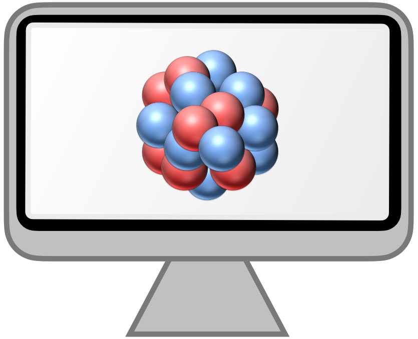
Nuclear Structure Theory

Ragnar Stroberg

Exotic Beam Summer School
July 10-14, 2023
Facility for Rare Isotope Beams
East Lansing, MI



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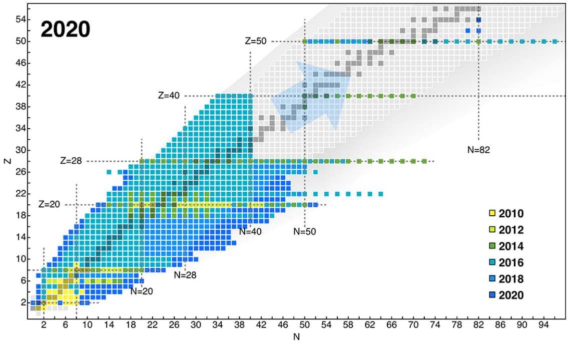
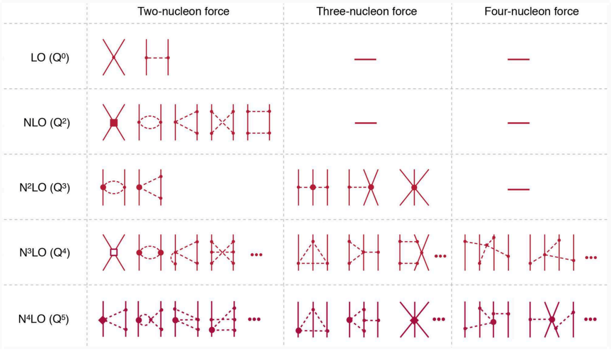
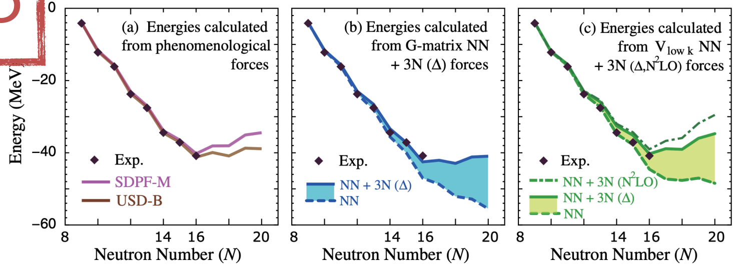
1


2020
Z=50
Z=40
Z=28
Z=20
N=82
N=40
N=50
N=28
N=20
2010
2012
2014
2016
2018
2020
Z
N

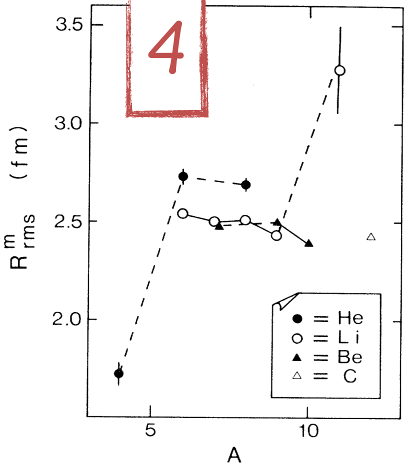
2


Two-nucleon force
Three-nucleon force
Four-nucleon force
LO (Q⁰)
NLO (Q²)
N²LO (Q³)
N³LO (Q⁴)
N⁴LO (Q⁵)

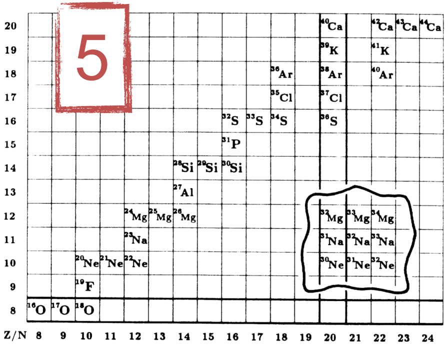
3


(a) Energies calculated from phenomenological forces
(b) Energies calculated from G-matrix NN + 3N (Δ) forces
(c) Energies calculated from V low k NN + 3N (Δ,N²LO) forces
Exp.
SDPF-M
USD-B
NN + 3N (Δ)
NN
NN + 3N (N²LO)
NN + 3N (Δ)
NN
Energy (MeV)
Neutron Number (N)

