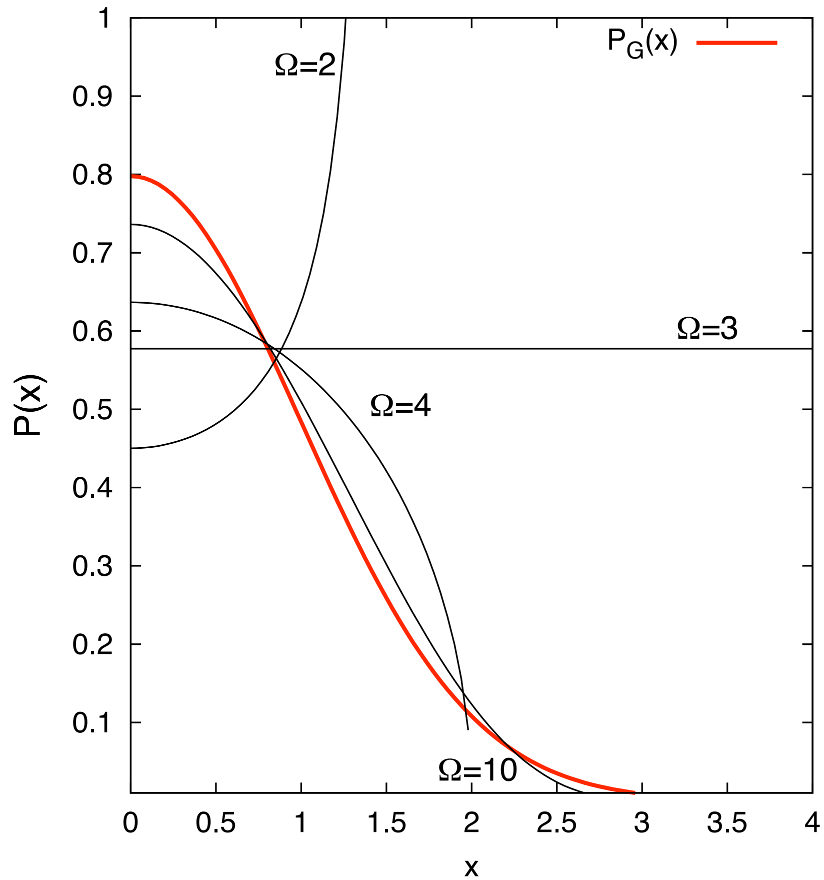
Average width $$\overline{\gamma} = \frac{1}{\Omega}\sum_I \gamma_I^c = \frac{\langle c|c\rangle}{\Omega}$$

Amplitude $$x_I = \sqrt{\gamma_I/\overline{\gamma}}$$

If any direction in the Ω-dimensional Hilbert space is equivalent

$$P(x_{I_1},\ldots x_{I_\Omega}) \sim \delta\left(\Omega - \sum_I x_I^2\right)$$

Why Porter-Thomas Distribution?



Projection of a randomly oriented vector in Ω-dimensional space

$$P(x) = \frac{V_{\Omega-1}}{\sqrt{\Omega} V_\Omega} \left(1 - x^2/\Omega\right)^{(\Omega-3)/2}$$

$$V_\Omega = \frac{\Omega \pi^{\Omega/2}}{\Gamma(\Omega/2+1)}$$

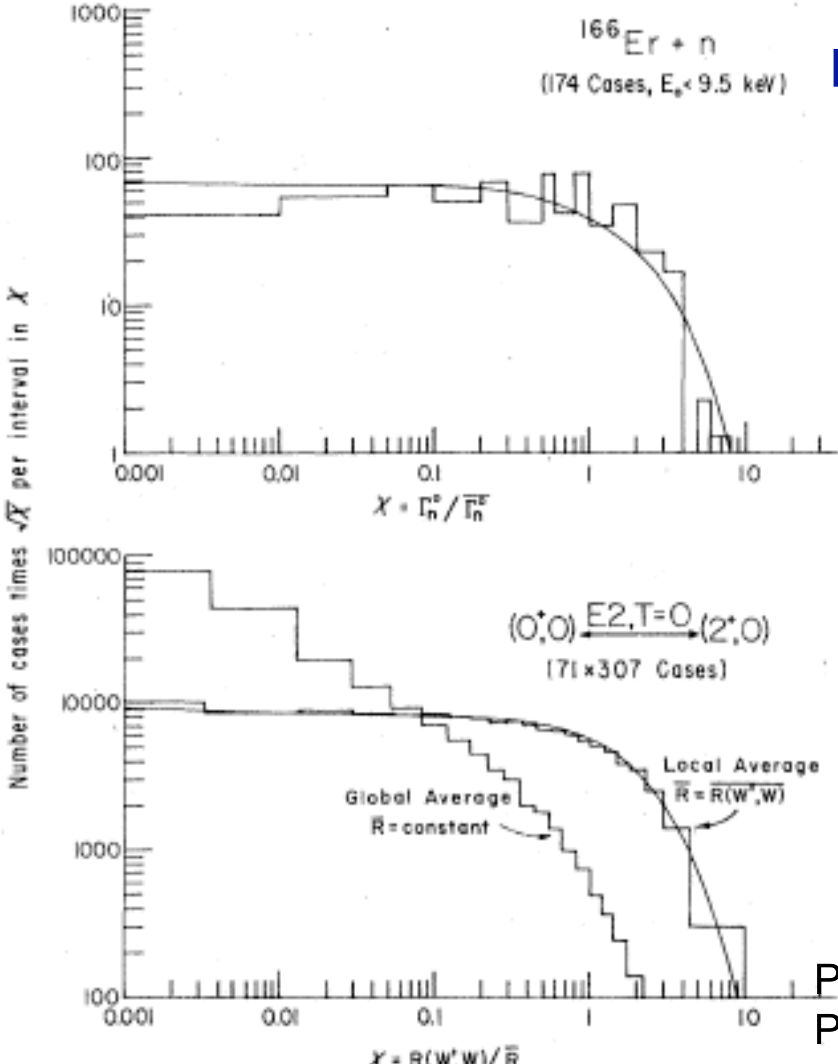
For large Ω this leads to Gaussian

$$P_G(x) = \sqrt{\frac{2}{\pi}} \exp\left(-x^2/2\right)$$

For large ν channels

$$P_\nu(\gamma) = \frac{1}{\gamma} \left(\frac{\nu\gamma}{2\overline{\gamma}}\right)^{\nu/2} \frac{1}{\Gamma(\nu/2)} \exp\left(-\frac{\nu\gamma}{2\overline{\gamma}}\right)$$

Observation of Porter-Thomas Distribution



Reduced neutron widths

E2 transitions in (sd)-shell model for 6 particles

Is PTD viloated?

P. E. Koehler, et.al Phys. Rev. Lett**. 105,** 072502 (2010)
P. E. Koehler, et.al, *Phys. Rev. C 76 (2007)***.**
J. F. Shriner, *Phys. Rev. C* ***32,*** *694 (1985)***.**
R. R. Whitehead, et.al*, Phys. Lett. B* ***76****, 149 (1978)***.**

Fig. from Brody, et.al.

Statistical treatment

- Microcanonical $\hat{\rho}(E,N) = \delta(E-\hat{H})\delta(N-\hat{N})$
- Canonical $\hat{\rho}(\beta,N) = \exp(-\beta\hat{H})\,\delta(N-\hat{N})$
- Grand canonical $\hat{\rho}(\beta,\mu) = \exp\left(-\beta(\hat{H}-\mu\hat{N})\right)$

Partition functions

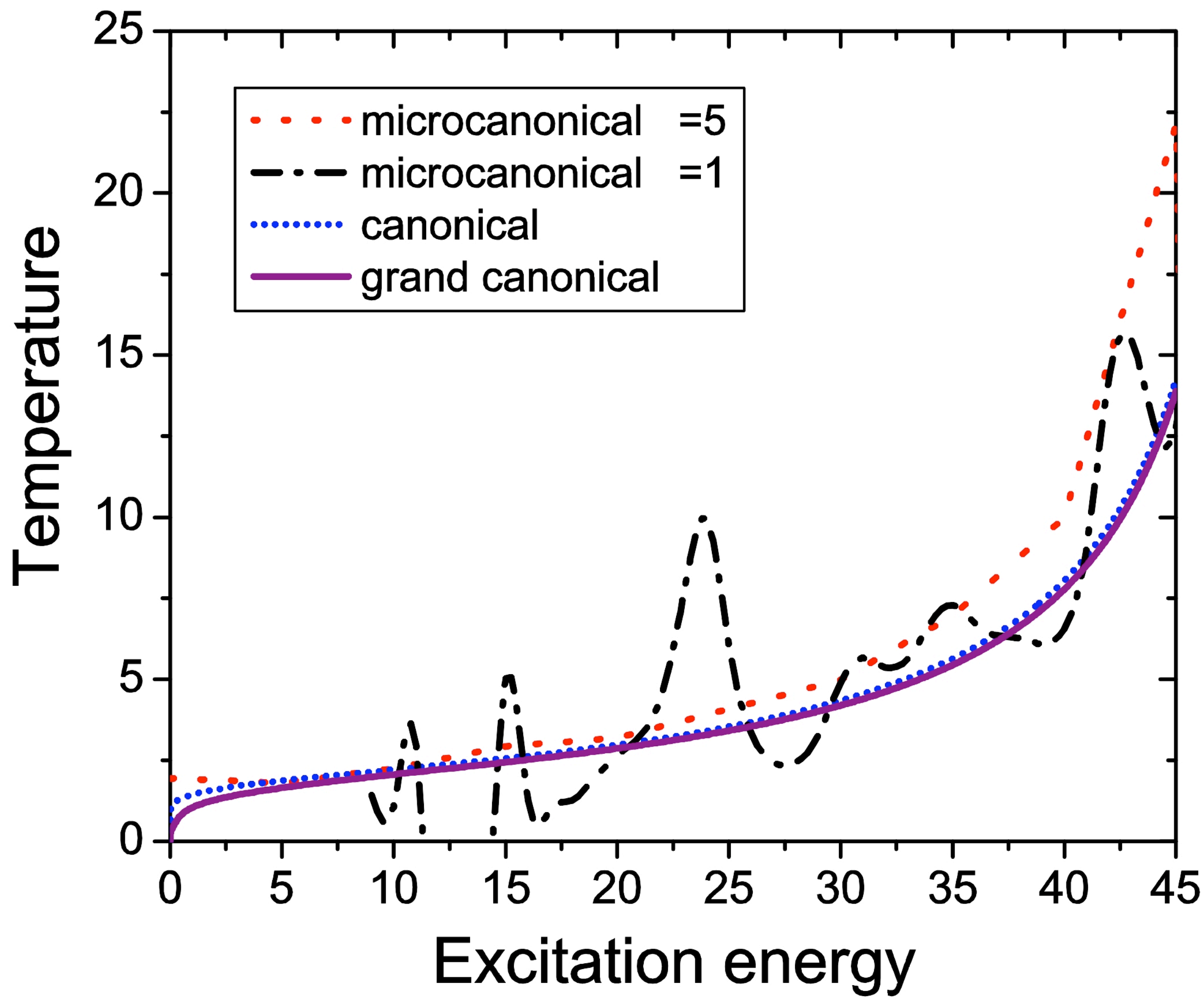
$$Z = \mathrm{Tr}(\hat{\rho}) \quad \text{and} \quad \hat{w} = \frac{\hat{\rho}}{Z}.$$

Statistical averages

$$\langle\hat{O}\rangle = \frac{\mathrm{Tr}(\hat{O}\hat{\rho})}{\mathrm{Tr}(\hat{\rho})} = \mathrm{Tr}(\hat{O}\hat{w})$$

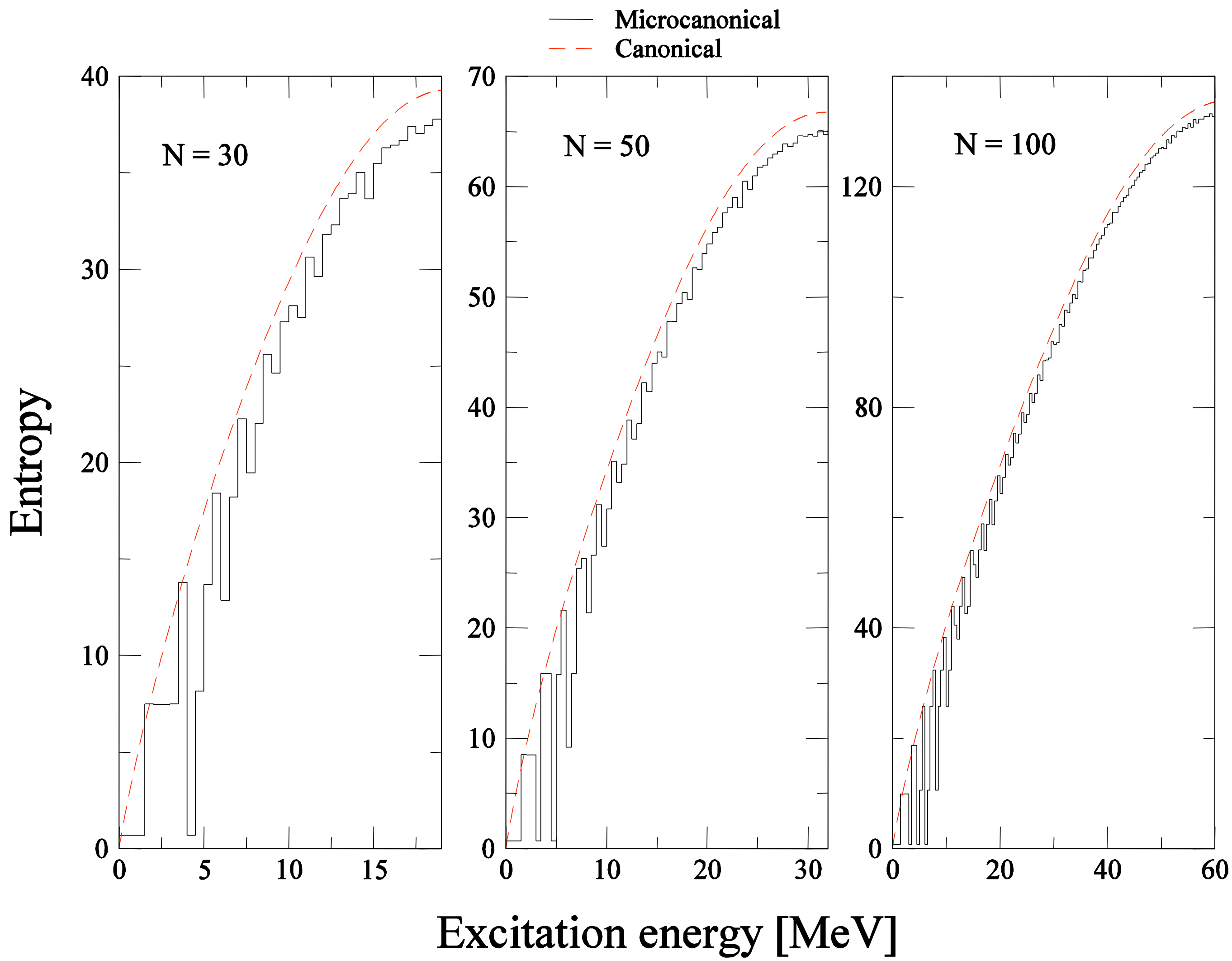
Entropy

$$S = -\langle\ln(\hat{w})\rangle = -\mathrm{Tr}(\hat{w}\,\ln\hat{w})$$



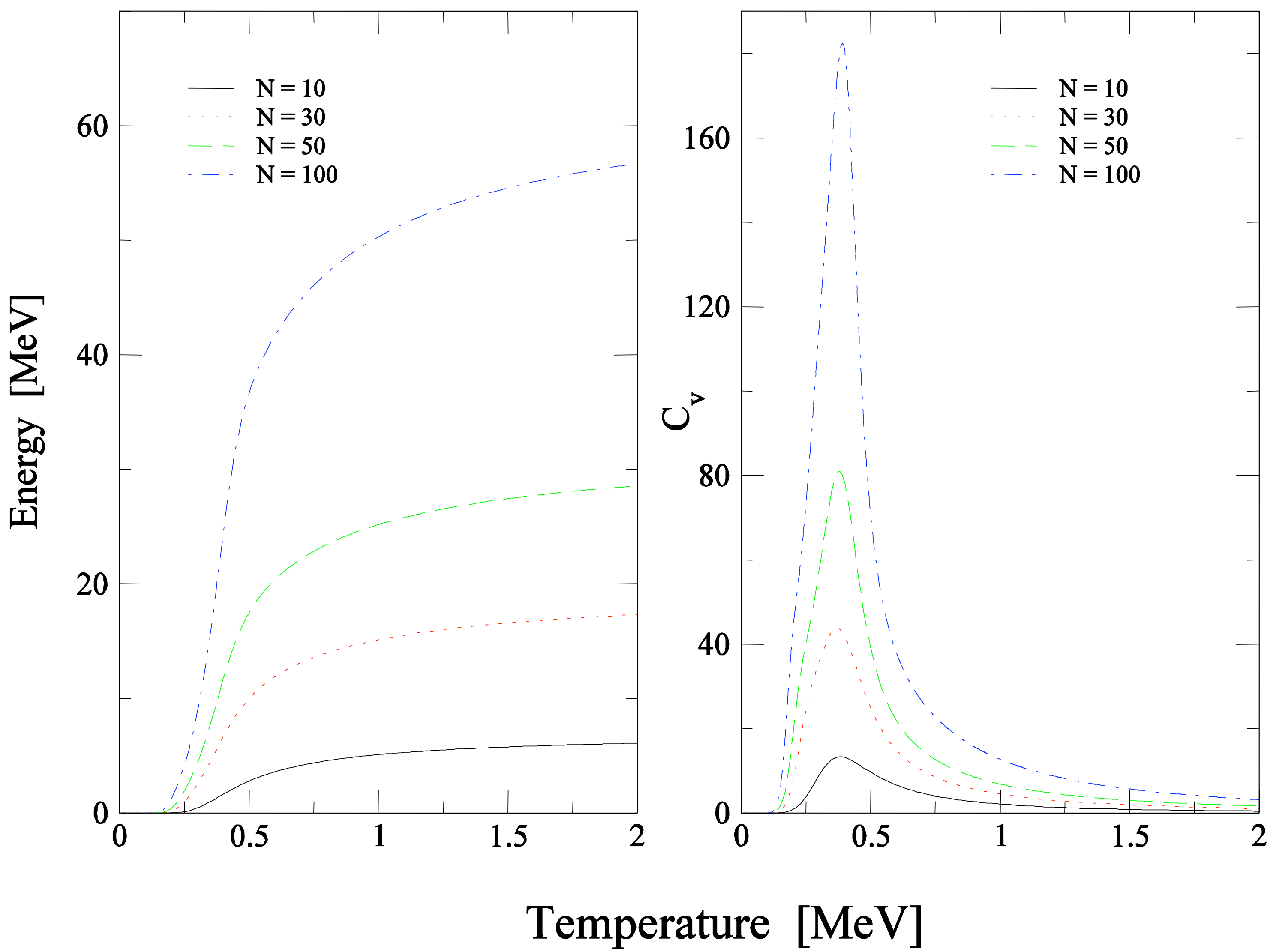
Temperature as a function of energy in three different statistical ensembles, for picket-fence model with 12 levels and 12 particles and V=1.00. For microcanonical we use σ =1 and 5

Microcanonical ensemble and thermodynamic limit



G=1

Pairing phase transition



Nuclear Structure Theory III

Open Systems

Alexander Volya

Florida State University

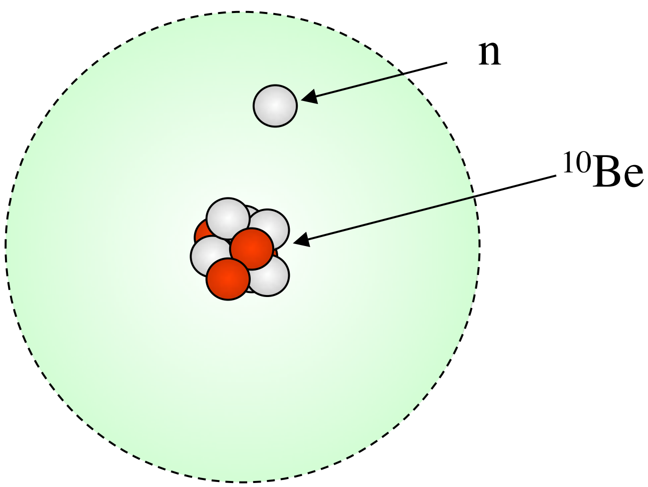
10th Exotic Beam Summer School - EBSS2011
East Lansing, Michigan. 25-30 July, 2011

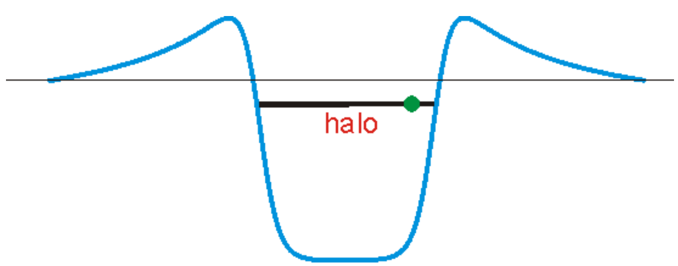
Halos and resonances

Halo phenomenon

-Weakly bound wave function is very big
-Rydberg atom
-^{11}Be nucleus

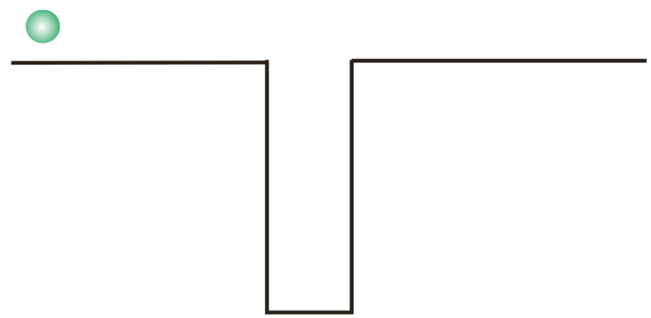
$$r = \frac{n^2}{Ze^2 m}$$

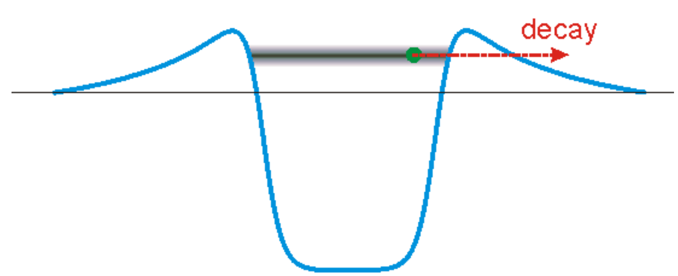




Resonance phenomenon

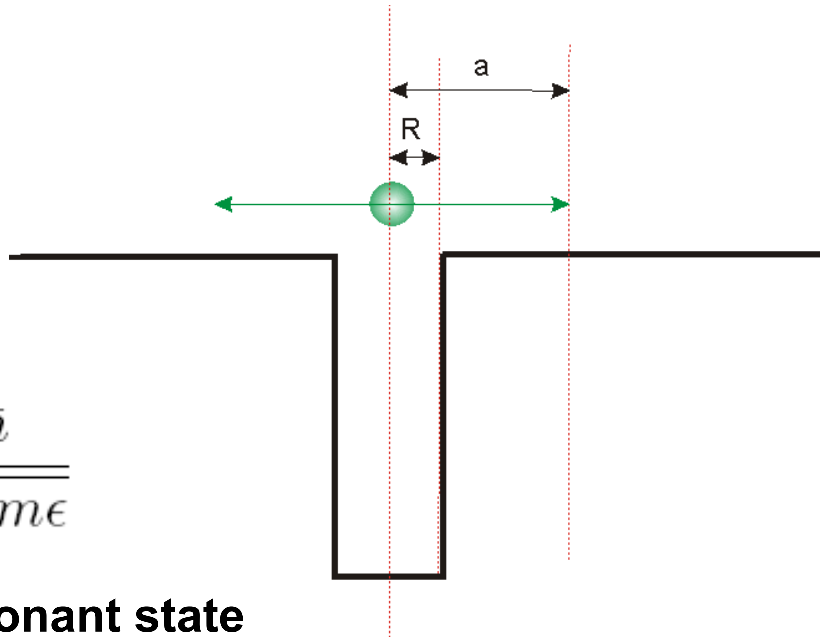
-Long-lived unbound state





Description of resonances and halo

R-interaction range
a-scattering length
$\sigma = \pi a^2$ cross section



a>0 and a>>R **bound Halo state**

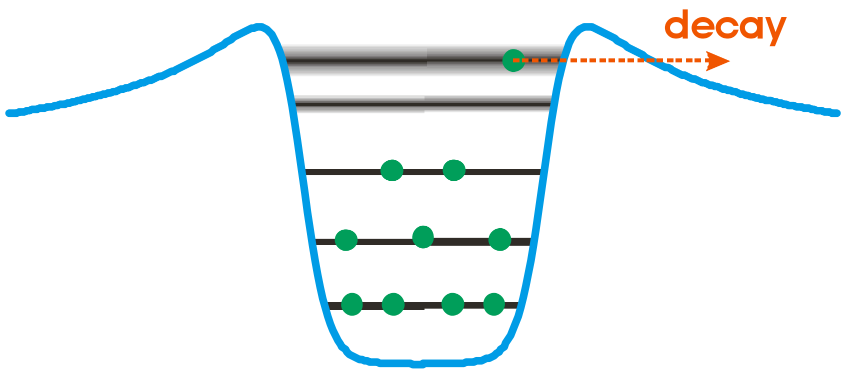
$$\psi(r) \sim e^{-kr} \quad k = \sqrt{\frac{2m\epsilon}{\hbar^2}} \qquad a = \frac{\hbar}{\sqrt{2m\epsilon}}$$

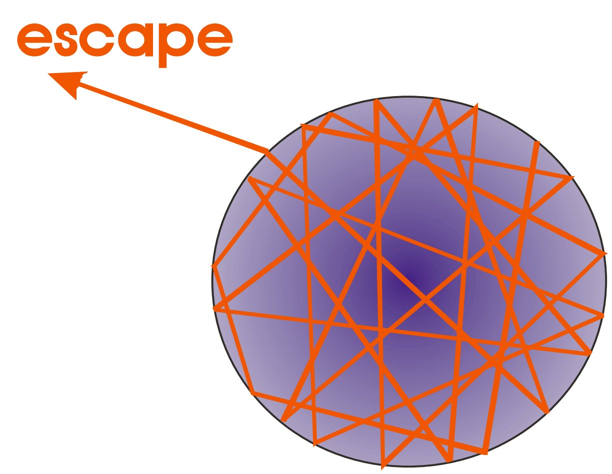
a<0 and |a|>>R **unbound long-lived resonant state**

$$\psi(r) \sim N(t)e^{-kr} \quad N(t) \sim e^{-i\gamma t/2}$$

Complex energy $\quad \epsilon \longrightarrow \epsilon - i\frac{\gamma}{2}$

Nuclear reaction theory
Quantum billiards with particle-leaks





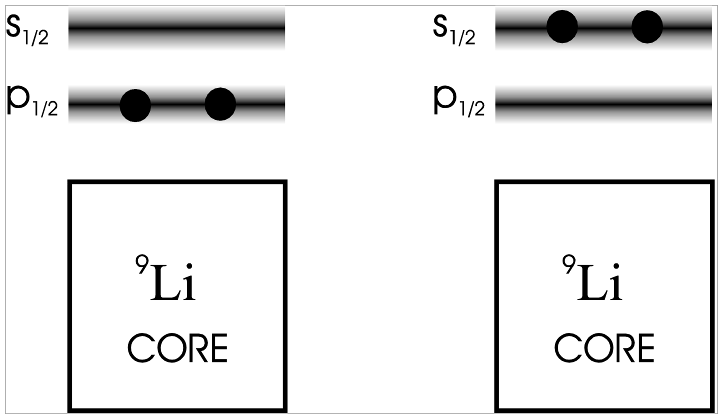
- Due to finite lifetime states acquire width
  (uncertainty in energy decay width)
  Complex Energies!!!

^{11}LI model

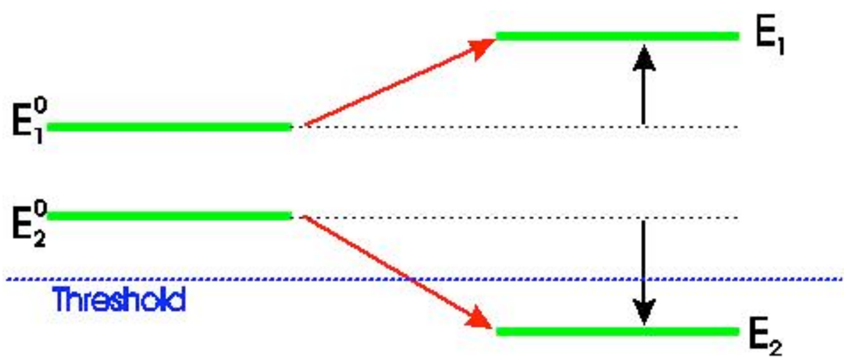
Dynamics of two states coupled to a common decay channel

- Model $\mathcal{H}$

$$\mathcal{H}(E) = \begin{pmatrix} \epsilon_1 - \frac{i}{2}\gamma_1 & v - \frac{i}{2}A_1 A_2 \\ v - \frac{i}{2}A_1 A_2 & \epsilon_2 - \frac{i}{2}\gamma_2 \end{pmatrix}$$



- Mechanism of binding by Hermitian interaction



Solutions with energy-dependent widths

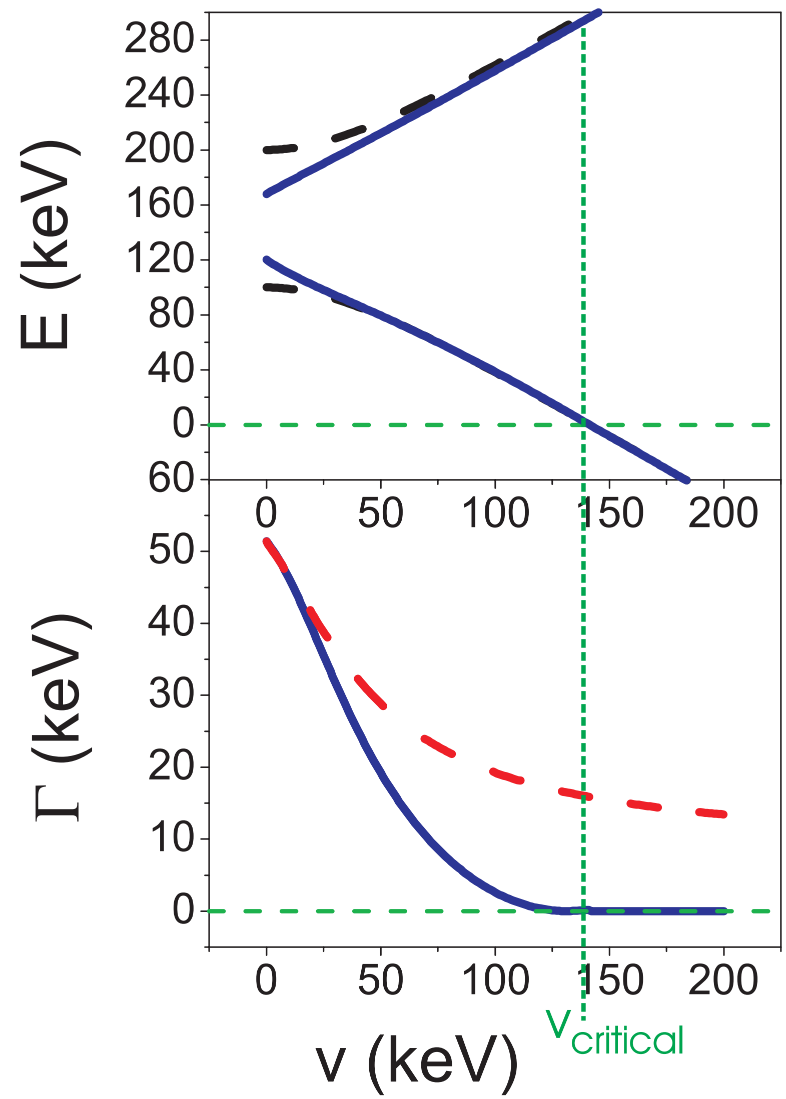
- Energy-independent width is not consistent with definitions of threshold

$$A_2^2 = \gamma_2(E) = \alpha\sqrt{E},$$

$$A_1^2 = \gamma_1(E) = \beta E^{3/2}$$

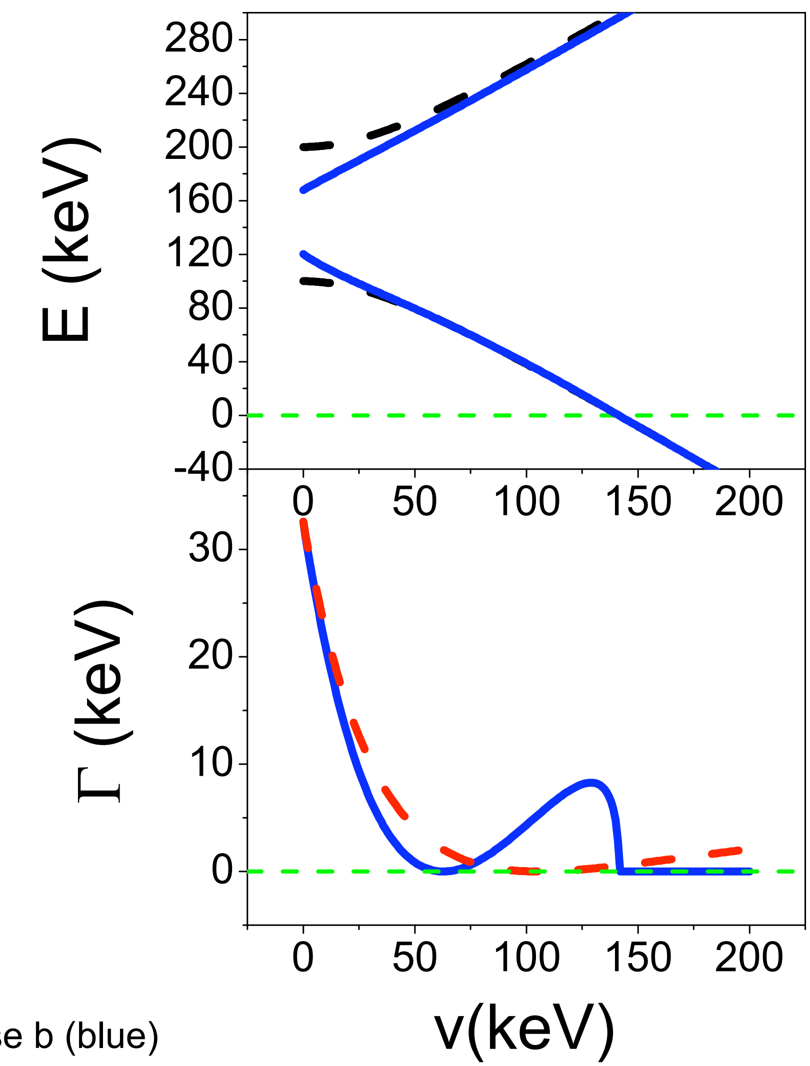
Squeezing of phase-space volume in s and p waves, Threshold E_c=0