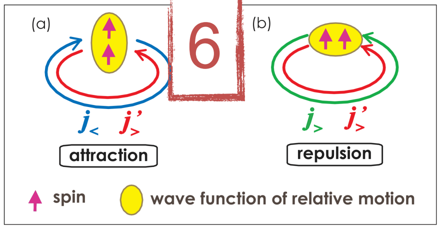
4


R m rms (fm)
A
= He
= Li
= Be
= C

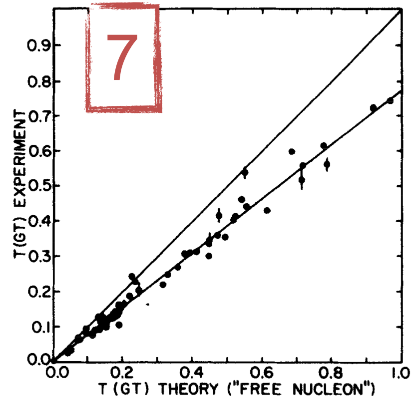
5


Z/N

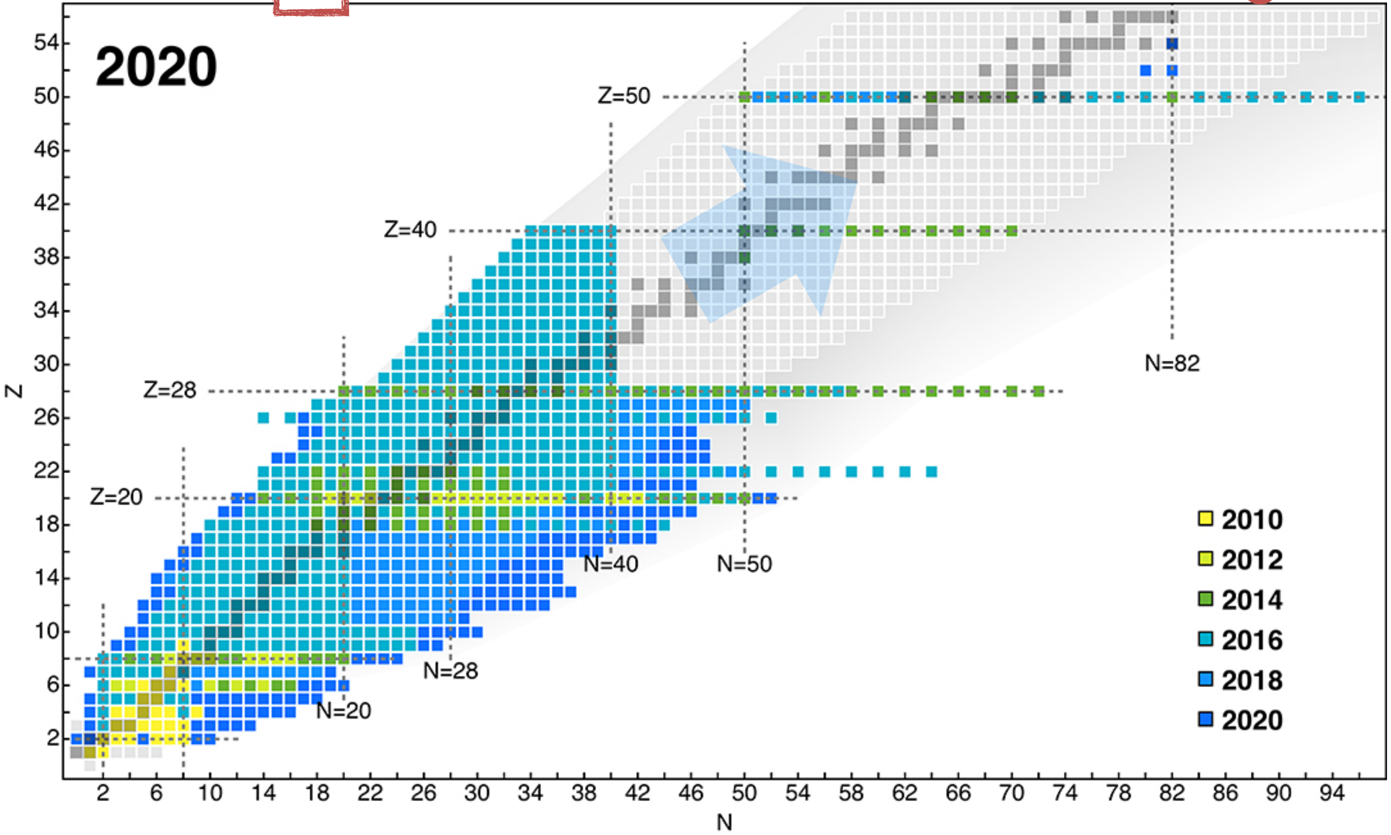
6


(a)
(b)
attraction
repulsion
spin
wave function of relative motion

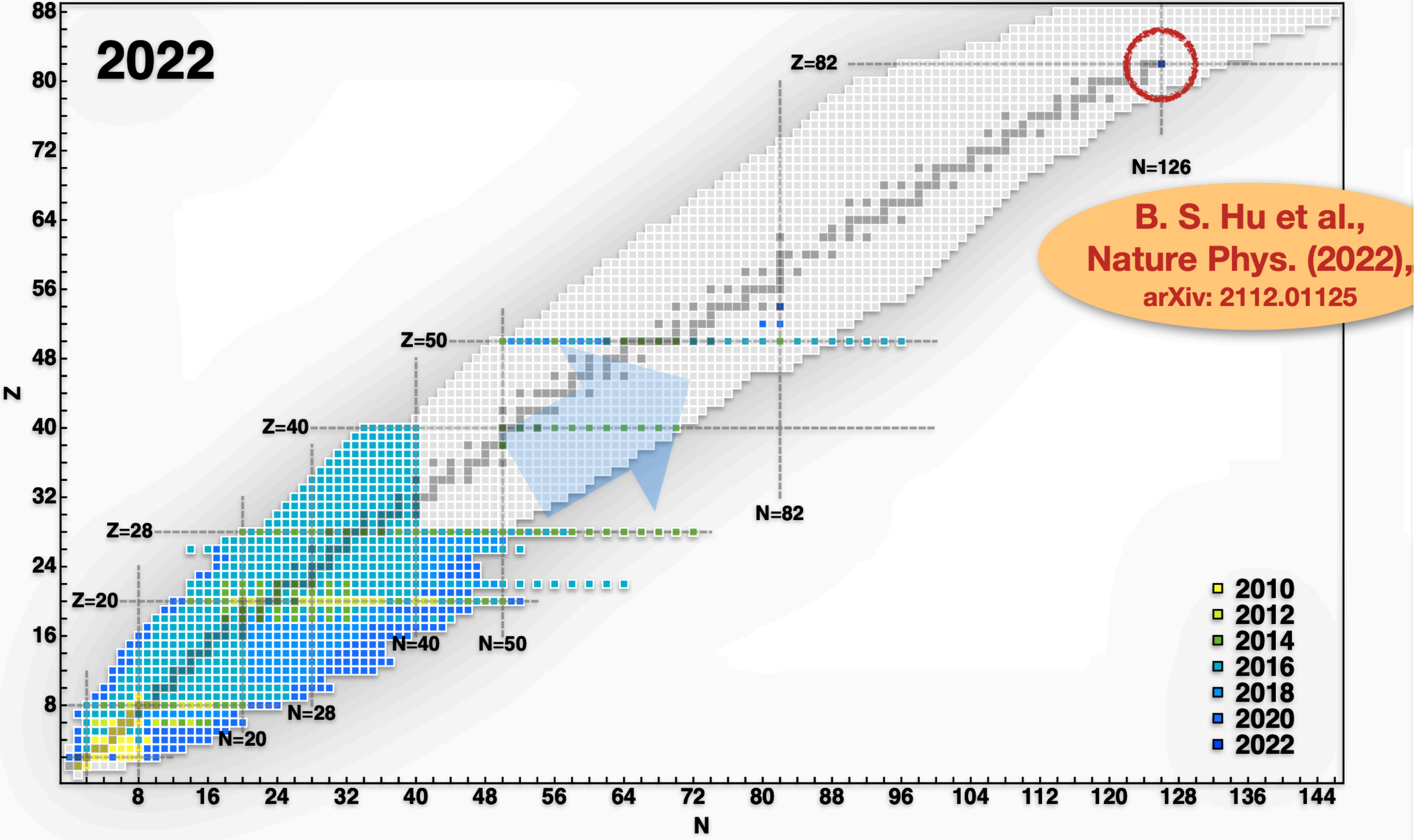
7


T(GT) EXPERIMENT
T (GT) THEORY ("FREE NUCLEON")

1 Ab initio nuclear theory

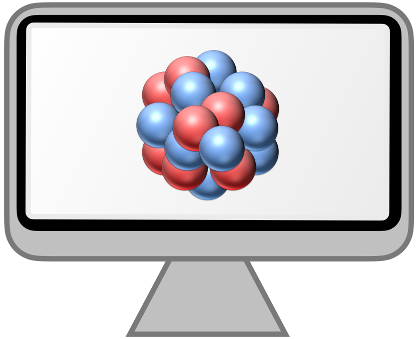


Hergert, Front. Phys. 8, 379 (2020)


2022
Z=82
N=126
Z=50
Z=40
Z=28
Z=20
N=82
N=40
N=50
N=28
N=20
B. S. Hu et al.,
Nature Phys. (2022),
arXiv: 2112.01125
2010
2012
2014
2016
2018
2020
2022
Z
N

Ab initio solution of the A-body problem

$$H|\Psi\rangle = E|\Psi\rangle$$



“Quasi-exact”

- No-core shell model
- Quantum Monte Carlo
- Lattice Effective Field Theory
- Hyperspherical hamonics

Polynomial-scaling

- Coupled cluster
- In-medium similarity renormalization group
- Self-consistent Green’s function
- Many-body perturbation theory
- …

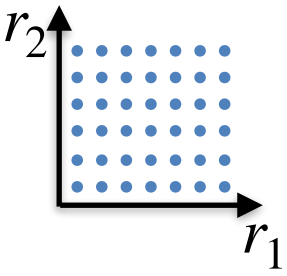
Variational Monte Carlo

Parameters $\vec{\alpha} = (\alpha_1, \alpha_2, \ldots)$

$$E_{\vec{\alpha}} = \frac{\langle \Psi_{\vec{\alpha}} | H | \Psi_{\vec{\alpha}} \rangle}{\langle \Psi_{\vec{\alpha}} | \Psi_{\vec{\alpha}} \rangle} \geq E_{\rm gs}$$

$$\iint \ldots \int dr_1 dr_2 \ldots dr_A \; \Psi^*_{\vec{\alpha}}(r_1, r_2, \ldots r_A) \Psi_{\vec{\alpha}}(r_1, r_2, \ldots r_A)$$

$3A \times 2$-dimensional integral $\Rightarrow (N_{\rm grid})^{6A}$ evaluations



Exponential scaling with A

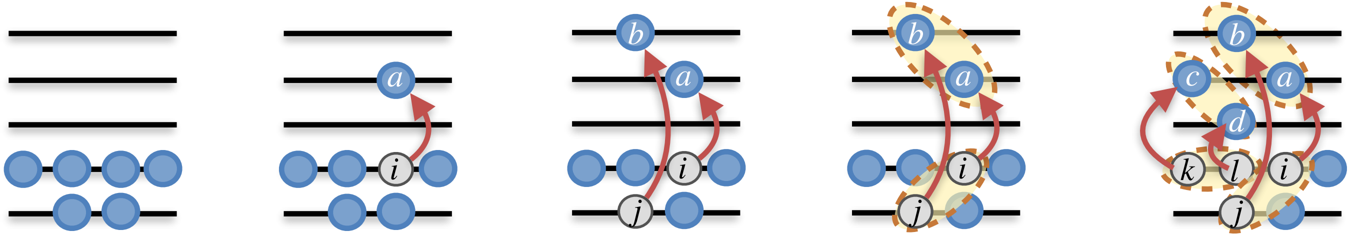
(Monte Carlo integration helps, but there's a "fermionic sign problem", and spin components still scale as 2^A.)

Coupled Cluster

$$|\Psi\rangle = e^{\hat{T}}|\Phi_0\rangle \qquad = (1 + \hat{T} + \frac{1}{2}\hat{T}^2 + \frac{1}{3!}\hat{T}^3 + \ldots)|\Phi_0\rangle$$

$$\hat{T} = \hat{T}_1 + \hat{T}_2 + \hat{T}_3 + \ldots$$

singles, doubles, triples



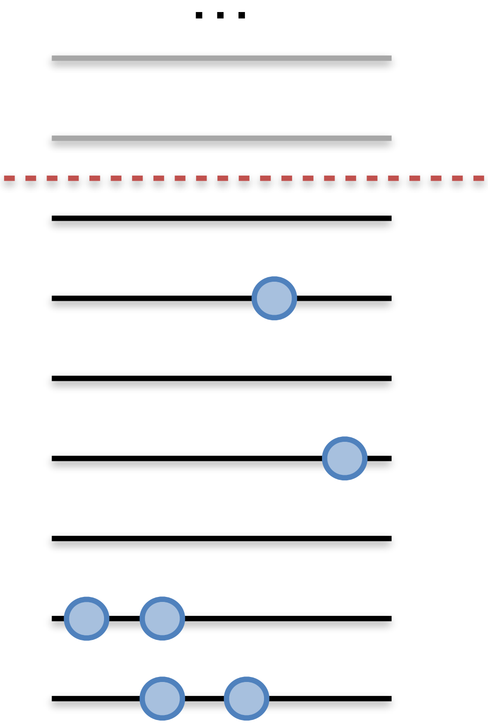
$|\Phi_0\rangle$ $\quad \hat{T}_1|\Phi_0\rangle$ $\quad (\hat{T}_1)^2|\Phi_0\rangle$ $\quad \hat{T}_2|\Phi_0\rangle$ $\quad (\hat{T}_2)^2|\Phi_0\rangle$

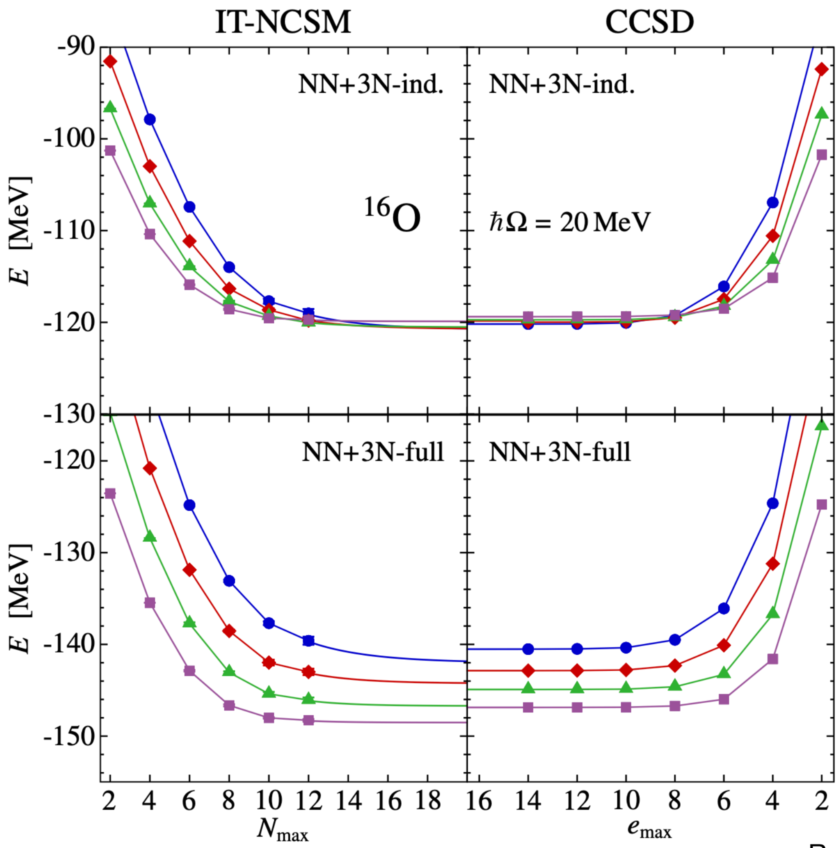
$\hat{T}_2$ specified by $(n_{\text{orb}})^4$ numbers t_{ij}^{ab}