Model parameters:
ε_1=100, ε_2=200,
A_1=7.1 A_2=3.1 (red); α=1, β=0.05 (blue)
(in units based on keV)
Upper panel: Energies with A_1=A_2=0 (black)



Bound state in the continuum effect: Γ=0, above threshold

$$v = A_1 A_2 \frac{\epsilon_1 - \epsilon_2}{\gamma_1 - \gamma_2}$$



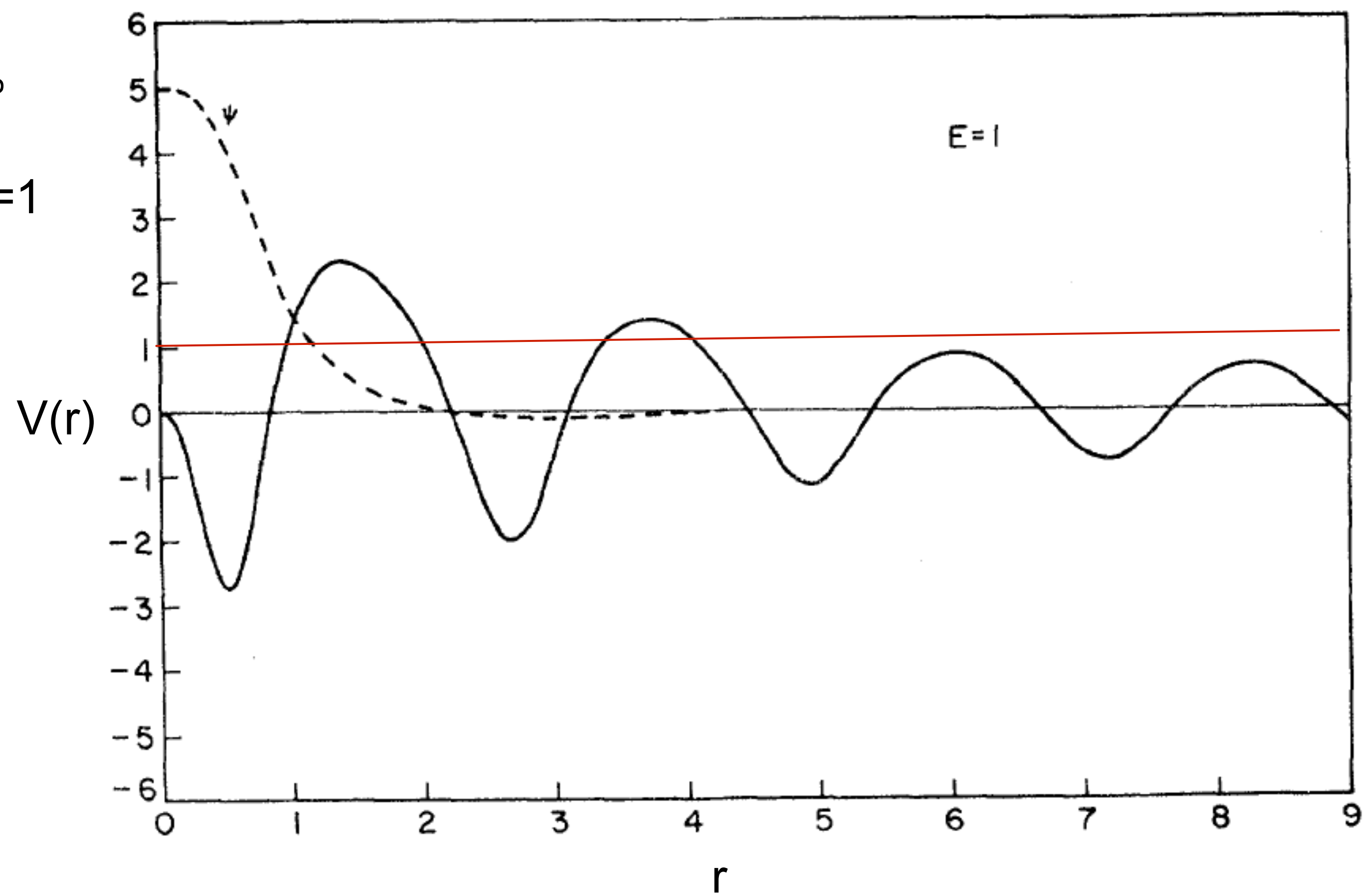
Model parameters:
ε_1=100, ε_2=200,
A_1=8.1 A_2=12.8 (red); α=15, β=0.05 (blue)
(in units based on keV)
Upper panel: Energies with A_1=A_2=0 (black) and case b (blue)

Bound States in the continuum

von Neumann, J. & Wigner, E. *Phys. Z.* **30**, 465−467 (1929).

V(r)=0 when r→∞

Bound state at E=1



Observation: Capasso, et.al. *Nature* **358**, 565 - 567

Level Crossing in two-level system

- Bound states, no level crossing if $v \neq 0$.
- System with decay, energy independent H [1]

$X=2Tr(H^2)-(Tr(H))^2=(E_1-E_2)^2$ determines picture

- Full level crossing $E_1=E_2$ if $X=0$
- $Im(X)=0$ partial level crossing, $\Delta E \Delta \Gamma=0$

  If $Re(X)<0$, energies cross, $\Delta E=0$

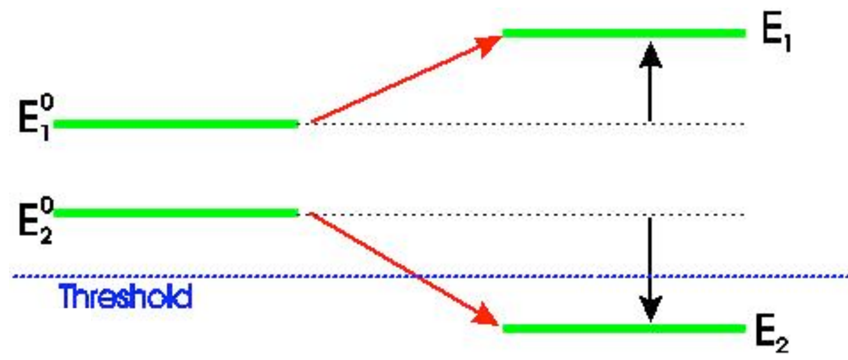
  If $Re(X)>0$, widths cross, $\Delta \Gamma=0$

- Open system, energy-dependent $H(E)$

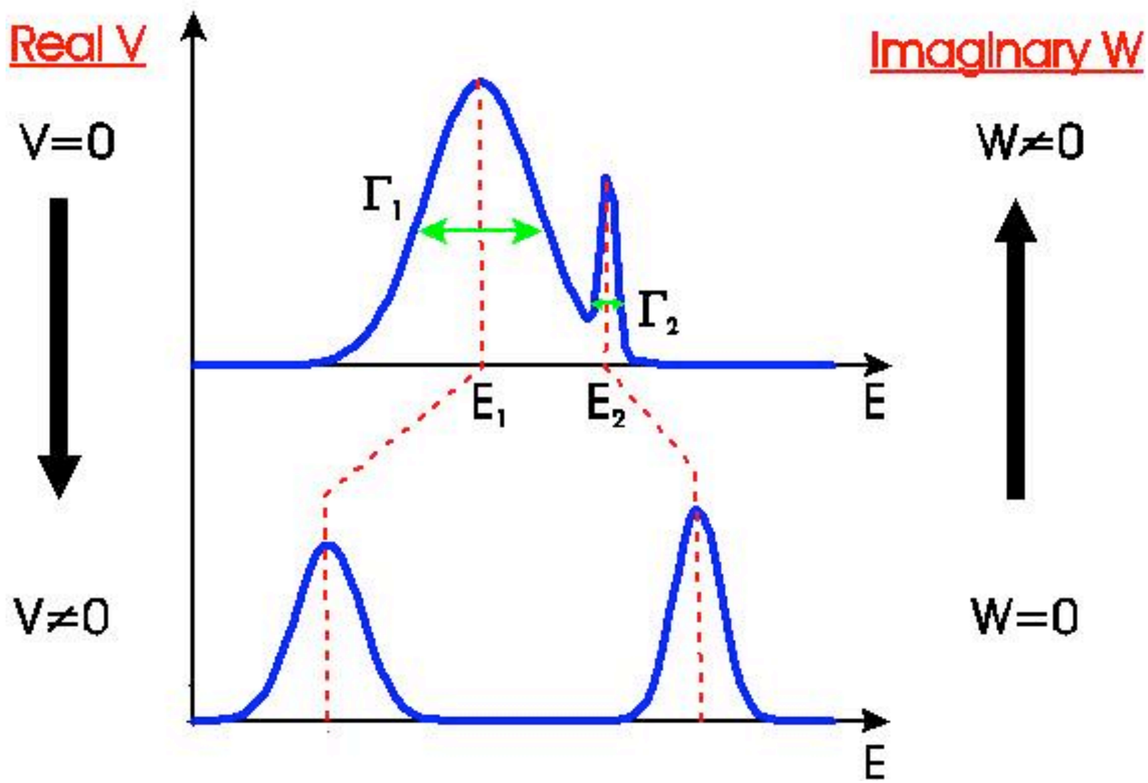
more complicated but features are similar

[1] P. von Brentano and M. Philipp, Phys. Lett B 454 (1999) 171

Interacting resonances

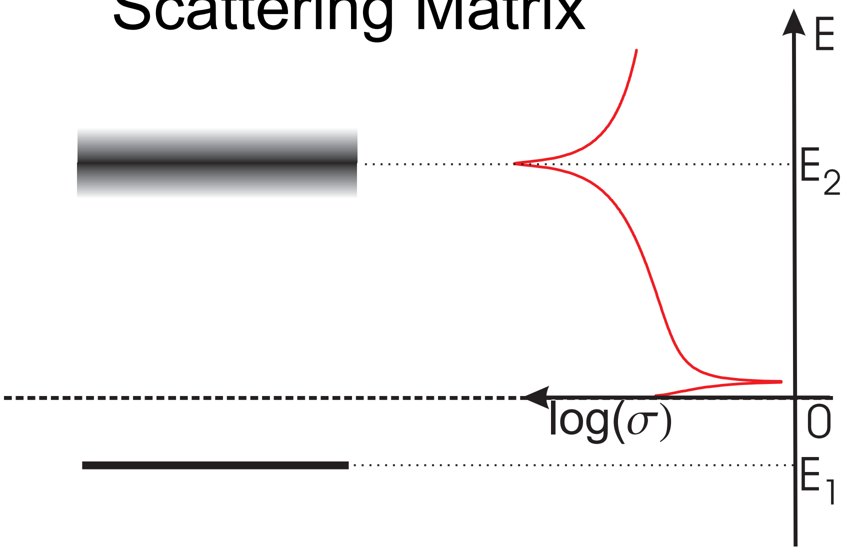


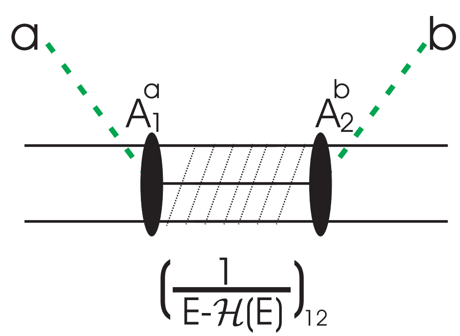
$$\mathcal{H} = H^0 + V - iW/2$$



Scattering and cross section near threshold

Scattering Matrix





Solution in two-level model

$$T(E) = \frac{E(\gamma_1 + \gamma_2) - \gamma_1\epsilon_2 - \gamma_2\epsilon_1 - 2vA_1A_2}{(E - \mathcal{E}_+)(E - \mathcal{E}_-)}$$

Cross section

$$\sigma(E) = \frac{\pi}{k^2}|S(E) - 1|^2$$

$$S^{ab} = (s^a)^{1/2}(\delta^{ab} - T^{ab})(s^b)^{1/2}$$

where $s^a = \exp(i\delta_a)$
is smooth scattering phase

$$T^{ab} = \sum_{12} A_1^{a*} \left(\frac{1}{E - \mathcal{H}}\right)_{12} A_2^b$$

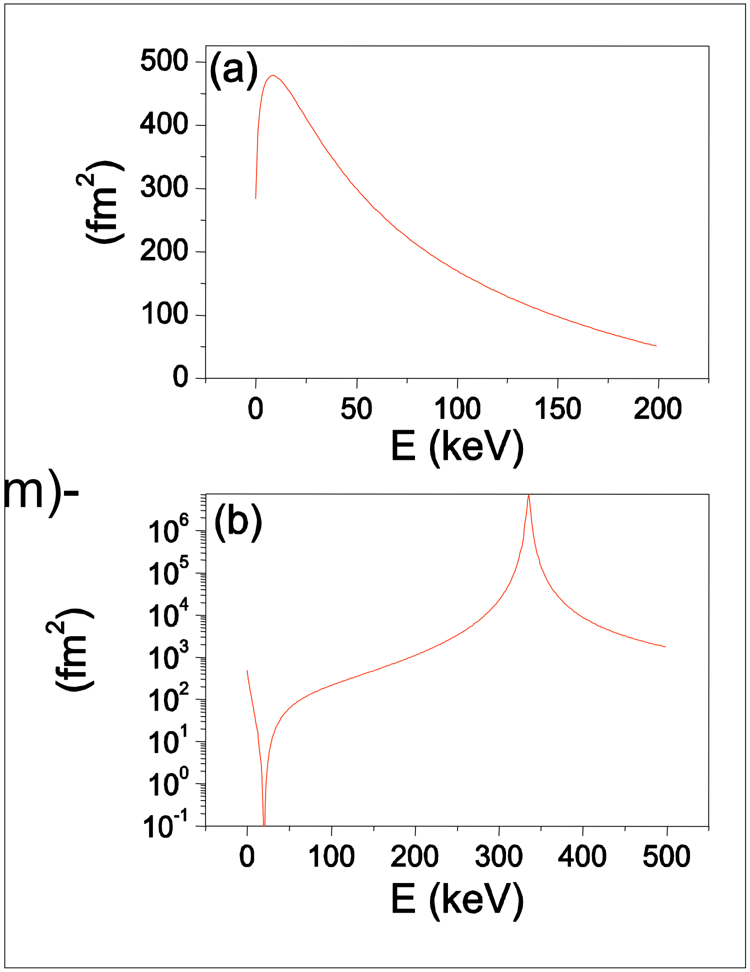
Cross section near threshold

- No direct interaction v=0
  Breit-Wigner resonance

$$T(E) = \frac{\gamma_1 + \gamma_2}{E - \epsilon + (i/2)(\gamma_1 + \gamma_2)}$$

- Below critical v (both states in continuum)- sharp resonances

- Above critical v
  - One state is bound- "attraction" to sub-threshold region fig (a)
  - Second state –resonance, fig (b)

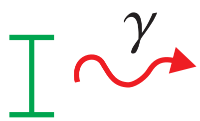


Model parameters:
ε_1=100, ε_2=200, v=180 (keV)
A_1^2=0.05 $(E)^{3/2}$, A_2^2=15 $(E)^{1/2}$

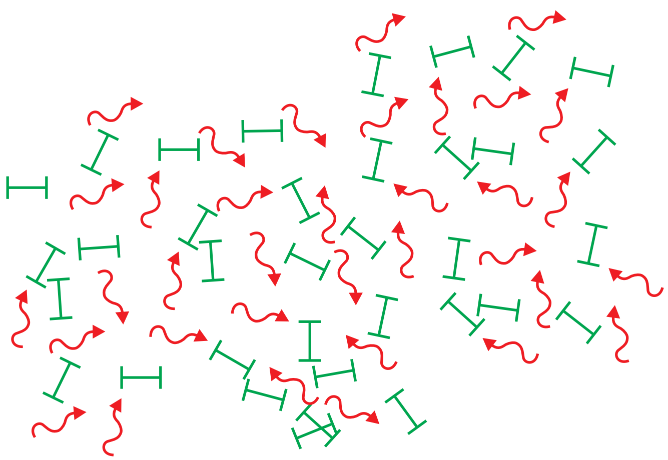
Superradiance, collectivization by decay

Dicke coherent state
N identical two-level atoms
coupled via common radiation

Single atom γ

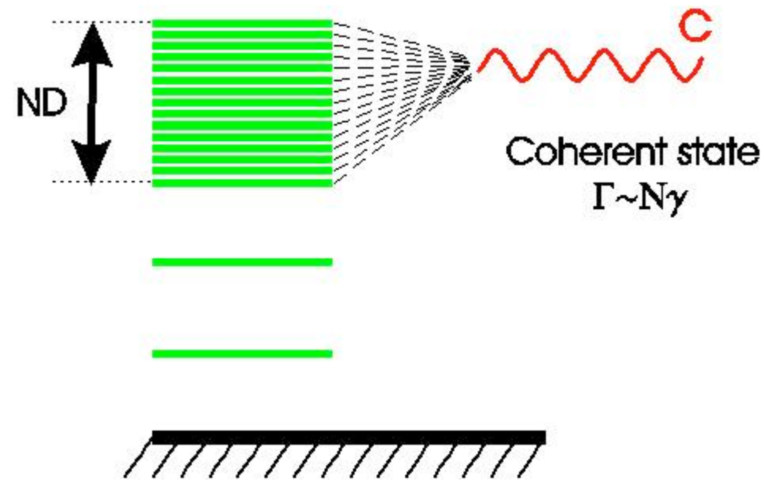


Coherent state $\Gamma \sim N\gamma$



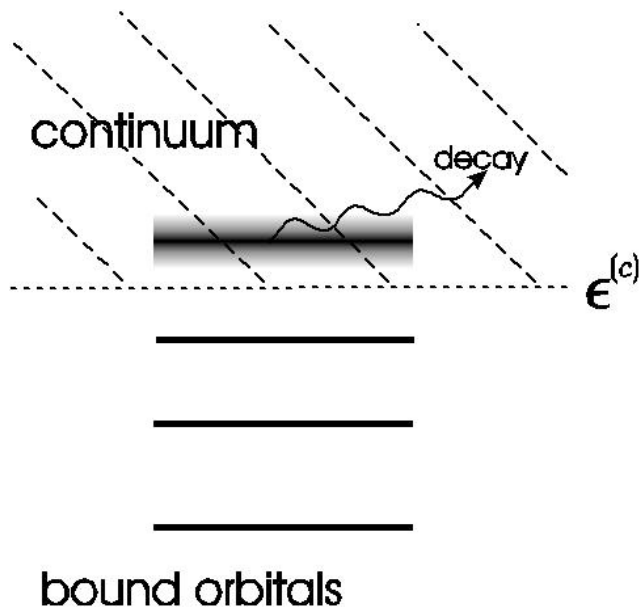
Volume << λ^3

Analog in nuclei
Interaction via continuum
Trapped states) self-organization

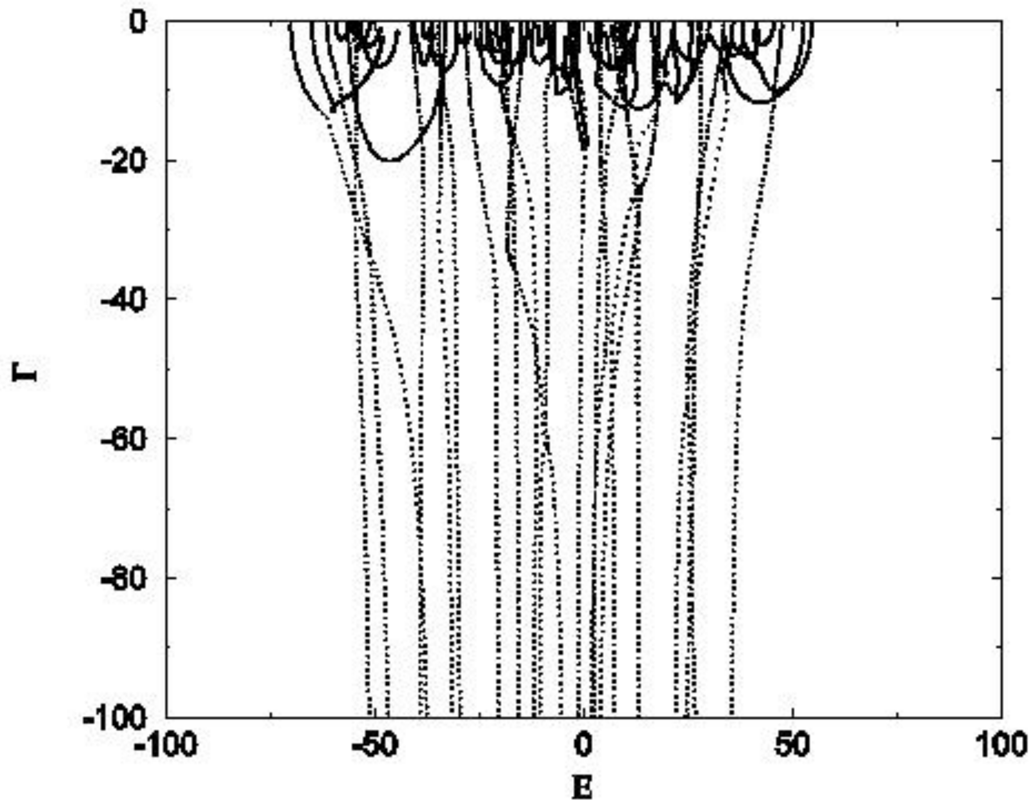


g ~ D and few channels
- Nuclei far from stability
- High level density (states of same symmetry)
- Far from thresholds

Single-particle decay in many-body system



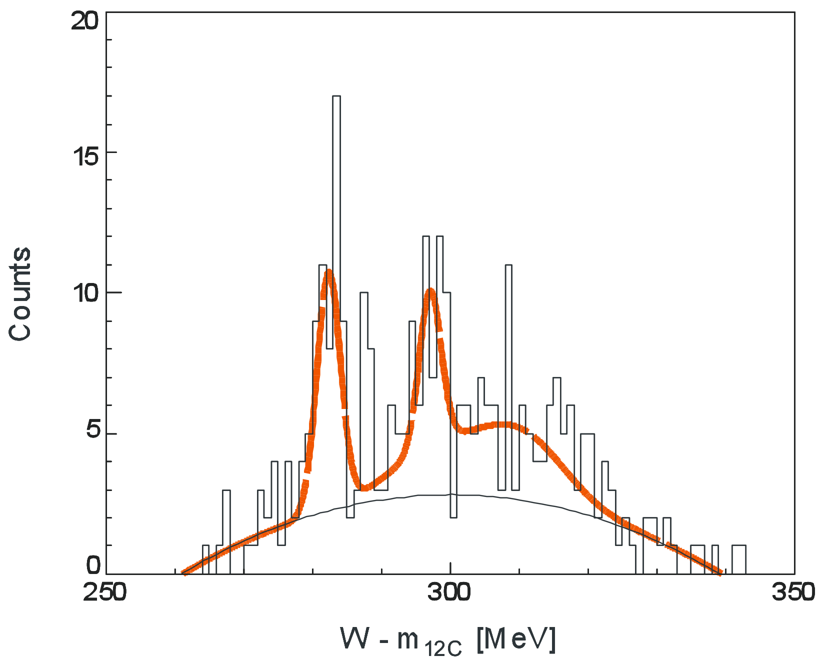
Evolution of complex energies E=E-i $\Gamma/2$ as a function of γ

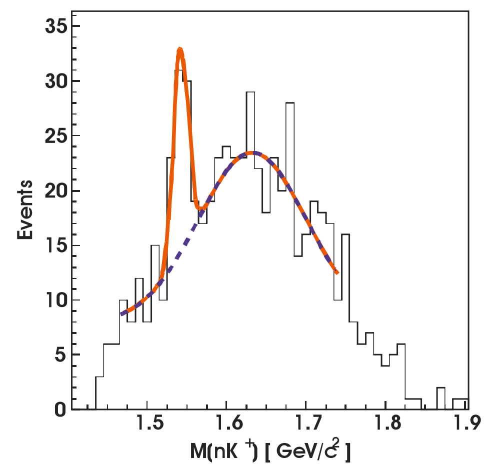


- Assume energy independent W
- Assume one channel $\gamma=A^2$
- System 8 s.p. levels, 3 particles
- One s.p. level in continuum $e=\varepsilon - i\gamma/2$

Total states 8!/(3! 5!)=56; **states that decay fast 7!/(2! 5!)=21**

Superradiance in resonant spectra





Narrow resonances and broad suparradiant state in ${}^{12}C\Delta$

Bartsch *et.al.* Eur. Phys. J. A **4**, 209 (1999)

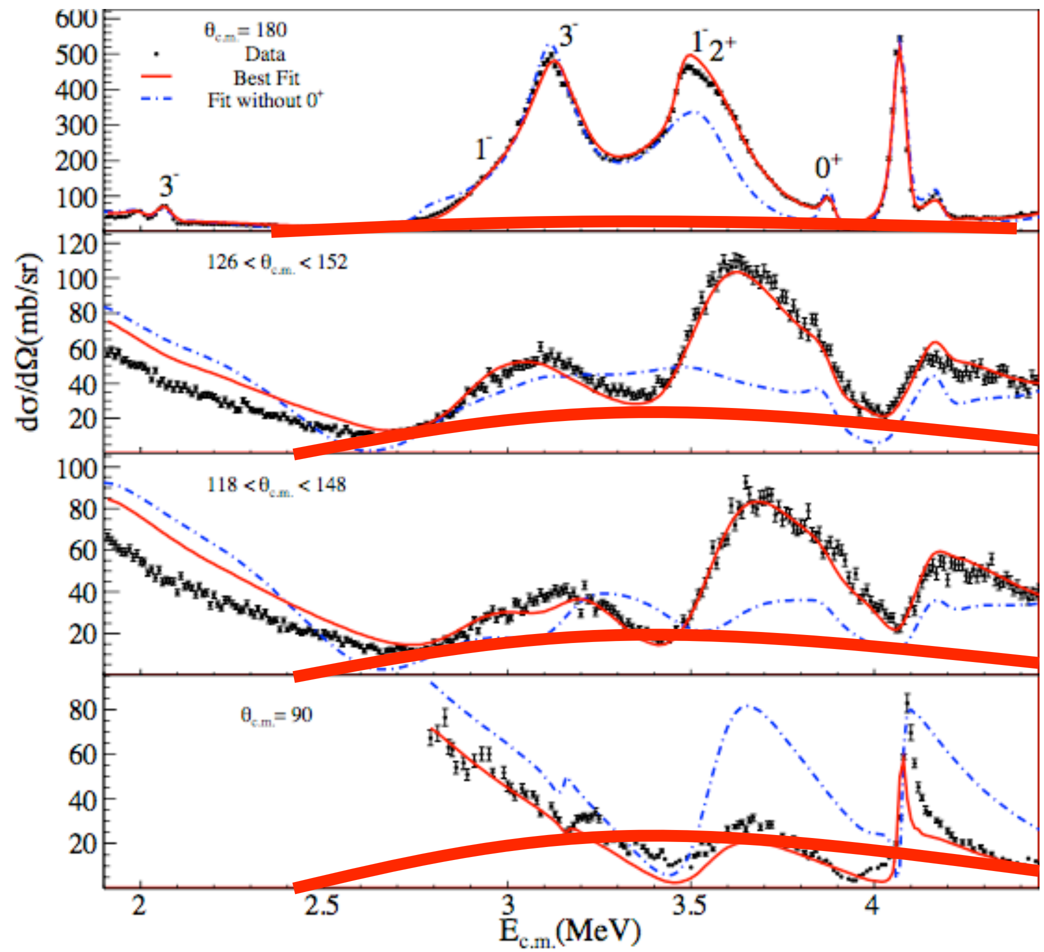
Pentaquark as a possible candidate for superradiance

Stepanyan *et.al.* hep-ex/0307018

Broad 0^+ alpha state at excitation energy of 9.9 MeV
$\alpha + {}^{14}C$

Very broad Γ≈2 MeV
0^+ state at 3.7+/-0.5 MeV
above the α decay
threshold was observed -
9.9 MeV excitation energy.

E.D. Johnson, et al.,
EPJA, 42 135 (2009)



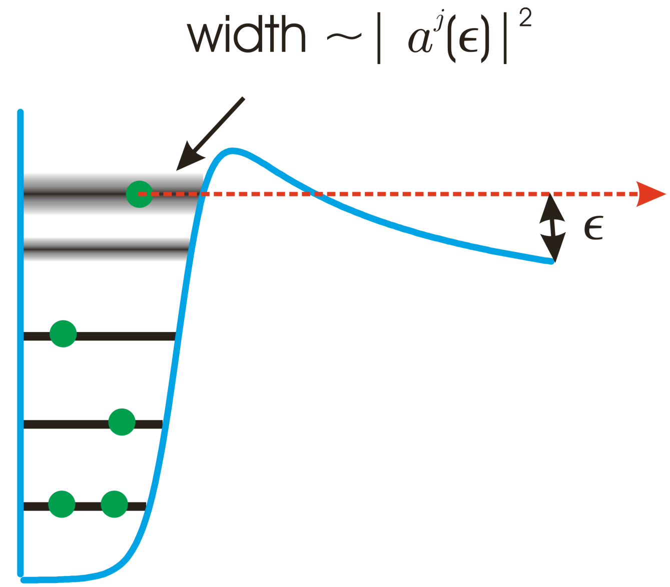
From G. Rogachev

Single-particle scattering problem

The same non-Hermitian eigenvalue problem

$$hu_l = \frac{1}{2\mu}\left\{-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} + 2\mu\left[V(r) + \alpha\frac{Zz}{r}\right]\right\} u_l(r) = \epsilon u_l(r),$$

Internal states: $u_l(r)$



External states: $\epsilon = \dfrac{k^2}{2\mu}$

$$F_l(r) = krj_l(kr)$$

$$G_l(r) = krn_l(kr)$$

Single-particle decay amplitude

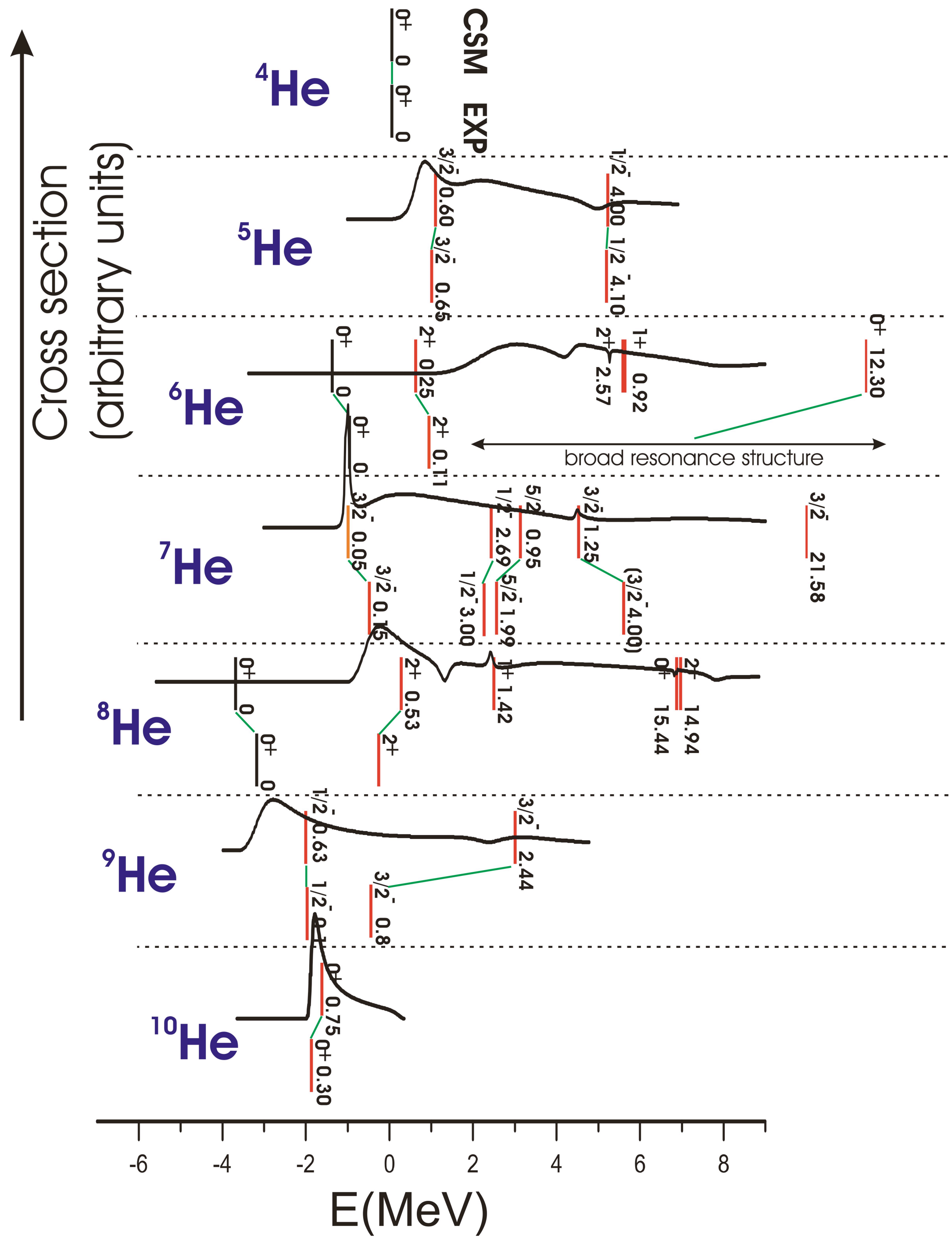
$$a^j(\epsilon) = \langle 0|c_j(\epsilon)Vb_j^\dagger|0\rangle = \sqrt{\frac{2\mu}{\pi k}}\int_0^\infty dr F_l(r)\,V(r)\,u_l(r)$$