$n_{\text{orb}} \sim A$

Polynomial scaling with A

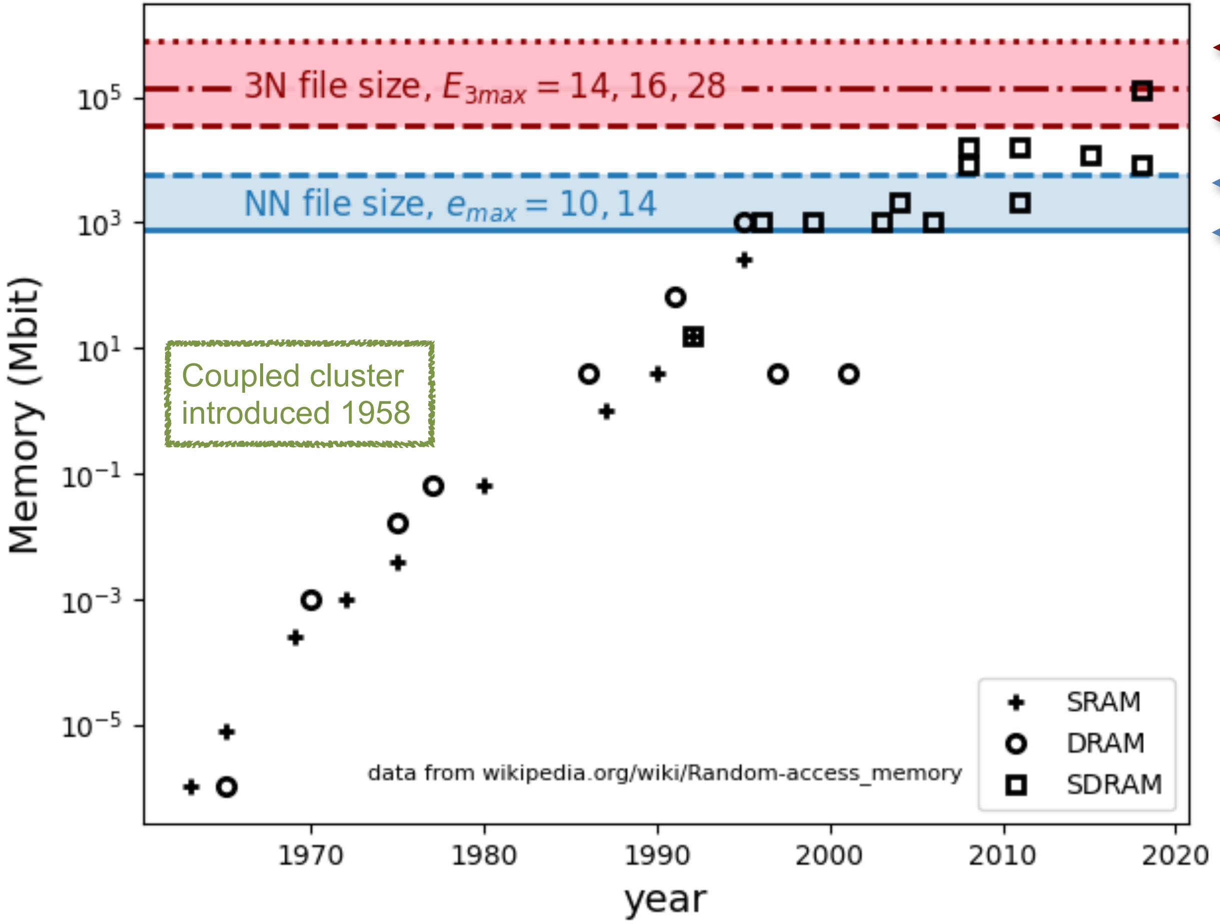
Why now?





Roth+ Phys. Rev. Lett. 109, 052501 (2012)

Why now?



Realistic computations of nuclei from chiral interactions

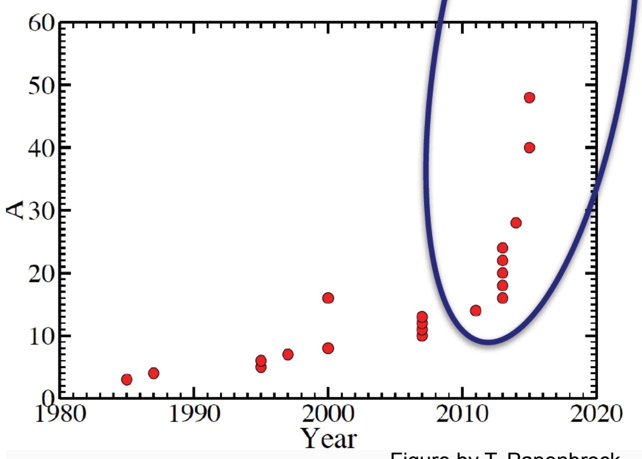
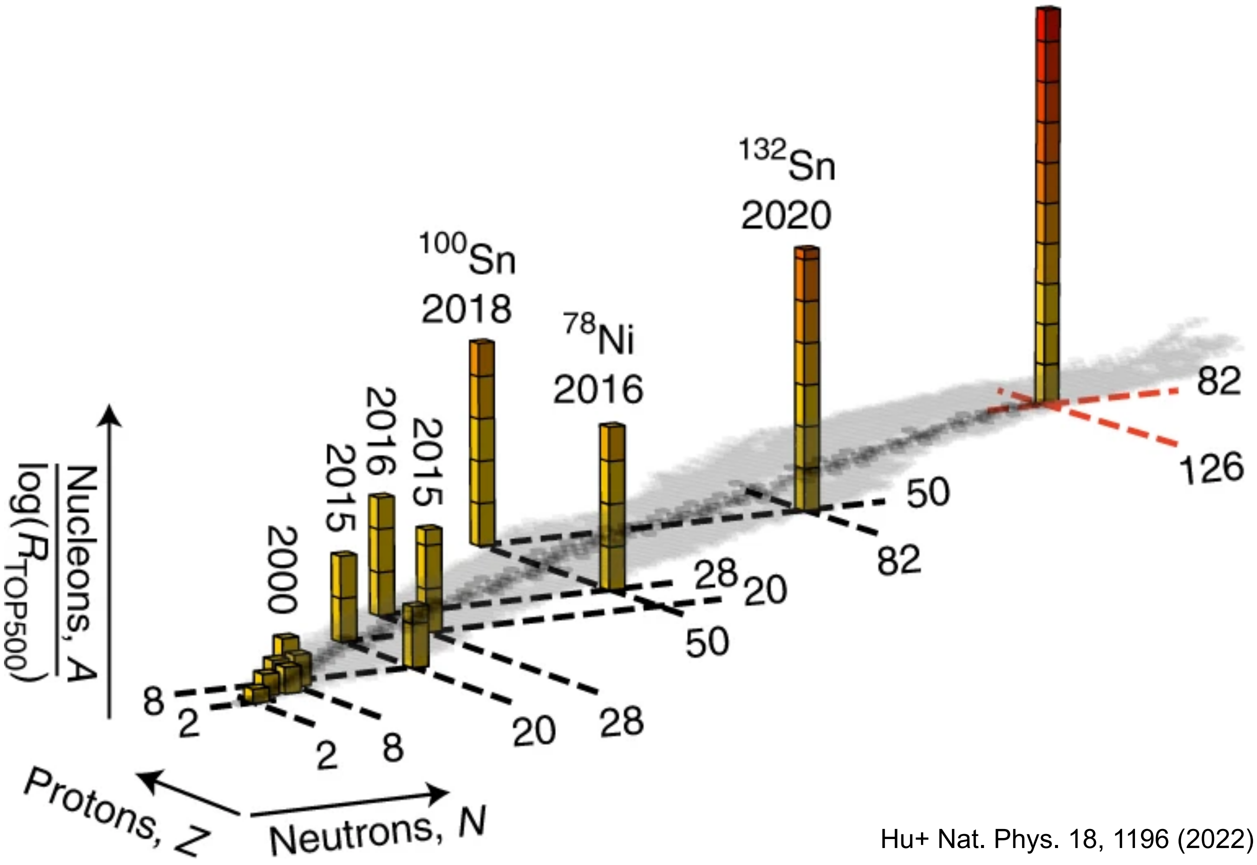
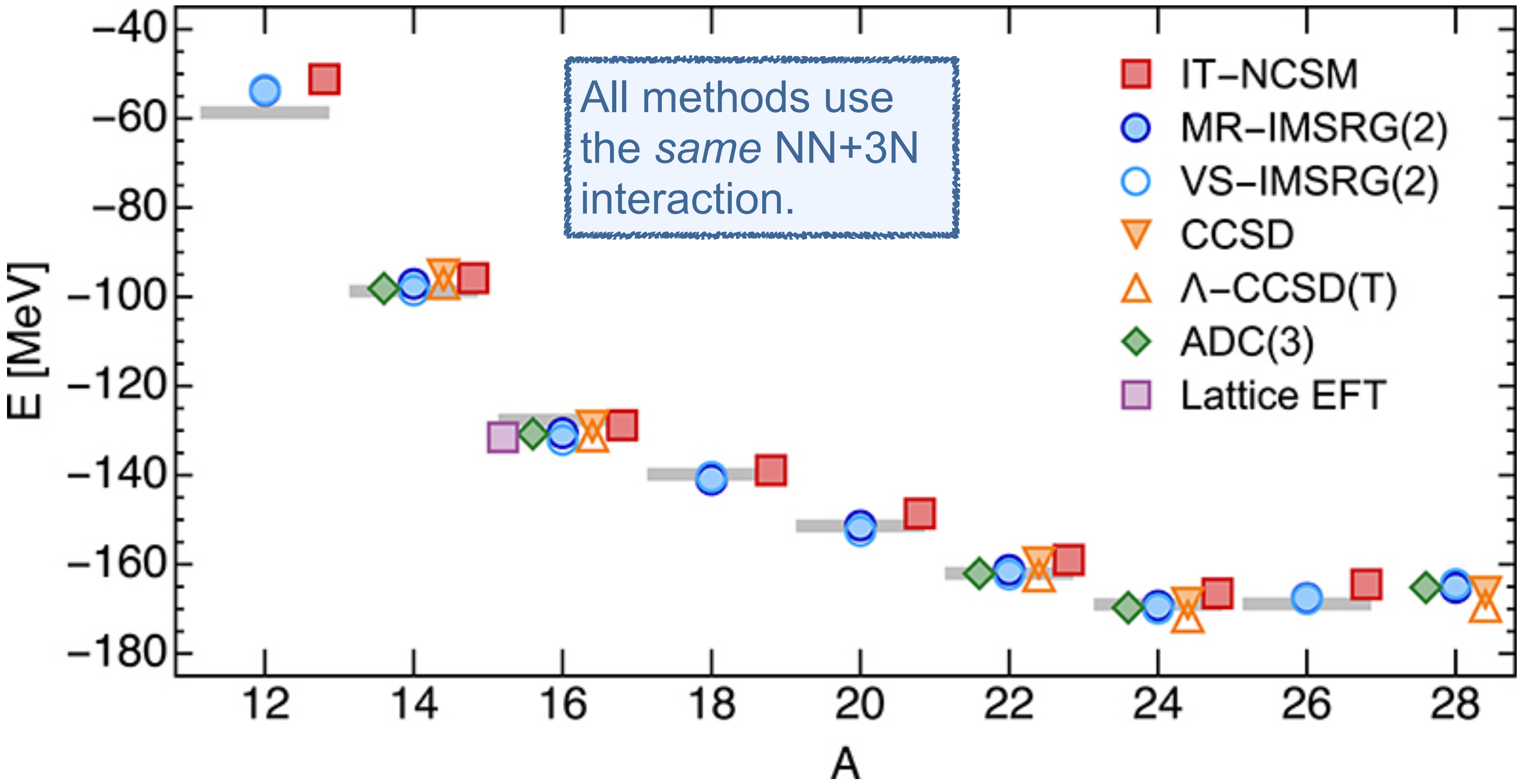