Single particle decay width: (requires definition and solution for resonance energy)

$$\gamma_j(\epsilon) = 2\pi\left|a^j(\epsilon)\right|^2$$

Continuum Shell Model
He isotopes

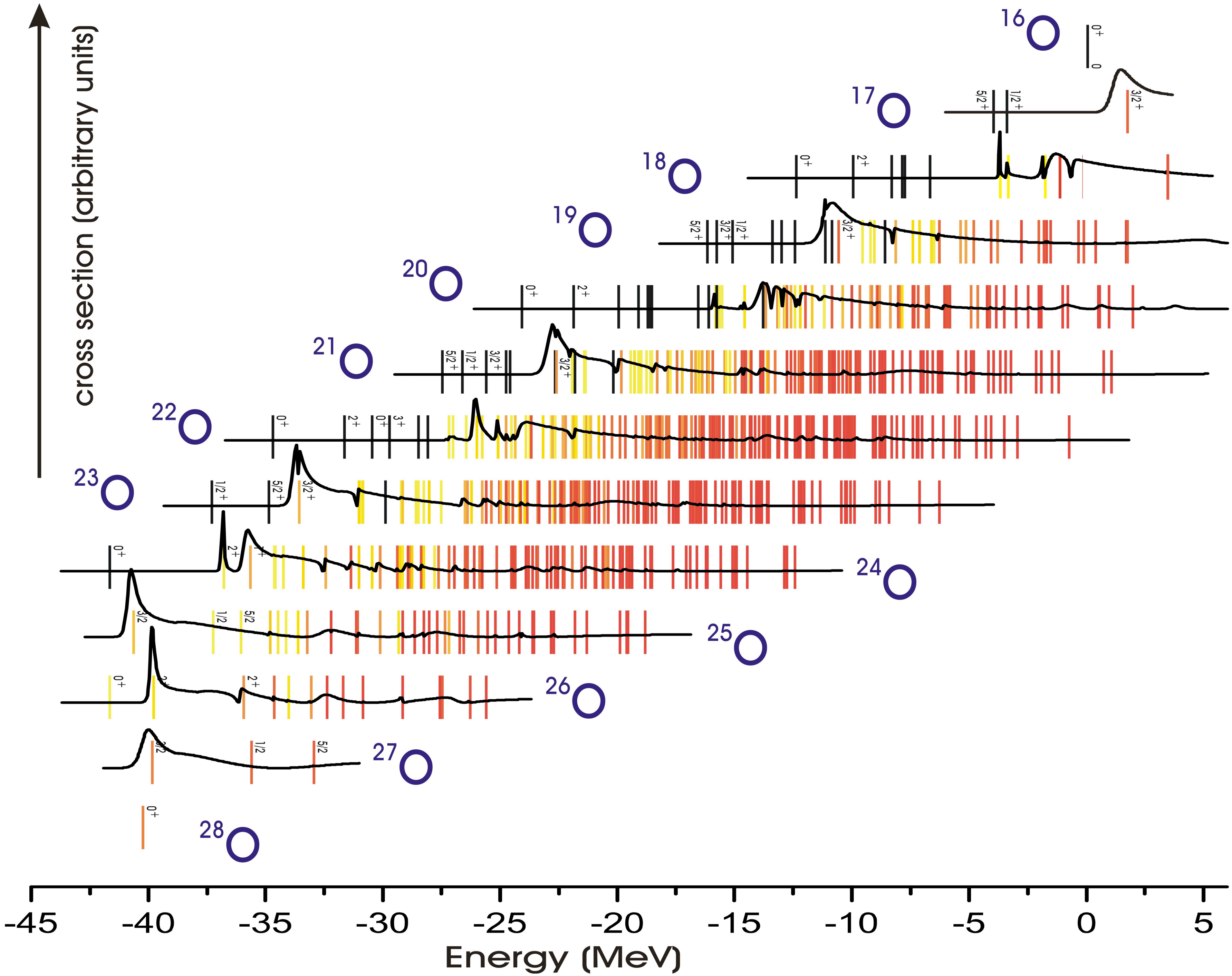
- Cross section and structure within the same formalism
- Reaction l=1 polarized elastic channel



References

[1] A. Volya and V. Zelevinsky, Phys. Rev. Lett 94 (2005) 052501.

[2] A. Volya and V. Zelevinsky, Phys. Rev. C 67 (2003) 54322


cross section (arbitrary units)
Energy (MeV)
-45
-40
-35
-30
-25
-20
-15
-10
-5
0
5
16
17
18
19
20
21
22
23
24
25
26
27
28

Scattering matrix and reactions

$$\mathbf{T}_{cc'}(E) = \langle A^c(E)| \left(\frac{1}{E - \mathcal{H}(E)} \right) |A^{c'}(E)\rangle$$

$$\mathbf{S}_{cc'}(E) = \exp(i\xi_c)\ \{\delta_{cc'} - i\,\mathbf{T}_{cc'}(E)\} \exp(i\xi_{c'})$$

Cross section:

$$\sigma = \frac{\pi}{k'^2} \sum_{cc'} \frac{(2J+1)}{(2s'+1)(2I'+1)} |\mathbf{T}_{cc'}|^2$$

Additional topics:

•Angular (Blatt-Biedenharn) decomposition

•Coulomb cross sections, Coulomb phase shifts, and interference

•Phase shifts from remote resonances.

Unitarity and flux conservation

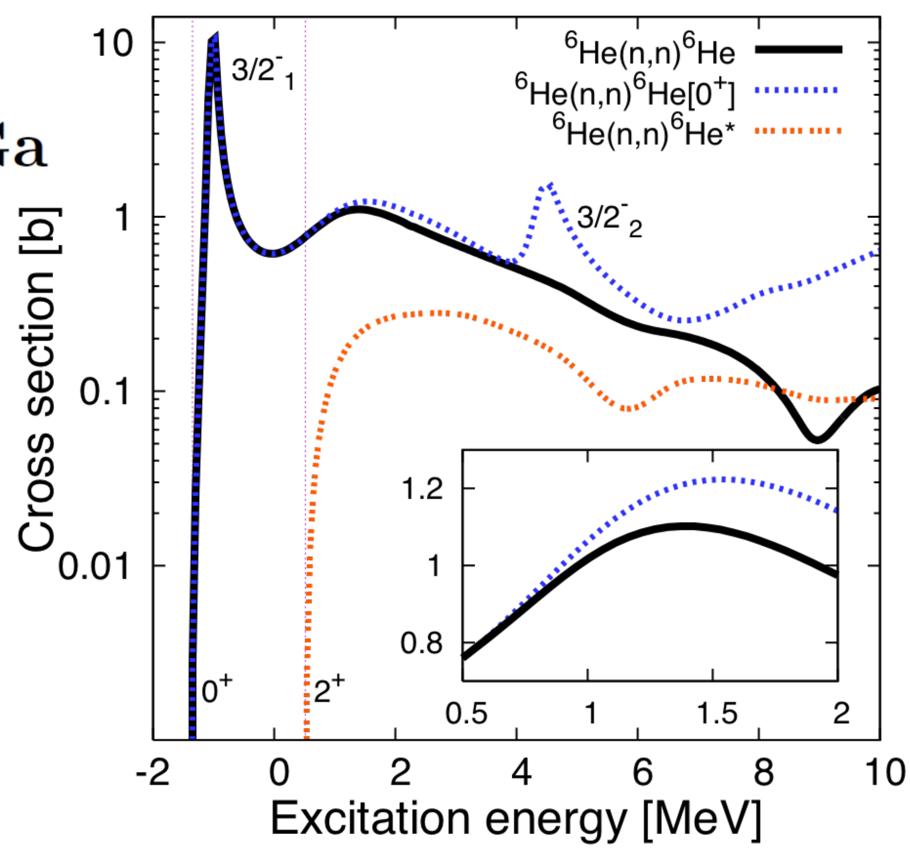
Take: $\mathbf{W} = \mathbf{a}\mathbf{a}^\dagger$

Exact relation:

$$\mathbf{S} = \frac{\mathbf{1} - i/2\,\mathbf{K}}{\mathbf{1} + i/2\,\mathbf{K}} \qquad \mathbf{K} = \mathbf{a}^\dagger \mathbf{G} \mathbf{a}$$

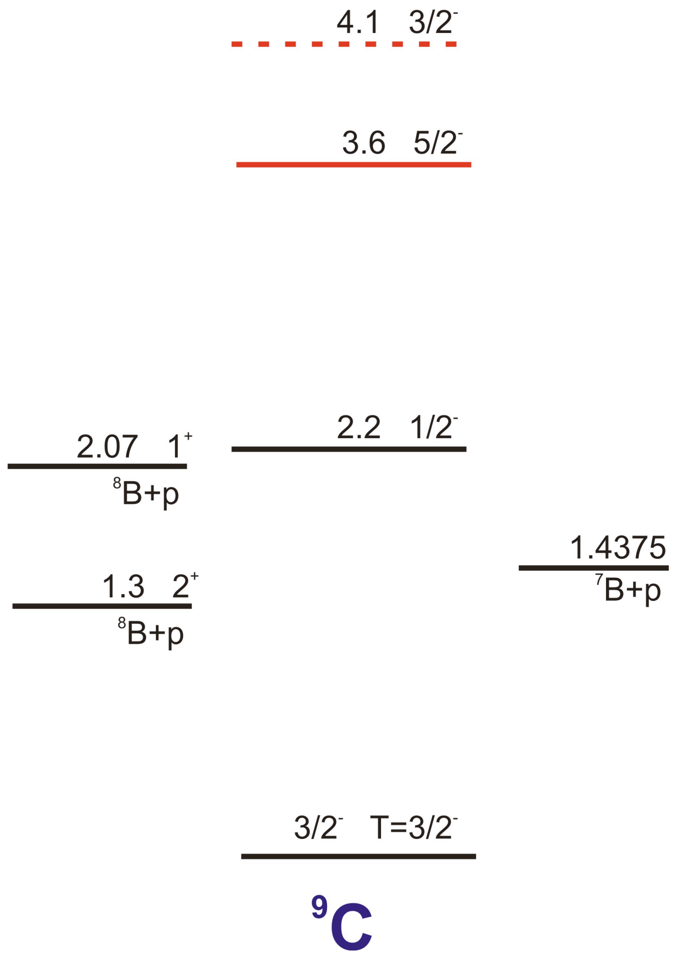
$$\mathbf{S}\mathbf{S}^\dagger = \mathbf{S}^\dagger \mathbf{S} = \mathbf{1}$$

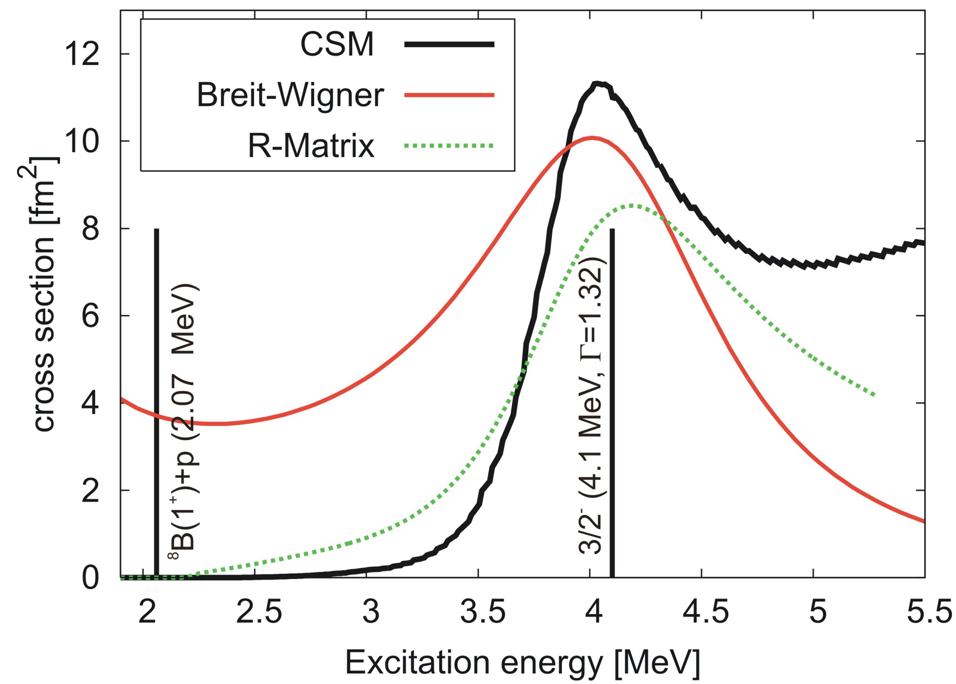
Figure: ^{6}He(n,n) cross section
- Solid curve-full cross section
- Dashed (blue) only g.s. channel
- Dotted (red) inelastic channel



- Cross section has a cusp when inelastic channels open
- The cross section is reduced due to loss of flux
- The p-wave discontinuity $E^{3/2}$

$^{8}B(p,p')^{8}B(1^{+})$

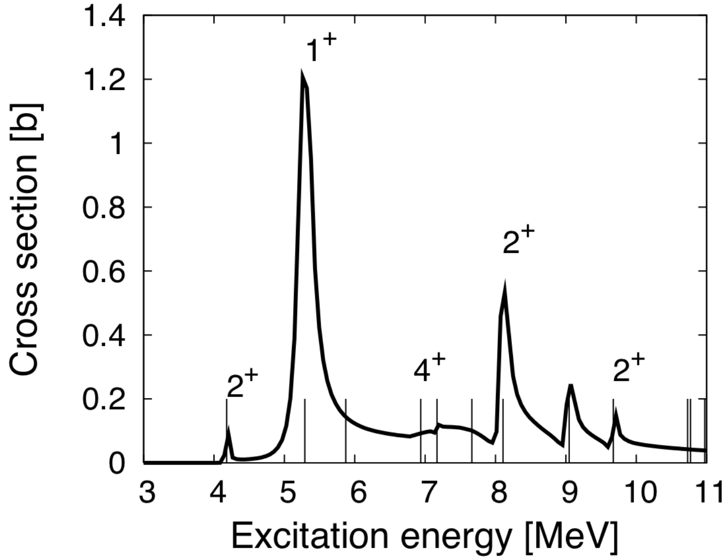


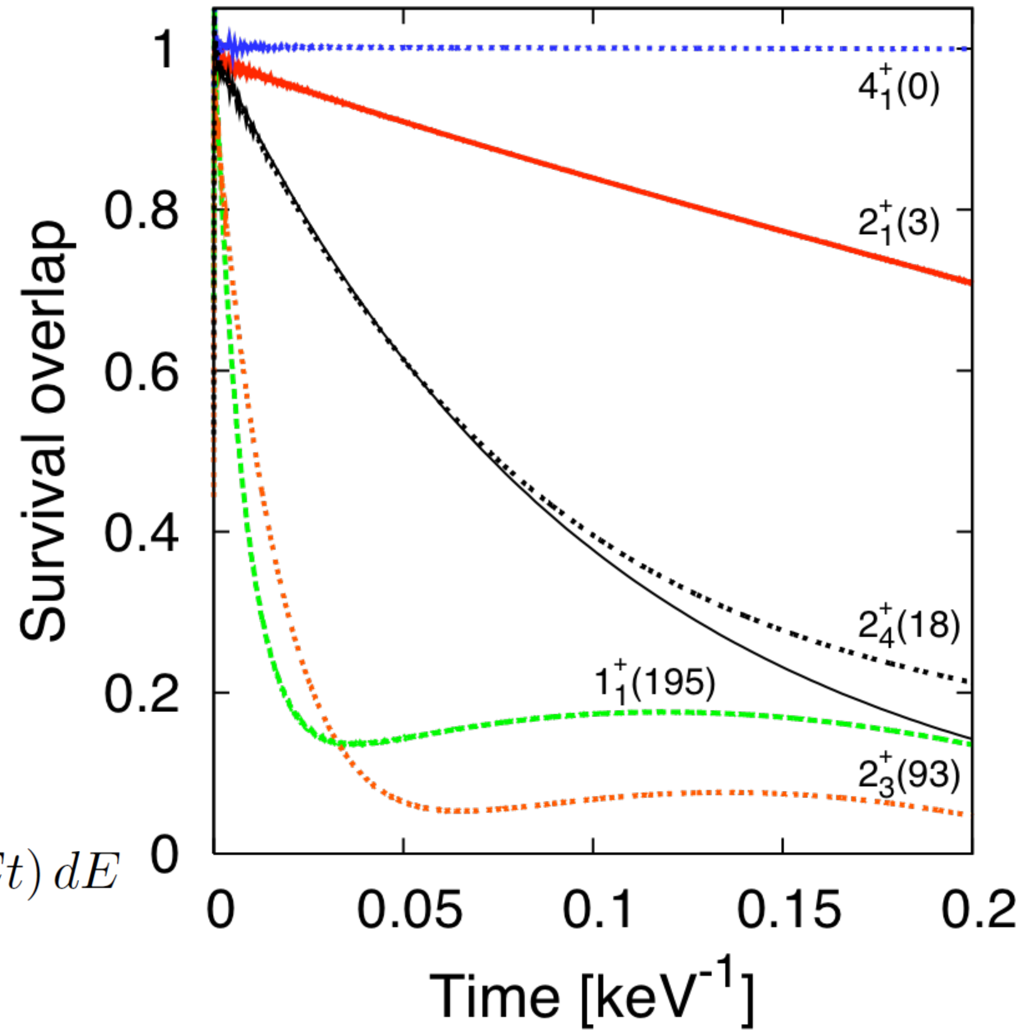


CSM:

- Automatically accounts for all states
- Includes interference
- Consistent with traditional theories

Time evolution of decaying states





Time evolution of several SM states in ^{24}O. The absolute value of the survival overlap is shown $|\langle\alpha|\mathcal{U}(t)|\alpha\rangle|$

$$\mathcal{U}(t) = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} \mathcal{G}(E)\ \exp\left(-iEt\right) dE$$

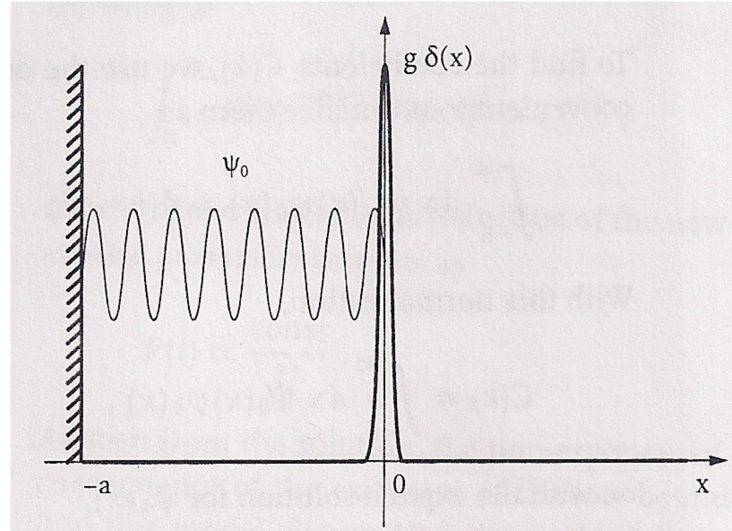
For an isolated narrow resonance

$$|\langle\alpha|\exp(-i\mathcal{E}_\alpha t)|\alpha\rangle| = \exp(-\Gamma_\alpha t/2)$$

Time dependendence of decay, Winter's model

Quasistationary state state $|n\rangle$

$$\langle x|n\rangle = \sqrt{2}\sin(n\pi x)$$



Continuum of reaction eigenstates $|k\rangle$

$$\langle x|k\rangle = A(k)\sin\left[k(x+1)\right] + \Theta(x)\,\frac{G}{k}\sin(k)\sin(kx)$$

Potential is formed by an infinite wall and a delta-barrier.

Time dependent decay

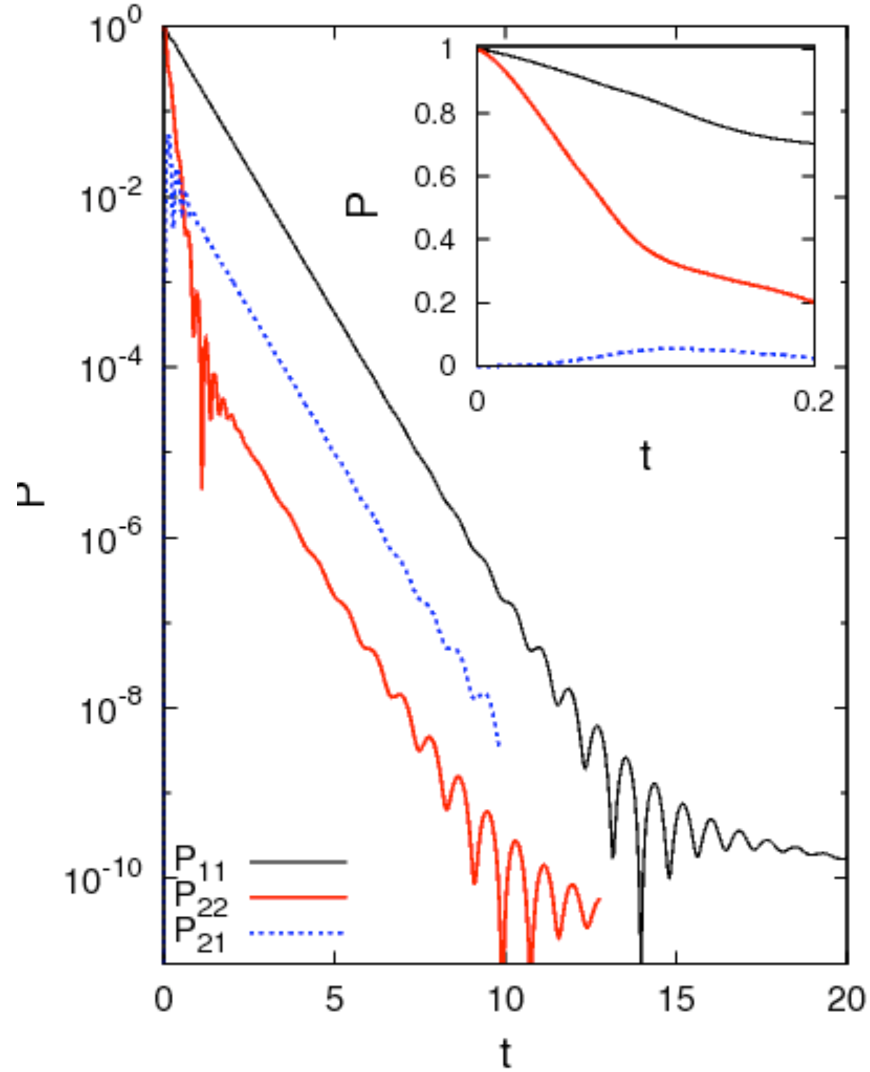
$$A_{nn'}(t) = \langle n|e^{-iHt}|n'\rangle = \int e^{-ik^2t}\langle n|k\rangle\langle k|n'\rangle dk$$

$$P_{nn'}(t) = |A_{nn'}(t)|^2$$

D. Dicus. at.al. Phys. Rev. A (2002) 65 032116

Time-dependent decay, Winter's model

- t^2 decay at small t
- Exponential decay
- Oscillations in decay
- remote decay as $1/t^3$



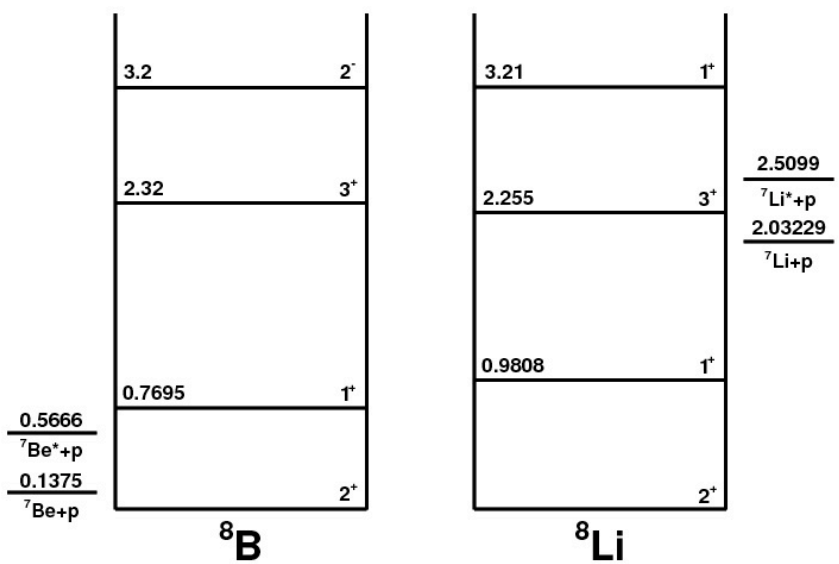
$$A_{nn'}(t) = \langle n|e^{-iHt}|n'\rangle$$

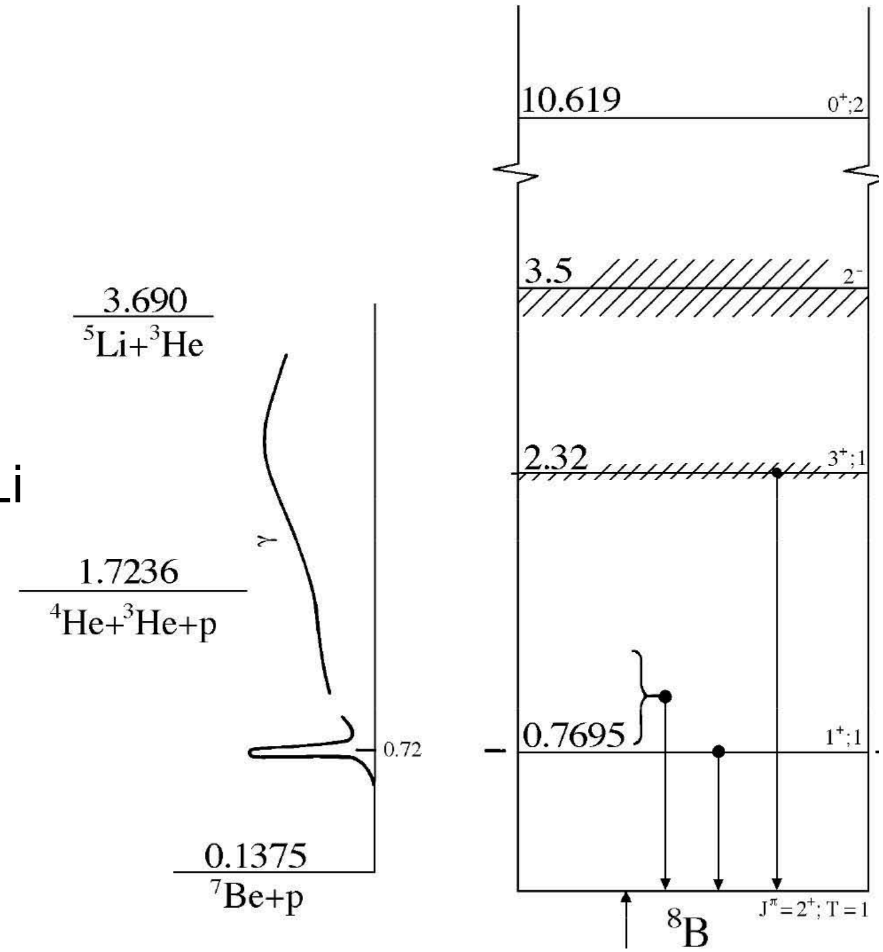
Probability $$P_{nn'}(t) = |A_{nn'}(t)|^2$$

D. Dicus. at.al. Phys. Rev. A (2002) 65 032116

States in ^8B

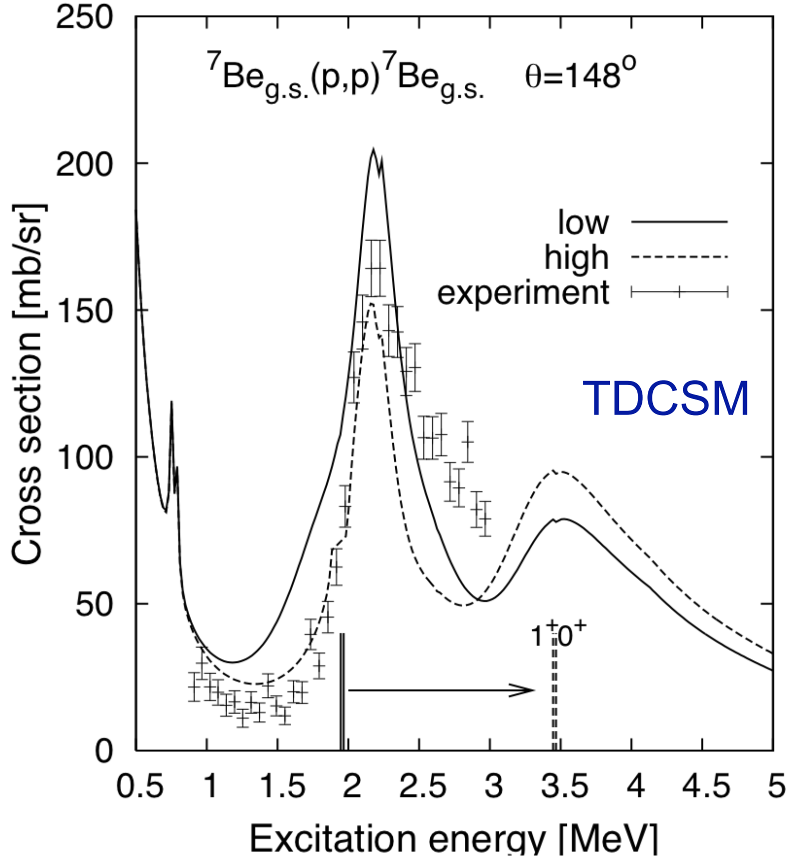
- Ab-initio and no core theoretical models predict low-lying 2^+, 0^+, and 1^+ states
- Recoil-Corrected CSM suggests low-lying states
- Traditional SM mixed results
- These states were not seen in ^8B and in ^8Li



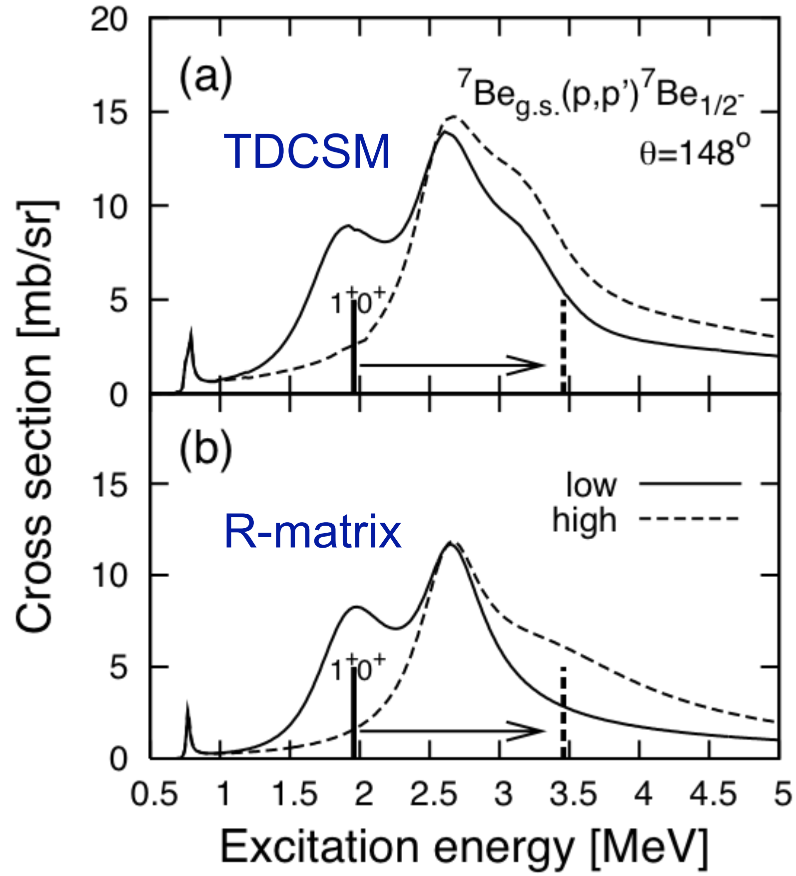


Experimental observation of 2⁺, 0⁺, and 1⁺ states can be done in inelastic reaction

Elastic cross section



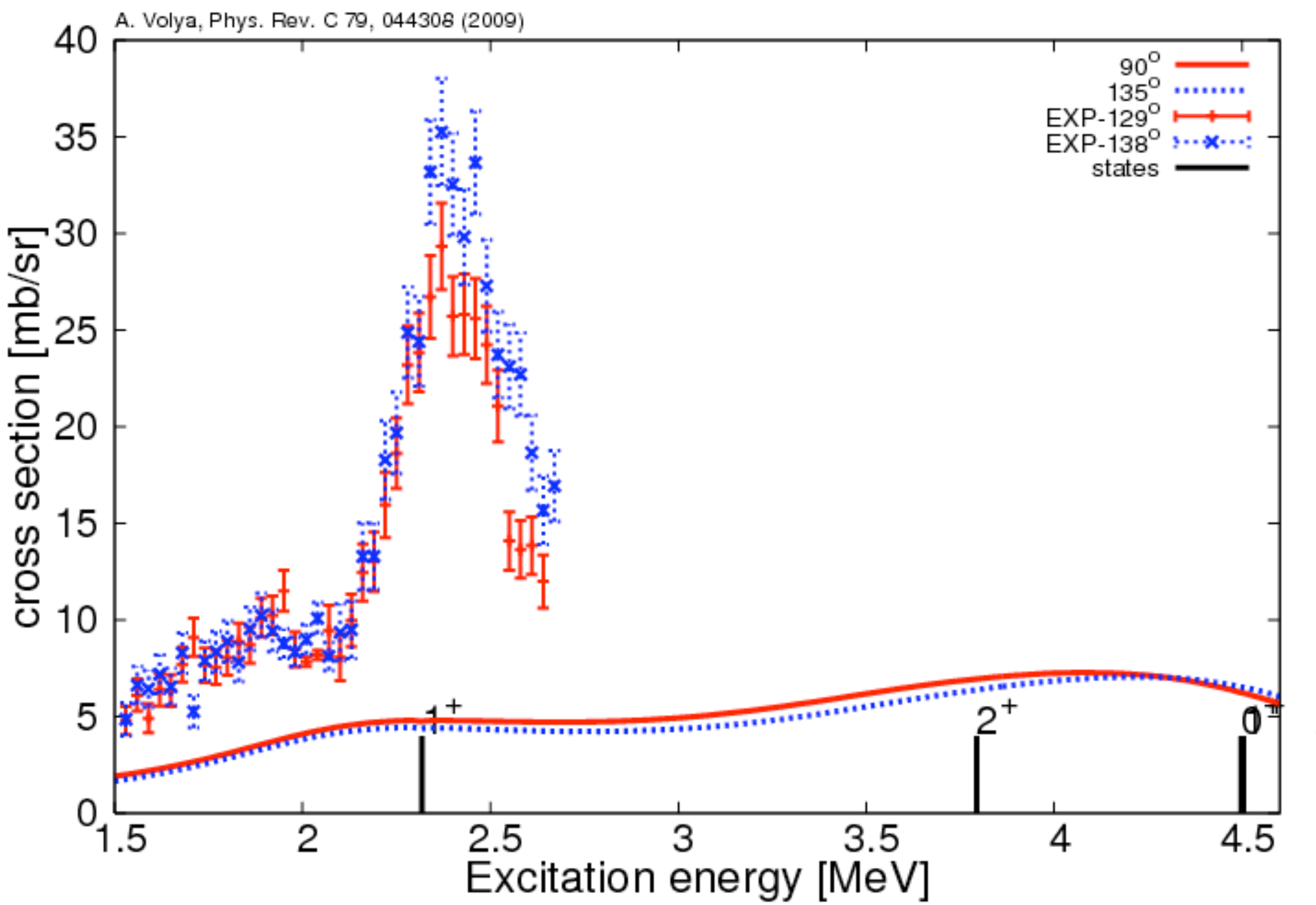
Inelastic cross section



TDCSM: WBP interaction +WS potential, threshold energy adjustment.
R-Matrix: WBP spectroscopic factors, R_c=4.5 fm, only 1^+ 1^+ 0^+ 3^+ and 2^+ l=1 channels
Experimental data from: G.Rogachev, et.al. Phys. Rev. C **64,** 061601(R) (2001).