Figure by T. Papenbrock

Not Just Moore's Law

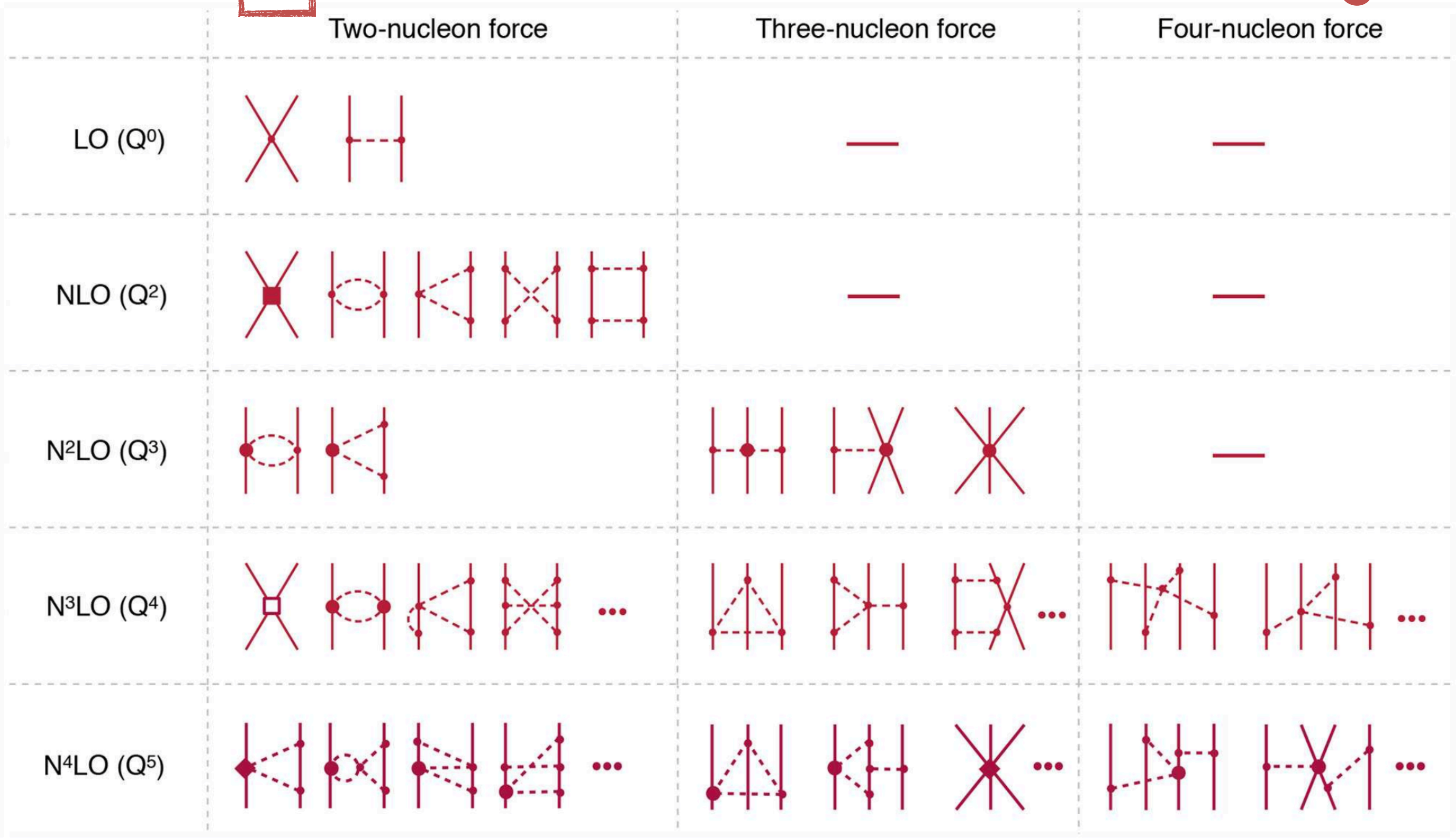


Hu+ Nat. Phys. 18, 1196 (2022)



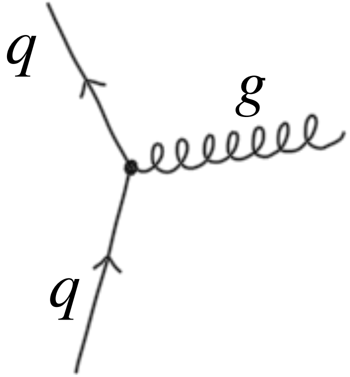
Hergert, Front. Phys. 8, 379 (2020)

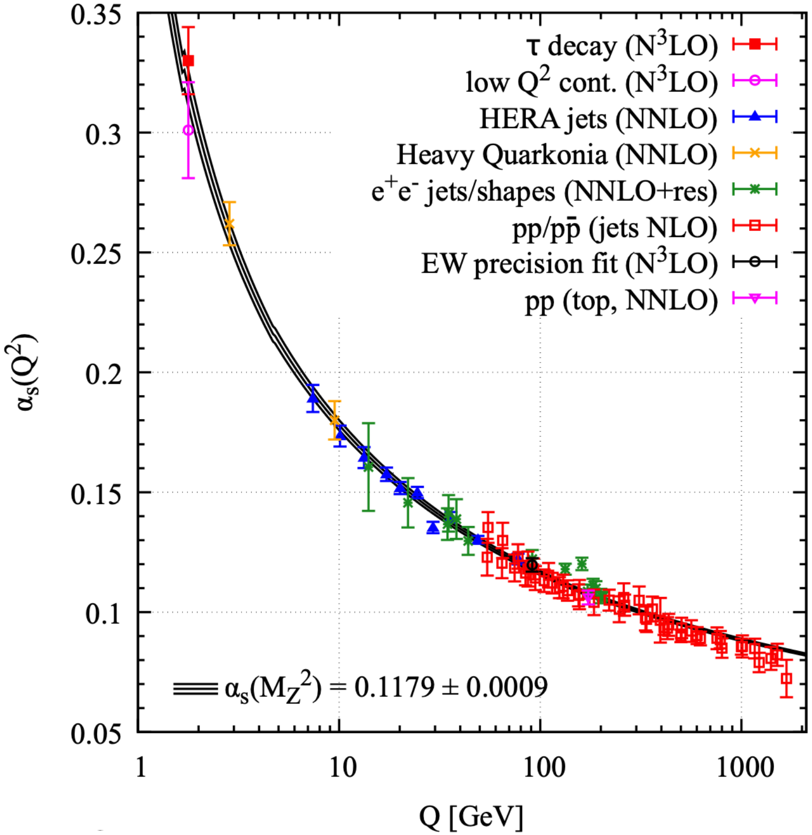
2 Chiral Effective Field Theory



Epelbaum+ Front. Phys. 8:98 (2020)

Quantum Chromodynamics (QCD)





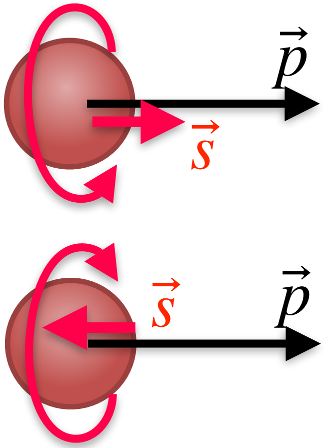
PDG Prog. Theo. Exp. Phys. 083C01 (2022)

perturbation theory breaks down below $\Lambda_{QCD} \sim 1$ GeV

At low energy, switch from quarks and gluons to nucleons and pions. Maintain connection to QCD via **chiral symmetry**.

Chirality and helicity

Helicity: $h = \vec{\sigma} \cdot \vec{p}$



$h = +1$ (right)

$h = -1$ (left)

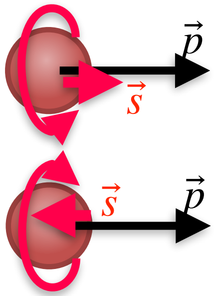
Chirality: Equivalent to helicity for $m = 0$. For $m \neq 0$,

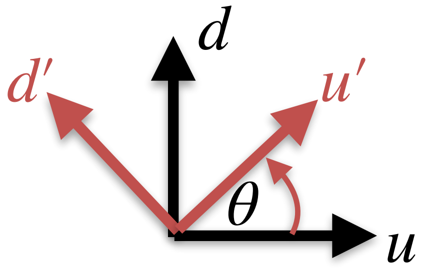
- Chirality is Lorentz invariant, but can change in time.
- Helicity is conserved, but is frame-dependent.

Chiral symmetry

Massless QCD is invariant under the change of basis

$$q \to q' = e^{i\vec{\theta}\cdot\vec{\tau}} q \qquad q = \begin{pmatrix} u \\ d \end{pmatrix}$$



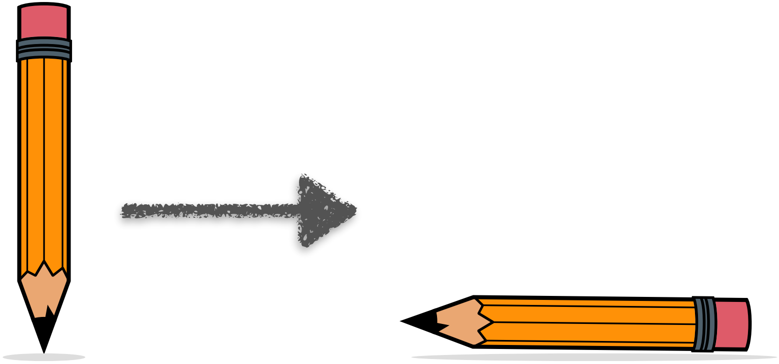


In fact, QCD is even invariant under different rotations for left- and right-chiral quarks.

$$q_L \to q_L' = e^{i\vec{\theta}_L\cdot\vec{\tau}} q_L$$

$$q_R \to q_R' = e^{i\vec{\theta}_R\cdot\vec{\tau}} q_R$$

Spontaneous symmetry breaking

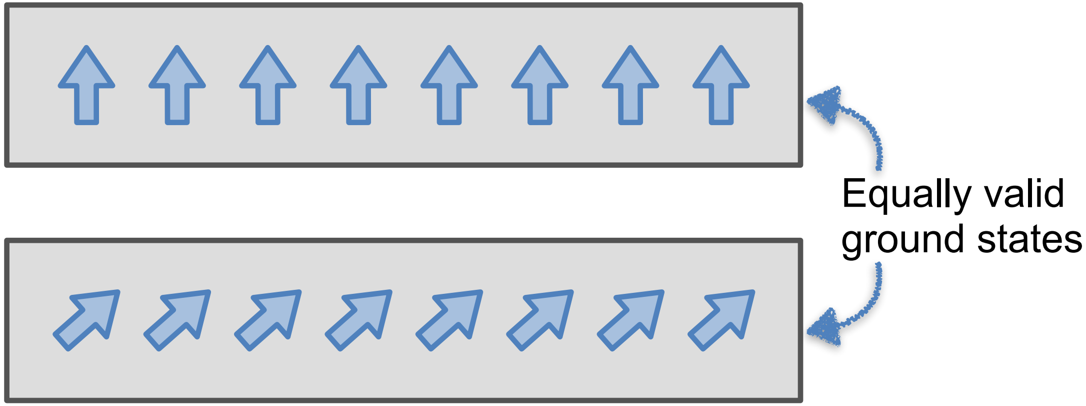


A symmetry of the Lagrangian/Hamiltonian is not a symmetry of the ground state.

Spontaneous symmetry breaking

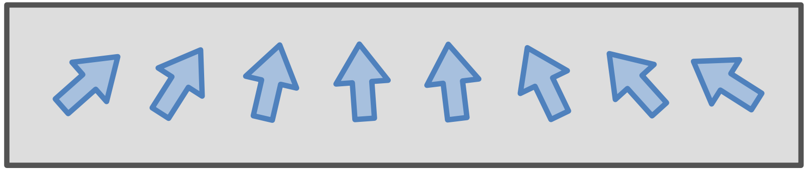
Analogy: Ferromagnet

The unmagnetized state has the rotational symmetry of the Hamiltonian, but at low temperature, the energy is reduced by a net magnetization pointing in *some direction*. But which?