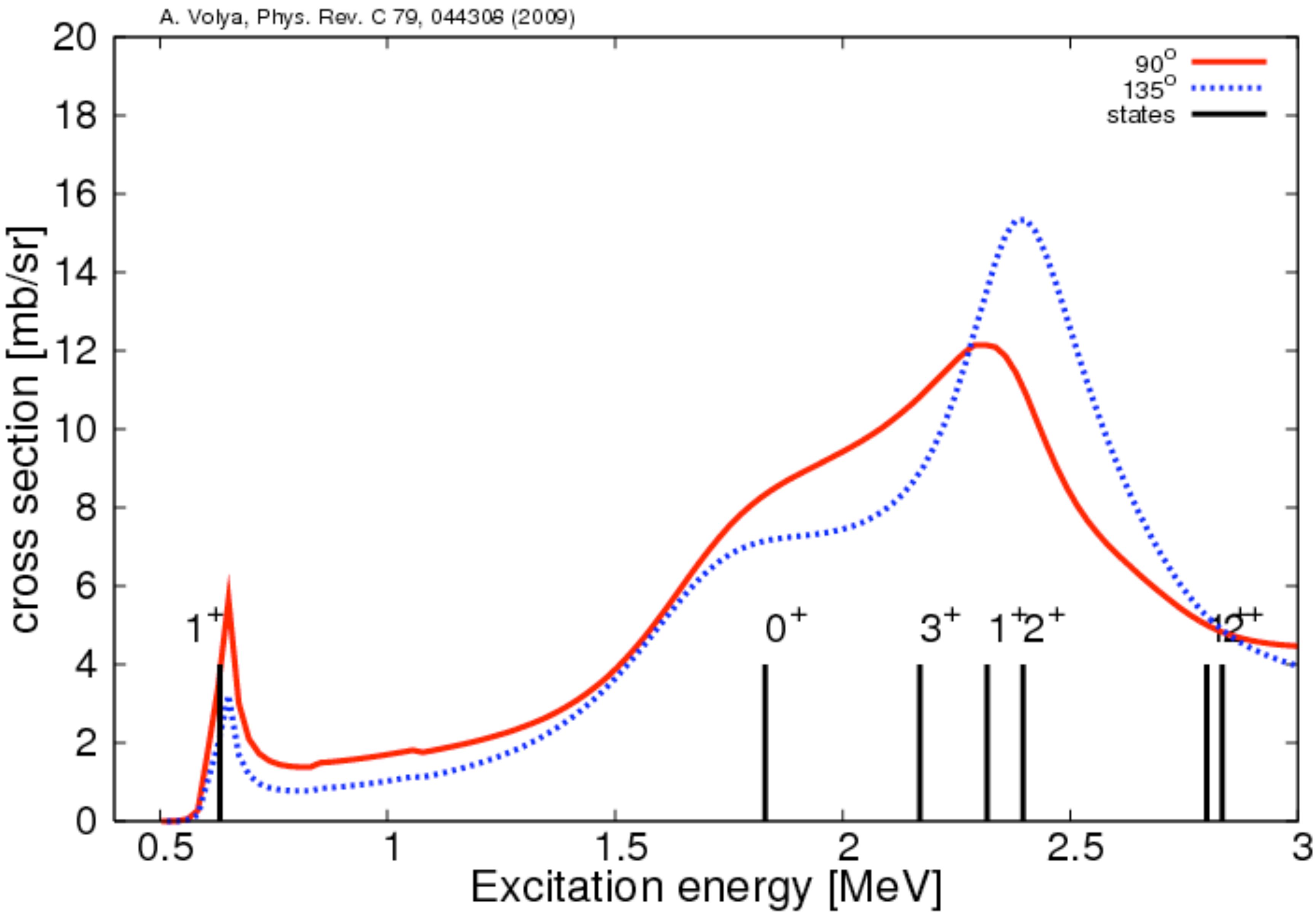
Resonances and their positions inelastic $^{7}Be(p,p')^{7}Be$ reaction in TDCSM

CKI+WS Hamiltonian

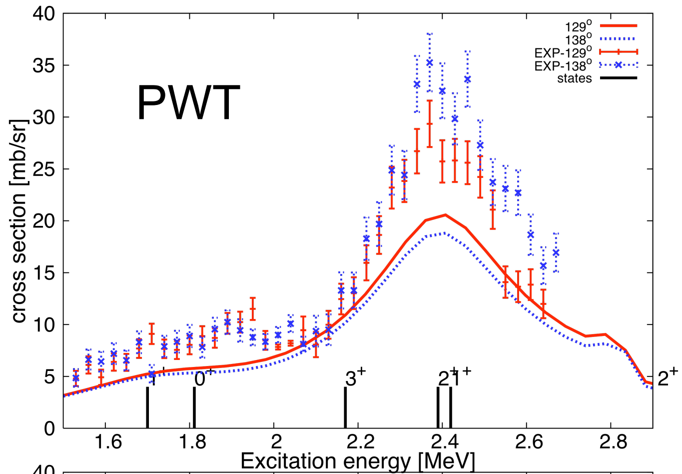


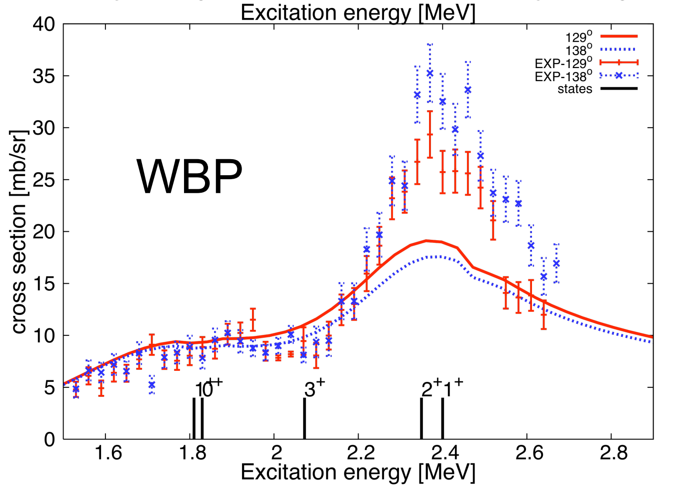
See animation at www.volya.net

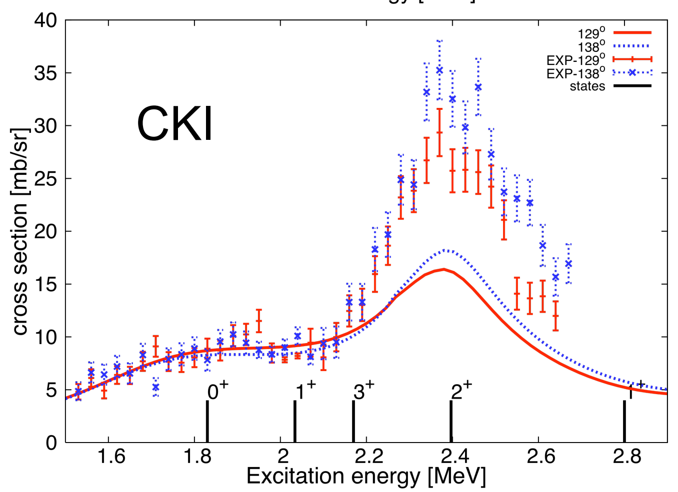
Position of the 2+ and its role in ^{7}Be(p,p)^{7}Be



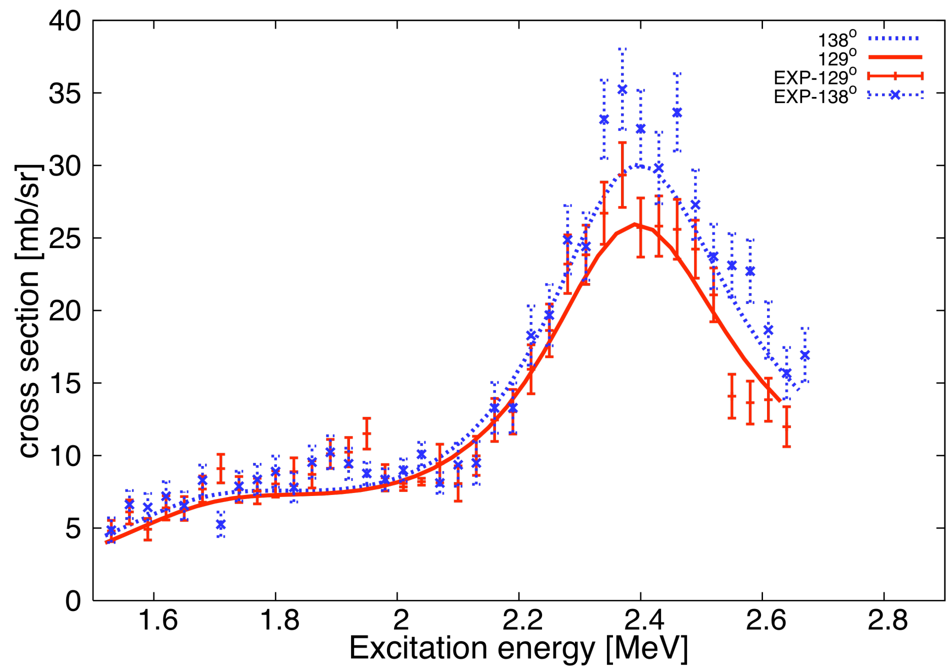
See animation at www.volya.net







R-matrix fit and TDCSM for $^7Be(p,p)^7Be$

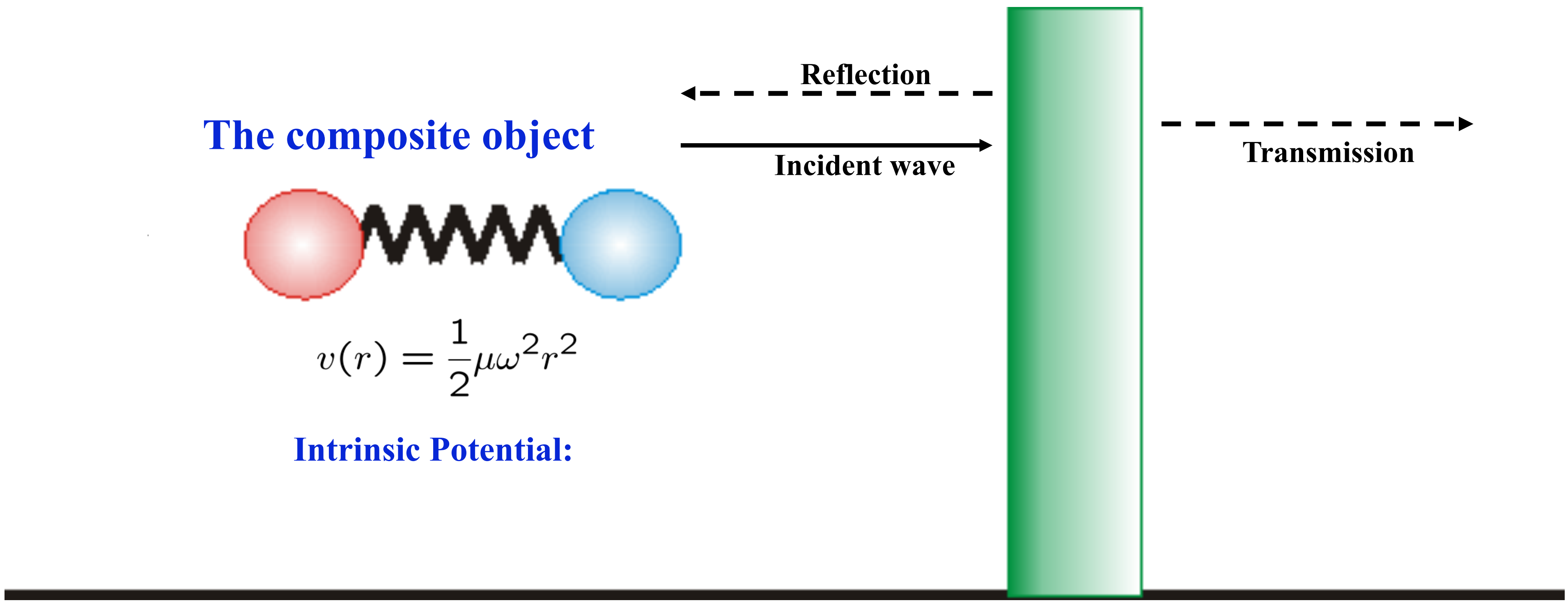


Chanel Amplitudes from TDCM and final best fit

	J^π	$p_{1/2}$, I=3/2	$p_{3/2}$, I=3/2	$p_{1/2}$, I=1/2	$p_{3/2}$, I=1/2
FIT	2^+	-0.293	0.293		0.534
CKI	2^+	-0.168	0.164		0.521
FIT	1^+	-0.821	-0.612	0.375	0.175
CKI	1^+	-0.840	-0.617	0.332	0.178

The role of internal degrees of freedom in scattering and tunneling

$$\Psi_-(r,R) = e^{iK_0R}\psi_0(r) + \sum_{n=0}^{\infty} C_{-,n}e^{-iK_nR}\psi_n(r) \qquad \Psi_+(r,R) = \sum_{n=0}^{\infty} C_{+,n}e^{-iK_nR}\psi_n(r)$$



$$v(r) = \frac{1}{2}\mu\omega^2 r^2$$

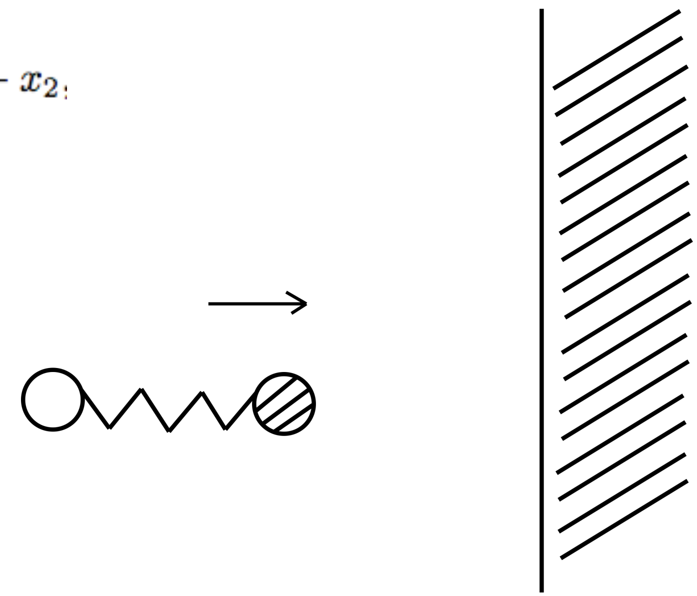
Reflection from the wall

Composite object $X = \frac{m_1 x_1 + m_2 x_2}{M}, \qquad x = x_1 - x_2;$

$$M = m_1 + m_2, \qquad m = \frac{m_1 m_2}{m_1+m_2}$$

$$h = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + v(x)$$

$$h\psi_n(r) = \epsilon_n \psi_n(r), \qquad n = 0, 1, 2, \ldots$$



Hamiltonian $H = -\frac{\hbar^2}{2M}\frac{\partial^2}{\partial X^2} + V(x_1, x_2) + h$

Assume that only one of the particles interacts with the potential!

$$V(x_1, x_2) \to V(x_2)$$

$$V(x_2) = \begin{cases} \infty & \text{when } 0 < x_2 \\ 0 & \text{otherwise} \end{cases}$$

Approach to solution

$$\Phi(X, x) = \frac{e^{iK_n X}}{\sqrt{|K_n|}}\psi_n(x) + \sum_{m=0}^{\infty} \frac{R_{mn}}{\sqrt{|K_m|}} e^{-iK_m X}\psi_m(x);$$

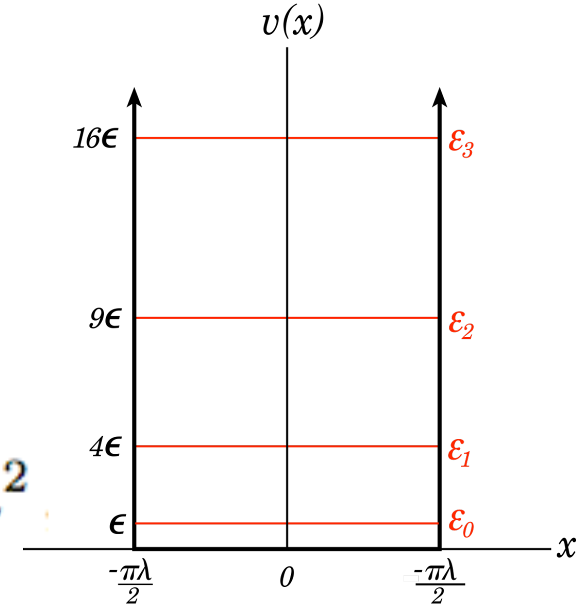
$$\Phi(X, x) = 0 \text{ at } x_2 = 0.$$

HO and Well models

"Well" model

$$v(x) = \begin{cases} 0 & \text{when } |x| < \pi\lambda/2 \\ \infty & \text{otherwise} \end{cases}$$

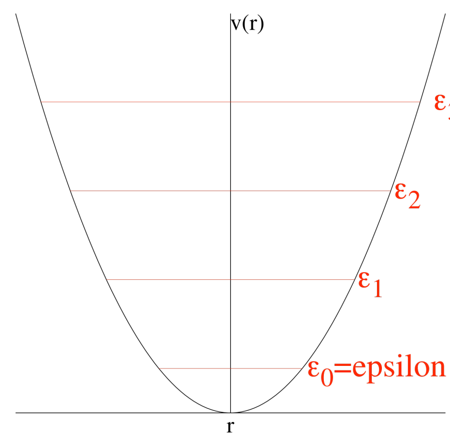
$$\psi_{n-1}(x) = \sqrt{\frac{2}{\pi}} \sin\left[\left(x + \frac{\pi}{2}\right) n\right], \quad \varepsilon_{n-1} = n^2$$



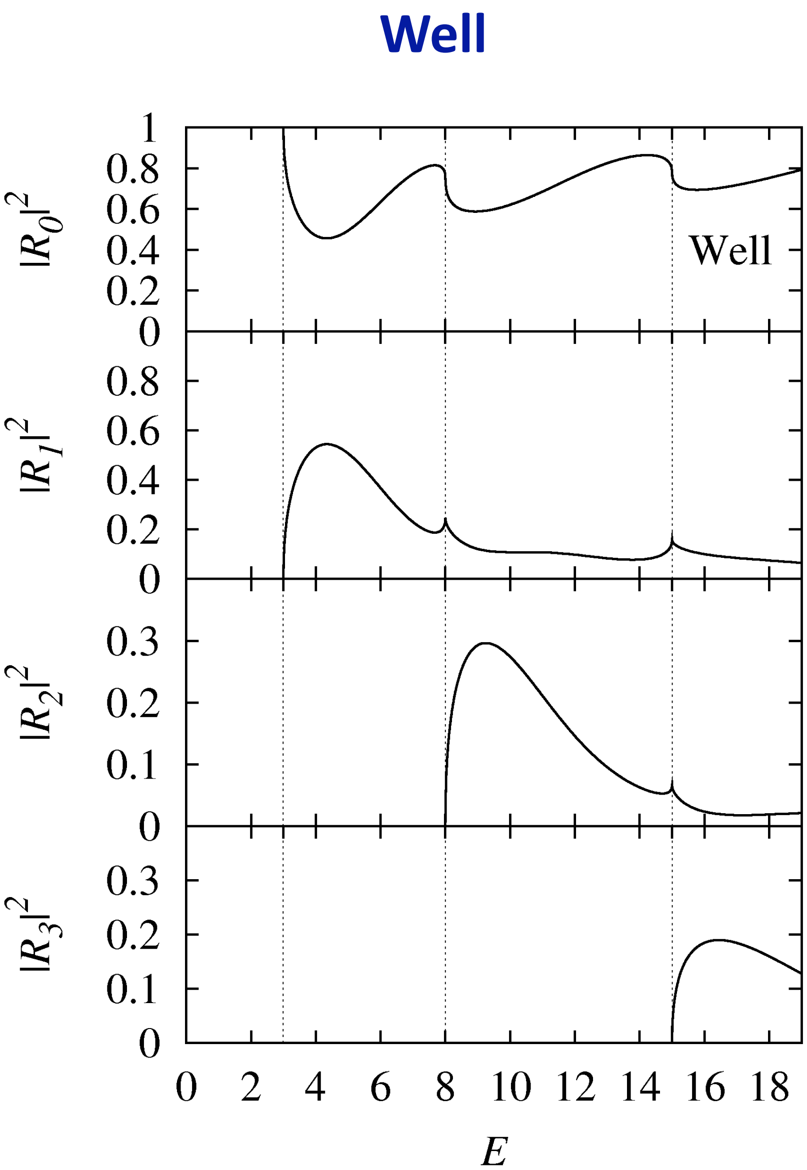
"HO" model

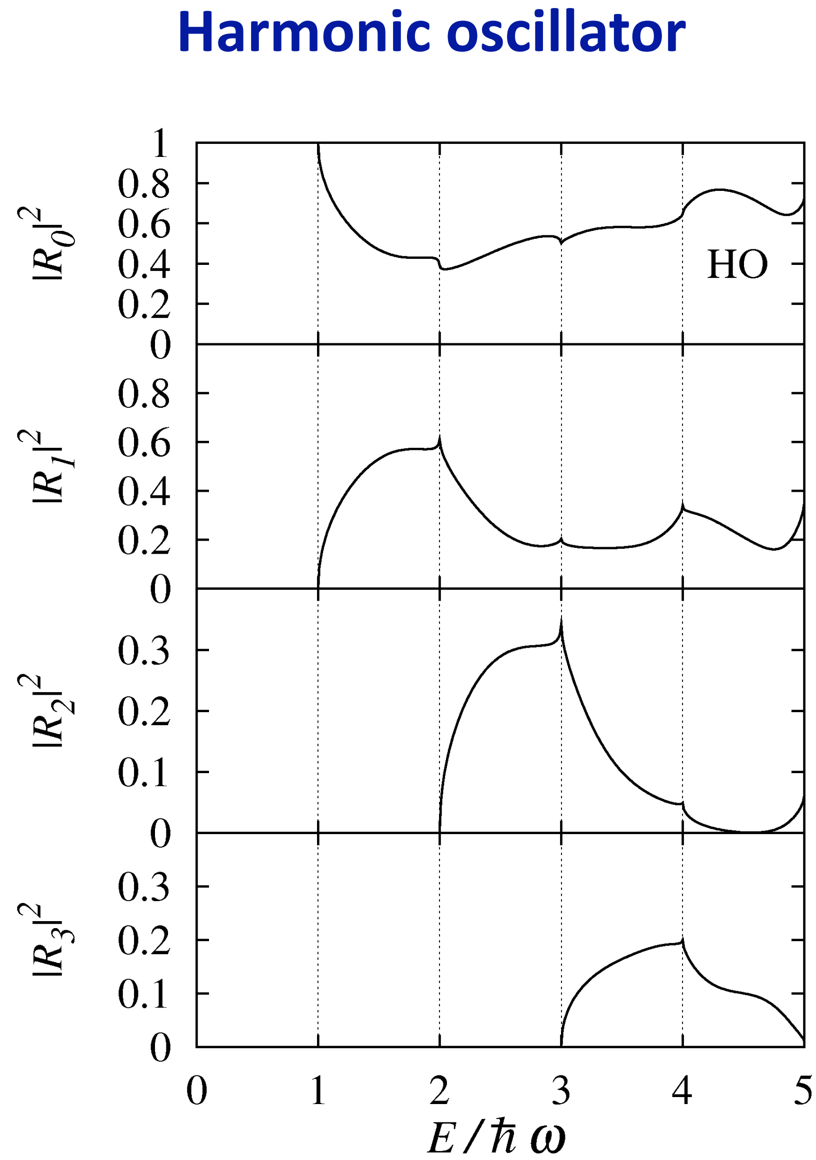
$$v(x) = m\omega^2 x^2/2$$

$$\psi_n(x) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} H_n(x) \exp\left(-\frac{x^2}{2}\right)$$

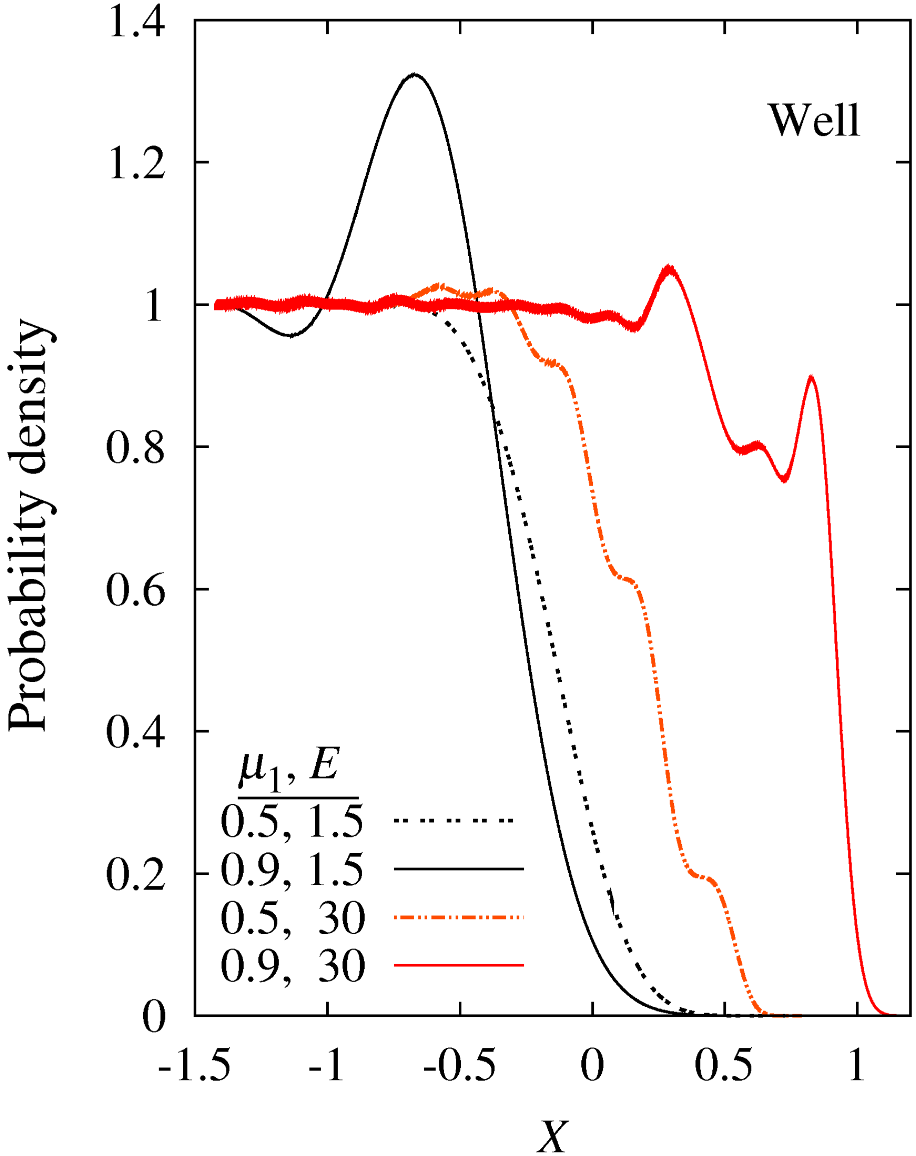


Results: scattering off an infinite wall





Center-of-mass penetration probability

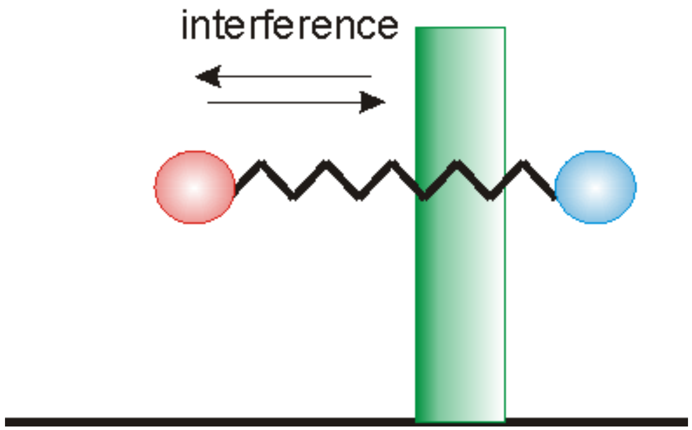


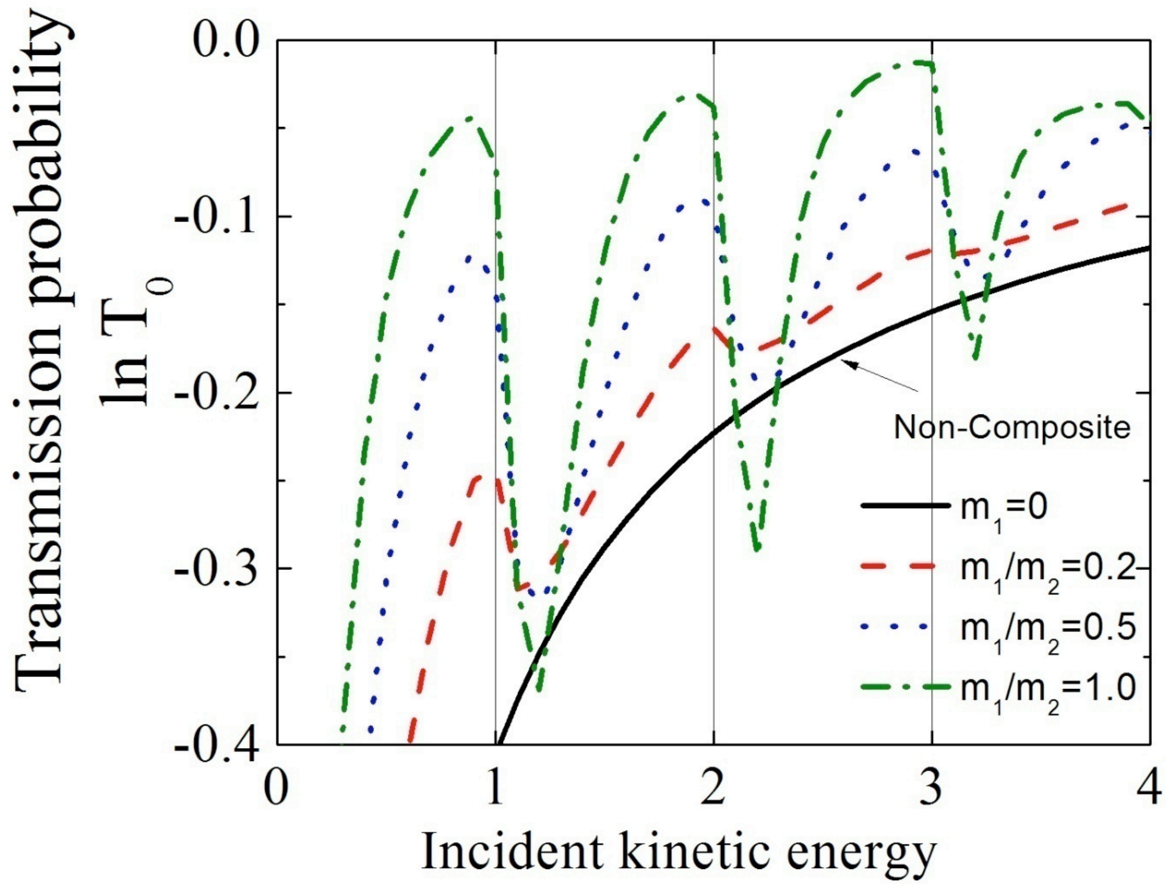
Wall is at X=0

Deep penetration for

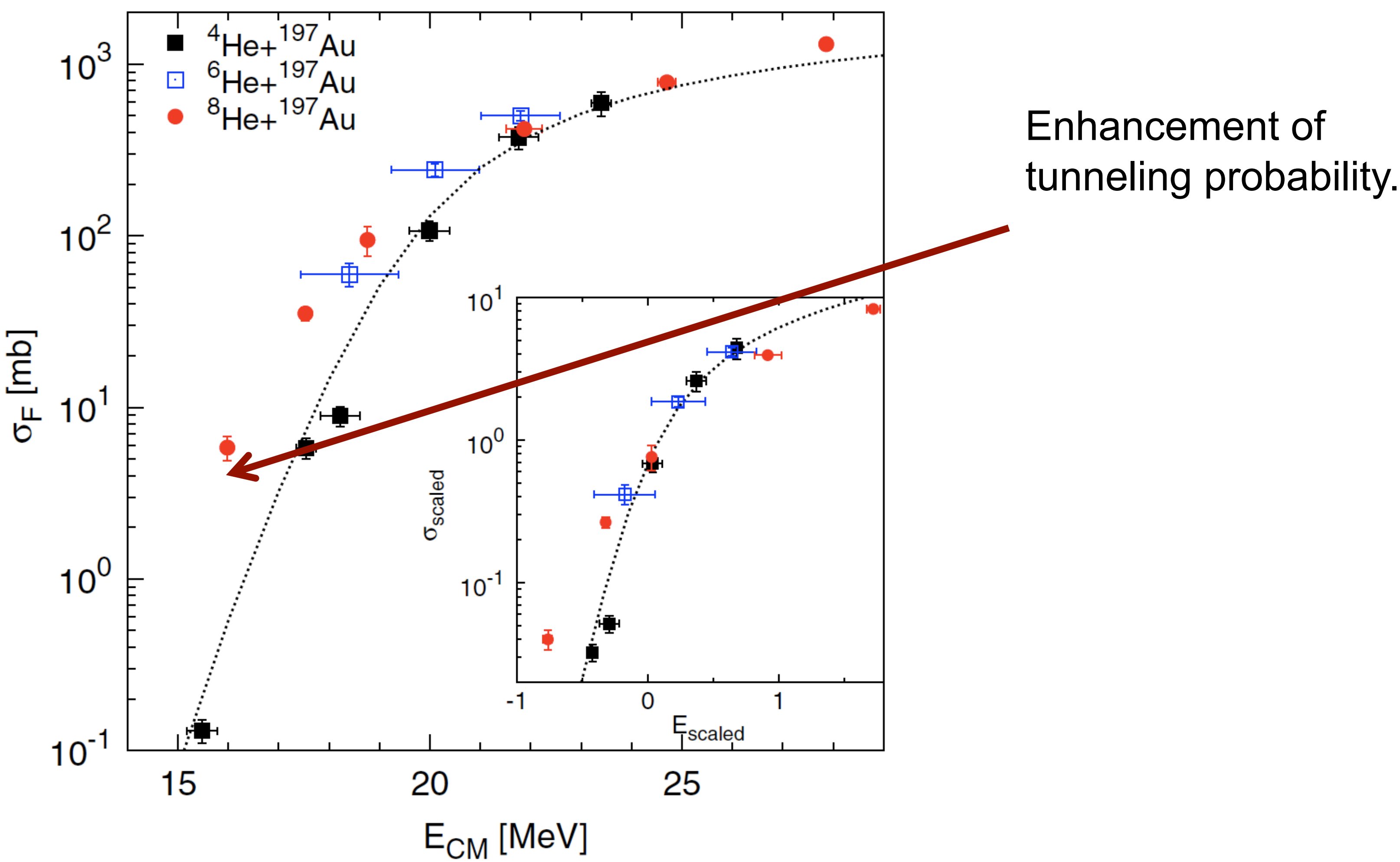
- -high energy
- -Massive non interactive particle

Resonant tunneling of composite objects





Enhanced tunneling probability for composite objects



A. Lemasson, et.al. PRL 103, 232701 (2009)