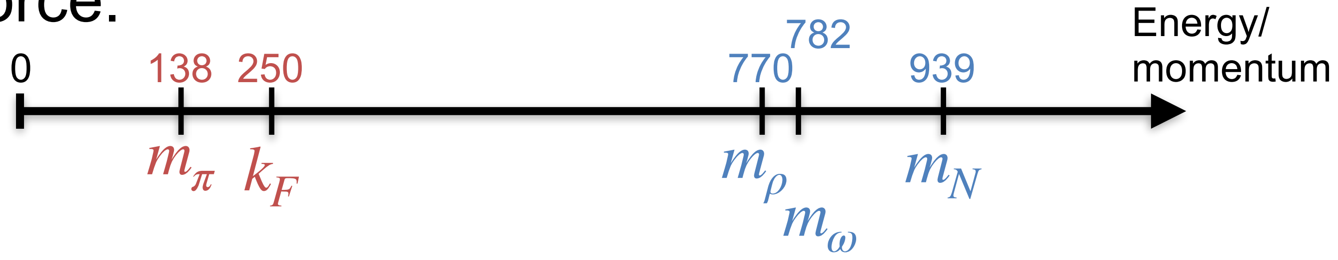
In QCD, the symmetry $\theta_A = \theta_R - \theta_L$ is spontaneously broken. The analog of magnetization is the "chiral condensate" $\langle \bar{q} q \rangle$. The low-energy excitation is the pion.
(The combination $\theta_V = \theta_R + \theta_L$ remains, and gives us isospin.)

Low-energy excitation: "magnon"

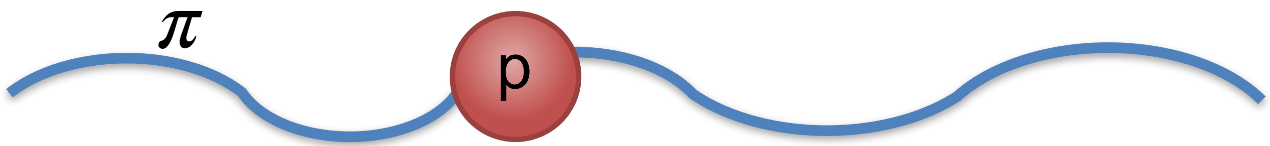


Two important consequences of spontaneously broken chiral symmetry

1. The pion is light, so it dominates the long-range part of the NN force.



2. The pion coupling always comes with a derivative $\partial_\mu \sim p_\mu$, so at low momenta it is weakly coupled.



Power counting: Naive Dimensional Analysis

Lagrangian has mass dimension $[\mathscr{L}] = M^4$

Building blocks:
- Nucleon field: $[N] = M^{3/2}$
- Pion field: $[\pi] = M$
- Derivative: $[\partial] = M$

Dimensonful parameters:
- pion mass $m_\pi \approx 140$ MeV
- pion decay const. $f_\pi \approx 92$ MeV
- $M_N \sim m_\rho \sim 4\pi f_\pi \equiv \Lambda_\chi \sim 700$ MeV

$$\mathscr{L} \sim c_{lmn} f_\pi^2 \Lambda_\chi^2 \left(\frac{\bar{N}N}{f_\pi^2 \Lambda_\chi} \right)^l \left(\frac{\pi}{f_\pi} \right)^m \left(\frac{\partial, m_\pi}{\Lambda_\chi} \right)^n$$

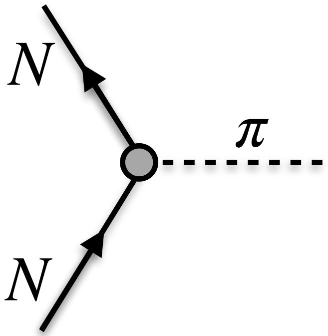
Low Energy Constant, $\mathcal{O}(1)$

Manohar & Georgi, Nuc. Phys. B 234,189 (1984), Friar, Few Bod. Syst. 22, 161 (1997)

Naive Dimensional Analysis

$$\mathscr{L} \sim c_{lmn} f_\pi^2 \Lambda_\chi^2 \left(\frac{\bar{N}N}{f_\pi^2 \Lambda_\chi}\right)^l \left(\frac{\pi}{f_\pi}\right)^m \left(\frac{\partial, m_\pi}{\Lambda_\chi}\right)^n$$

Example: πNN vertex, $l = m = n = 1.$



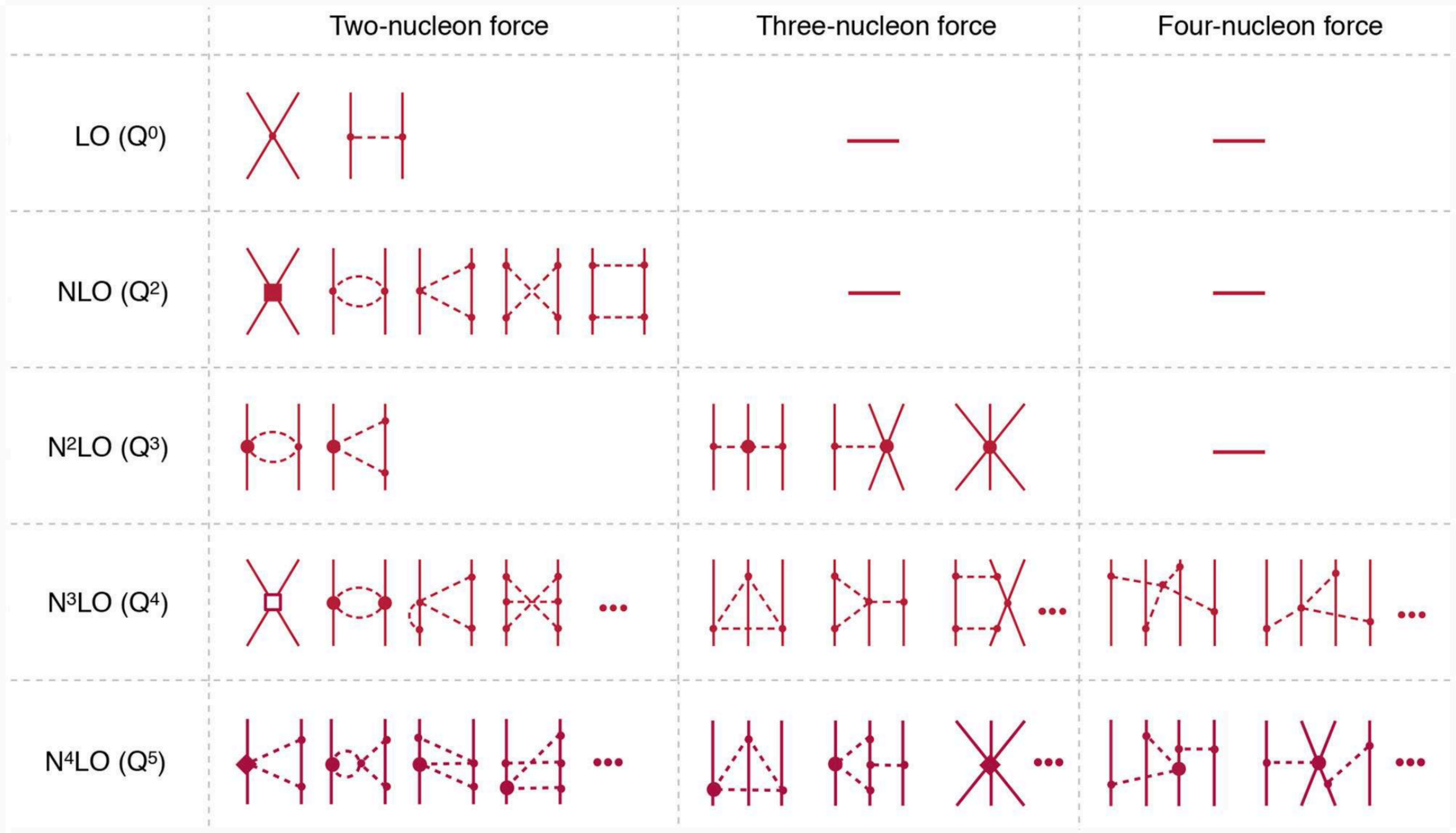
$$\mathscr{L}_{\pi NN} = \frac{c_{\pi NN}}{f_\pi} \left(\bar{N}\partial N\right) \pi$$

$$\mathscr{L}_{\pi NN} = \frac{g_A}{2f_\pi} \bar{N}\sigma \cdot \partial\tau N\pi$$

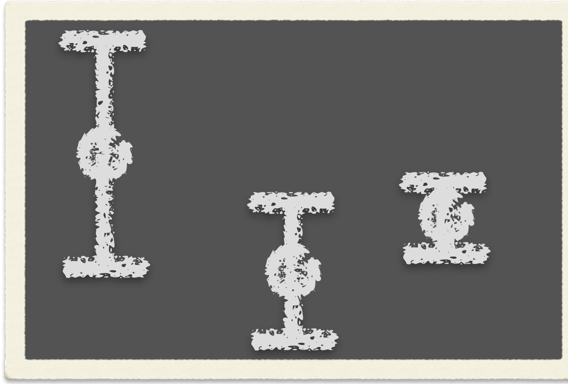
(Goldberger-Treiman)

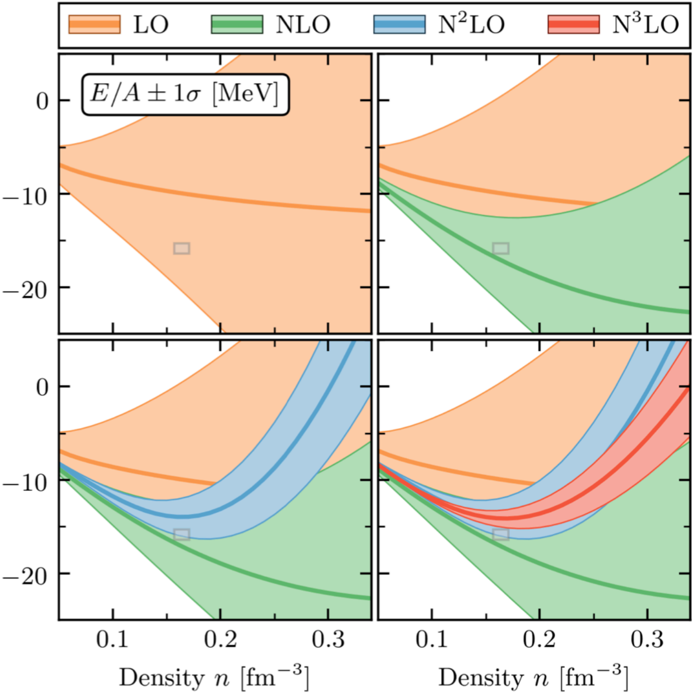
$$c_{\pi NN} = \frac{g_A}{2} = \frac{1.27}{2} = \mathscr{O}(1)$$

✓


Two-nucleon force
Three-nucleon force
Four-nucleon force
LO (Q^0)
NLO (Q^2)
N^{2}LO (Q^3)
N^{3}LO (Q^4)
N^{4}LO (Q^5)

If you need to fit the low energy constants, isn't this just an elaborate form of phenomenology? What's the point??

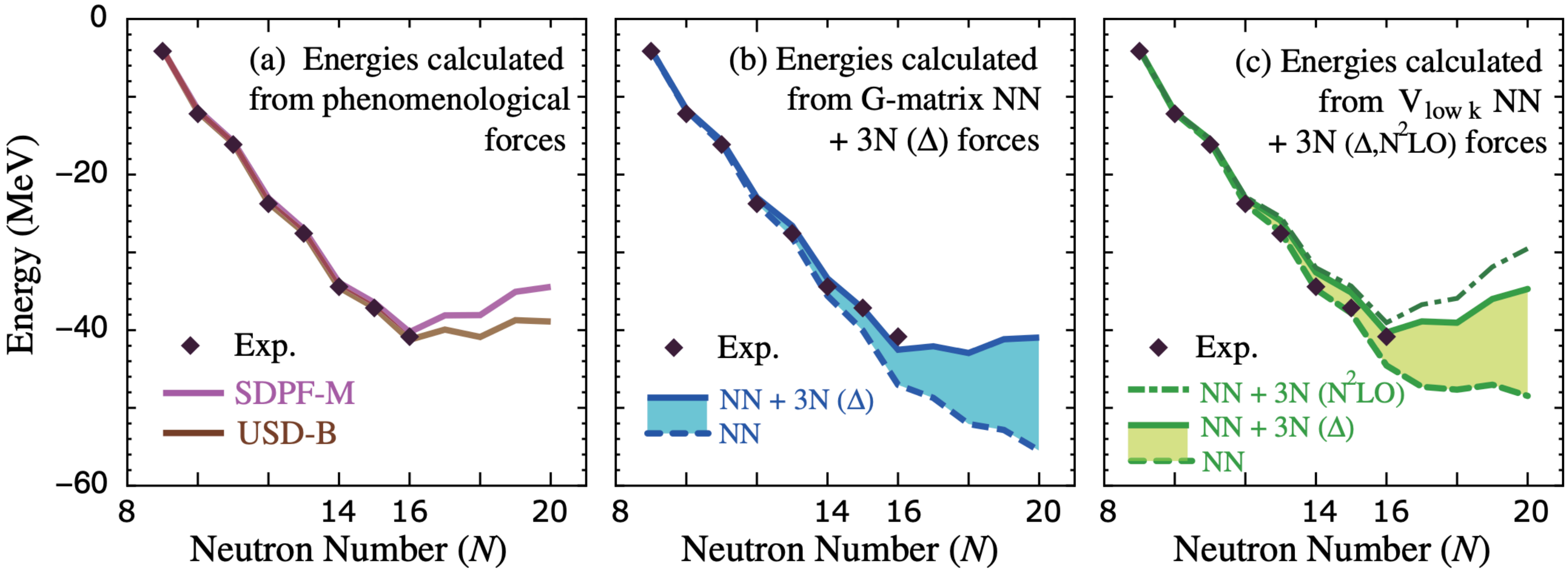




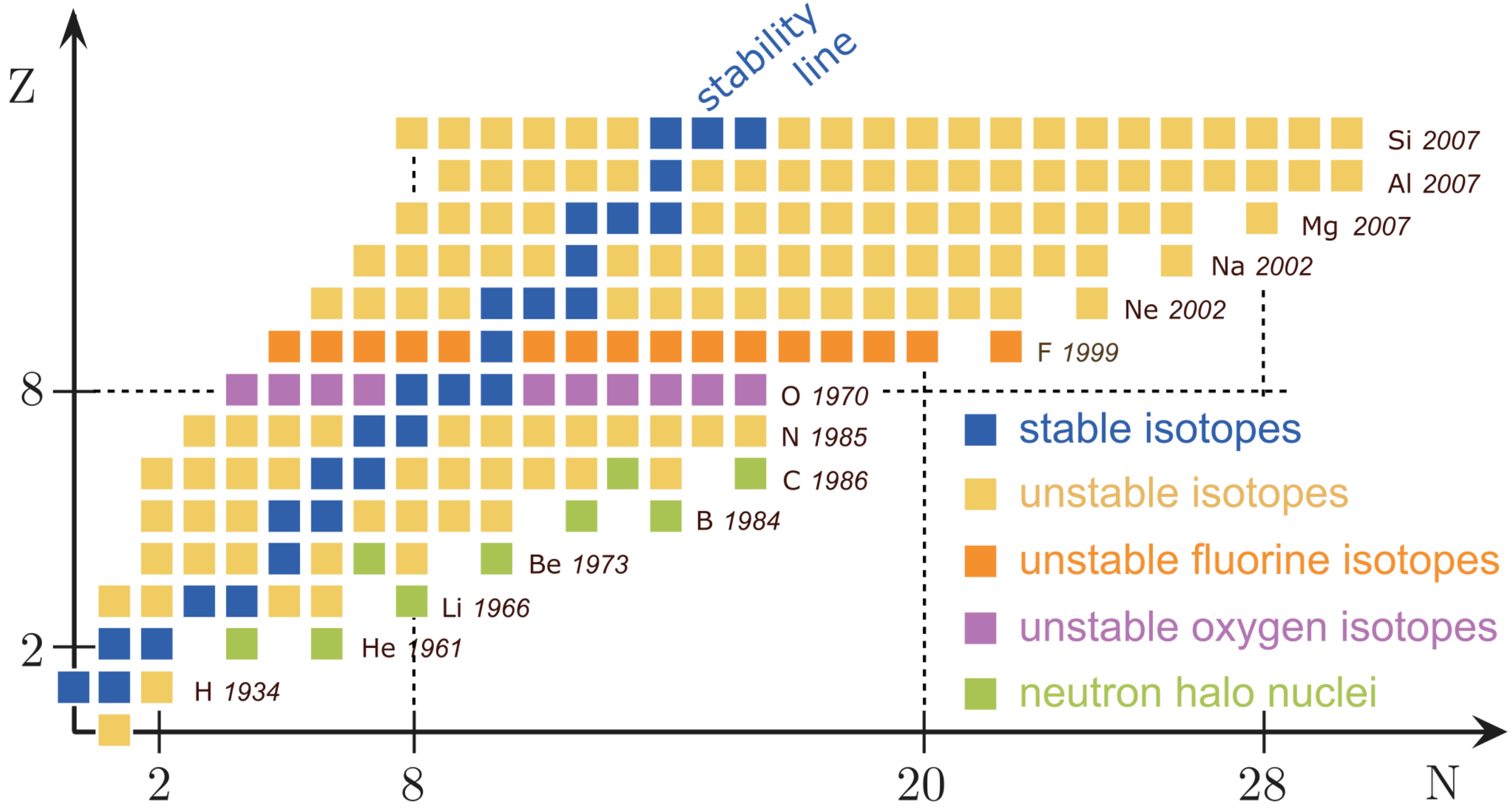
Drischler+ Phys. Rev. C 102 054315 (2020)

The point is that, with the assumption of ***naturalness***, i.e. $c_{lmn} \sim \mathcal{O}(1)$, we can estimate the size of omitted terms, so we can make a ***theory error bar***. We include all possible terms, so we don't have to worry that we left something out.

3 3N forces and the oxygen drip line



Otsuka+ Phys. Rev. Lett. 105, 032501 (2010)



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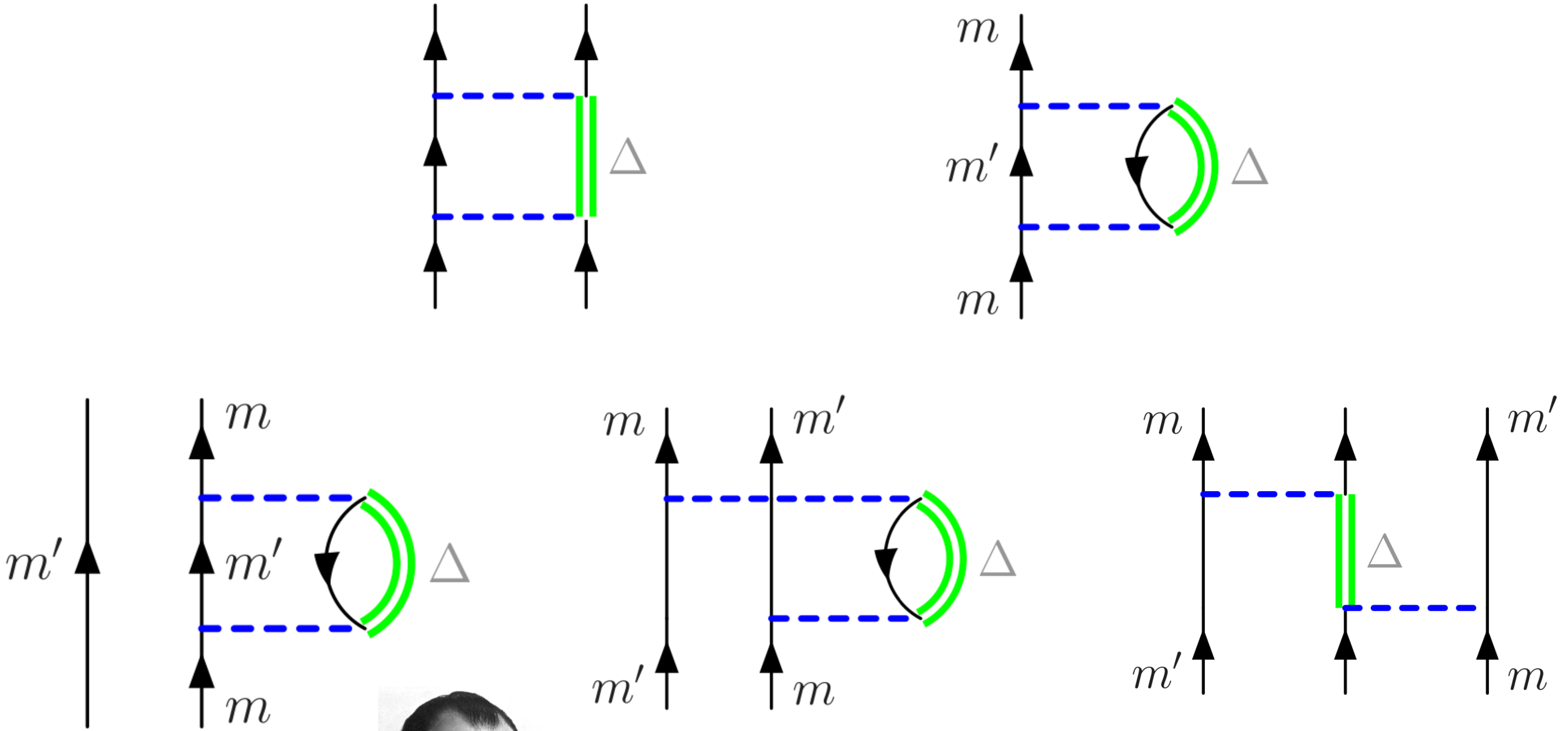
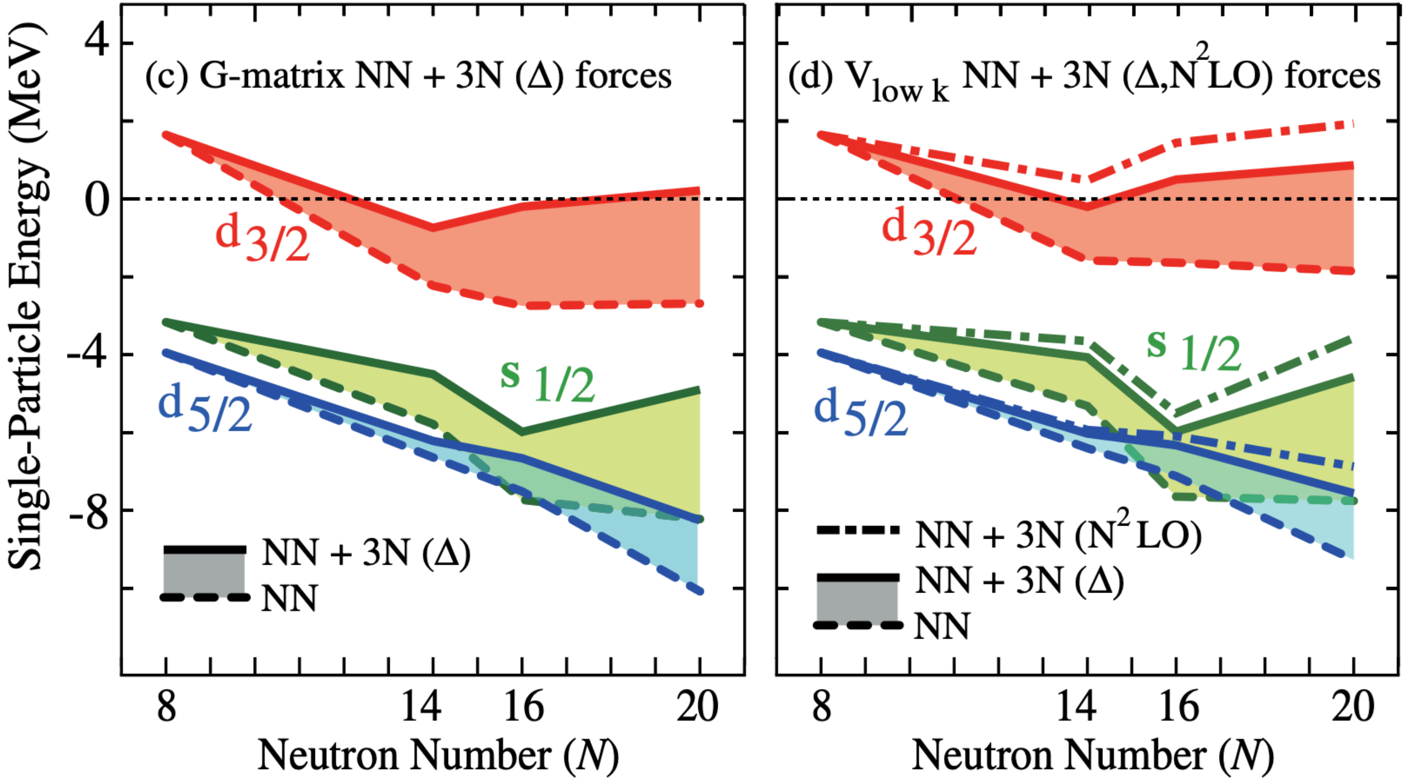
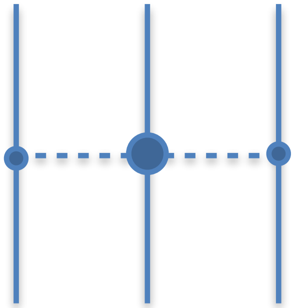


Otsuka+ Phys. Rev. Lett. 105, 032501 (2010)



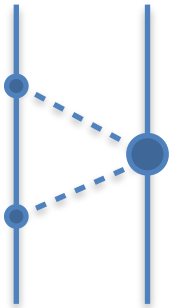
Otsuka+ Phys. Rev. Lett. 105, 032501 (2010)

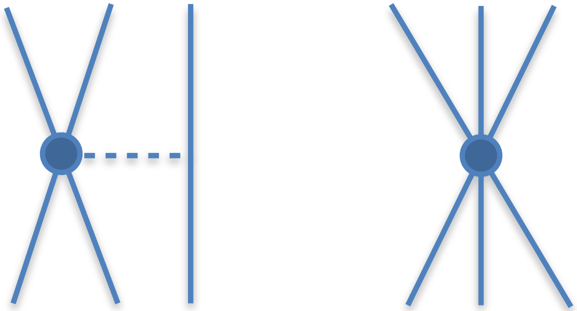
3N forces from chiral EFT at N²LO



c_1, c_3, c_4

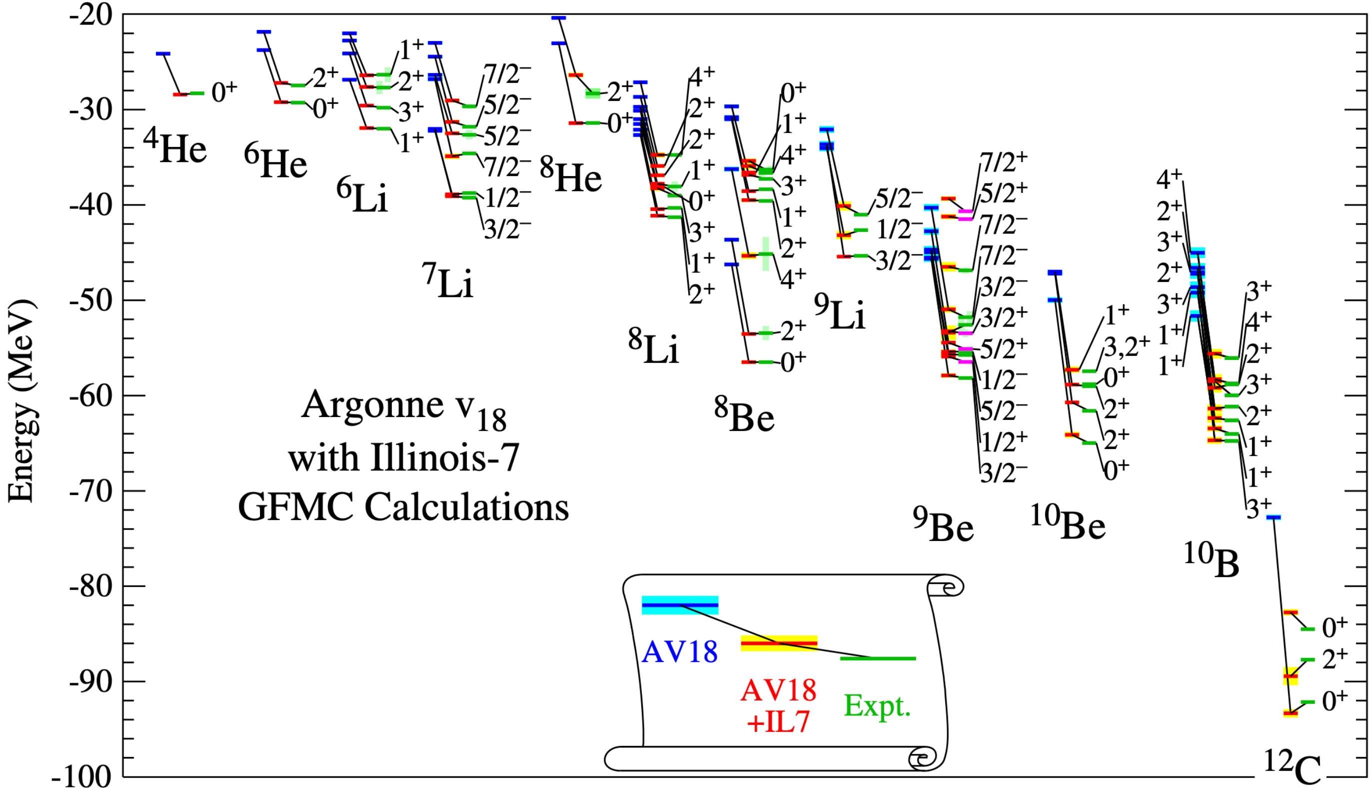
Fixed by the πN or NN sectors





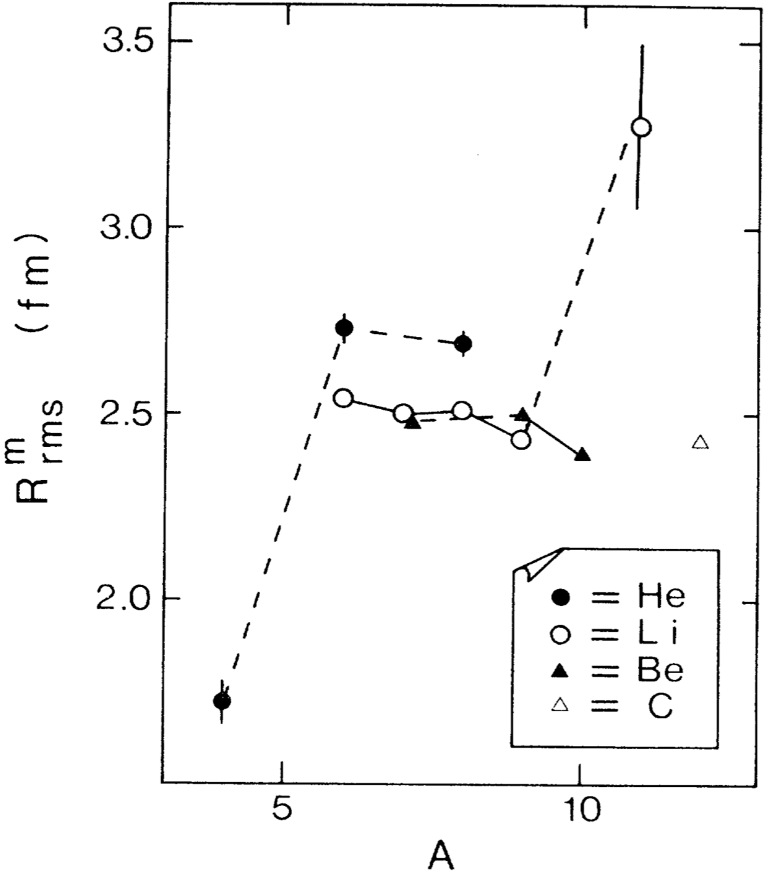
c_D c_E

No contribution for 3-neutron system

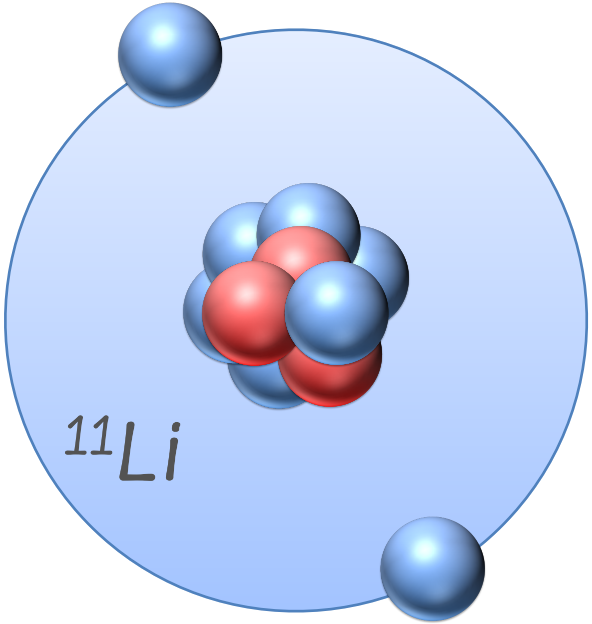


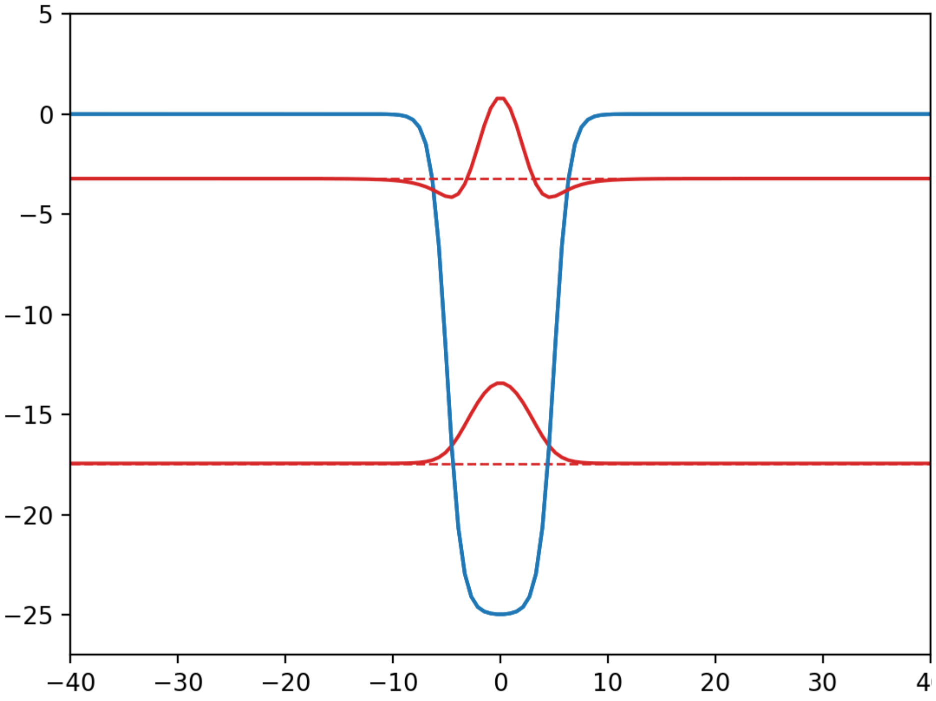
Carlson+, Rev. Mod. Phys. 87, 1067 (2015)

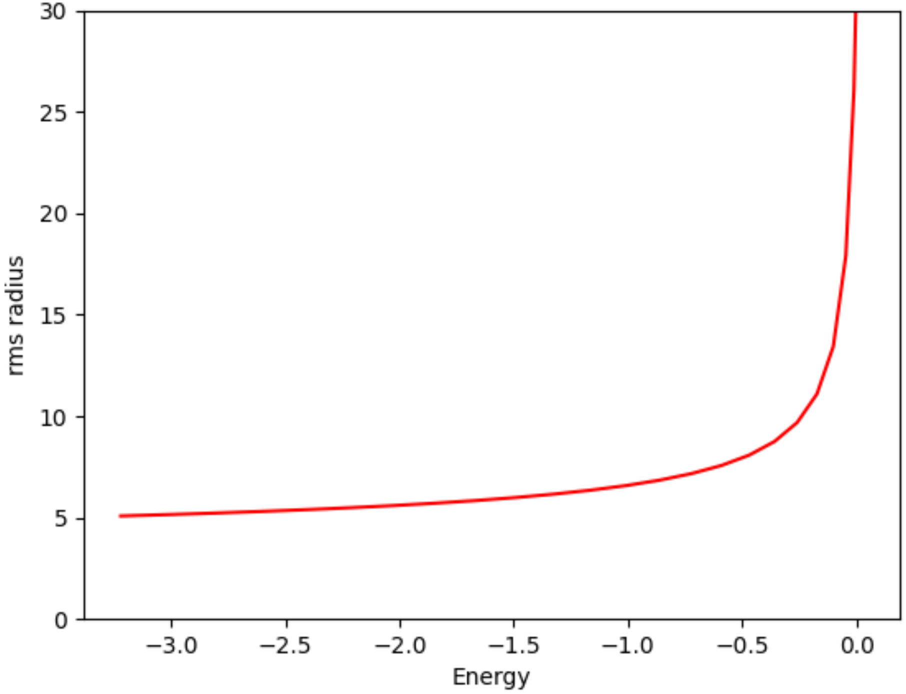
4 *Halo nuclei*



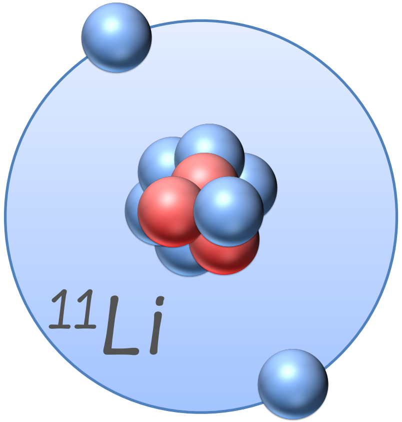
Tanihata+ Phys. Rev. Lett. 55 2676 (1985)

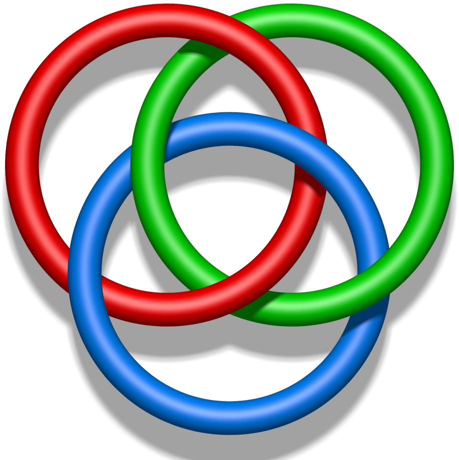



5
0
−5
−10
−15
−20
−25
−40
−30
−20
−10
0
10
20
30

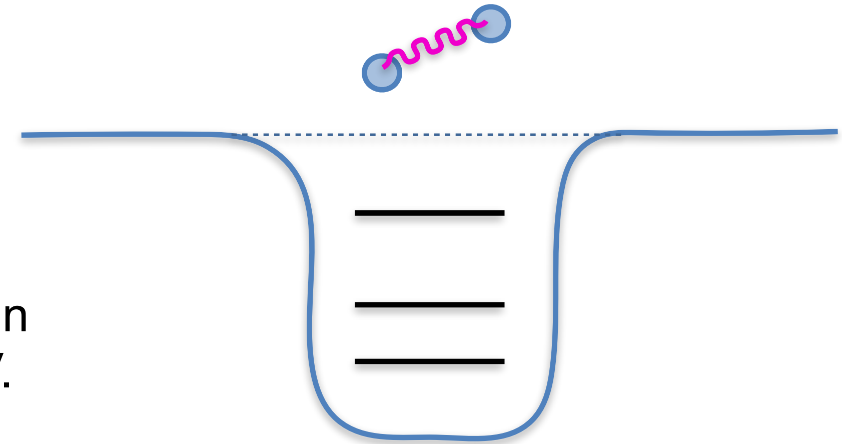

rms radius
Energy
30
25
20
15
10
5
0
−3.0
−2.5
−2.0
−1.5
−1.0
−0.5
0.0

A Borromean Nucleus



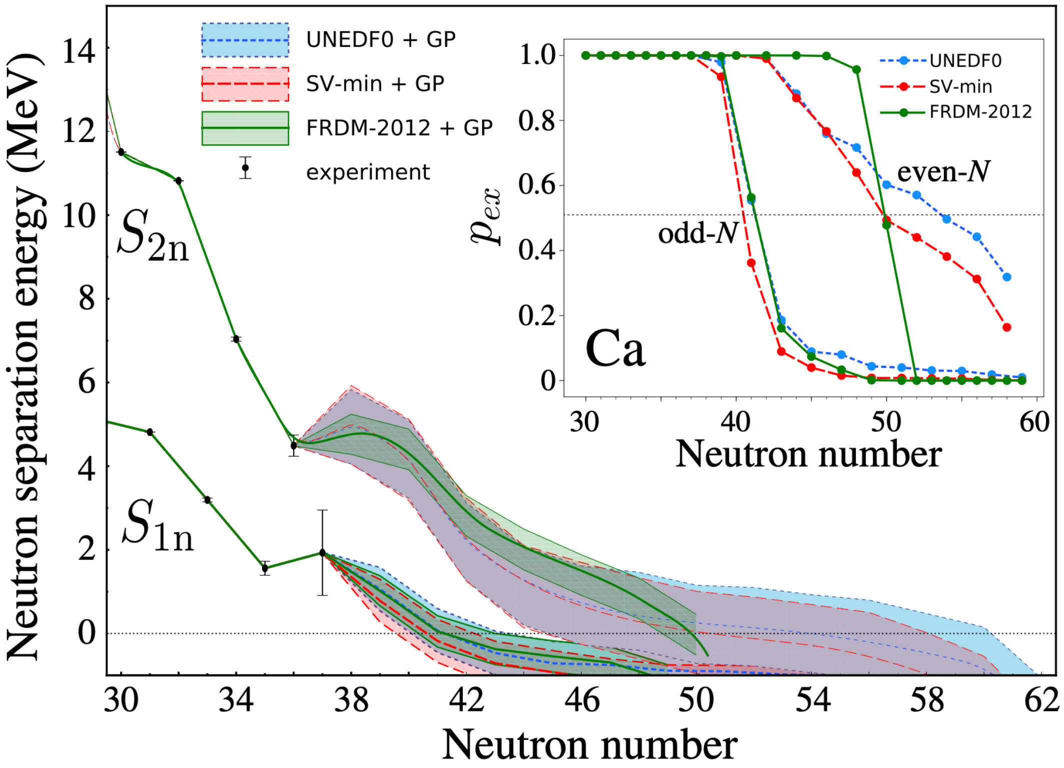


wikipedia.org/wiki/Borromean_rings

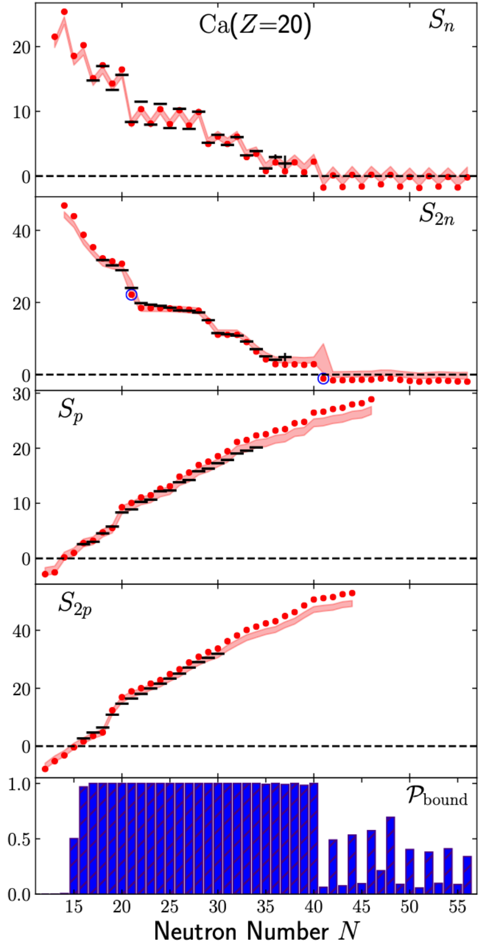


^{10}Li is unbound to 1n emission by 26 keV.

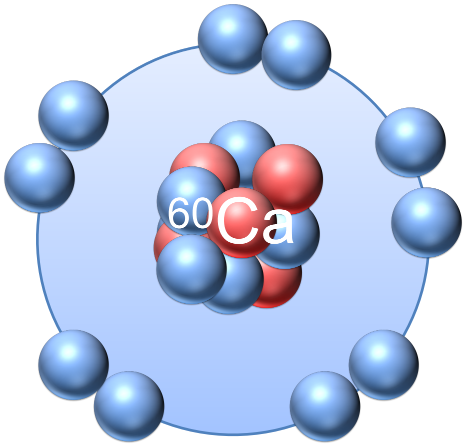
Neutron-rich Calcium isotopes



Neufcourt+ Phys. Rev. Lett 122 062502 (2019)

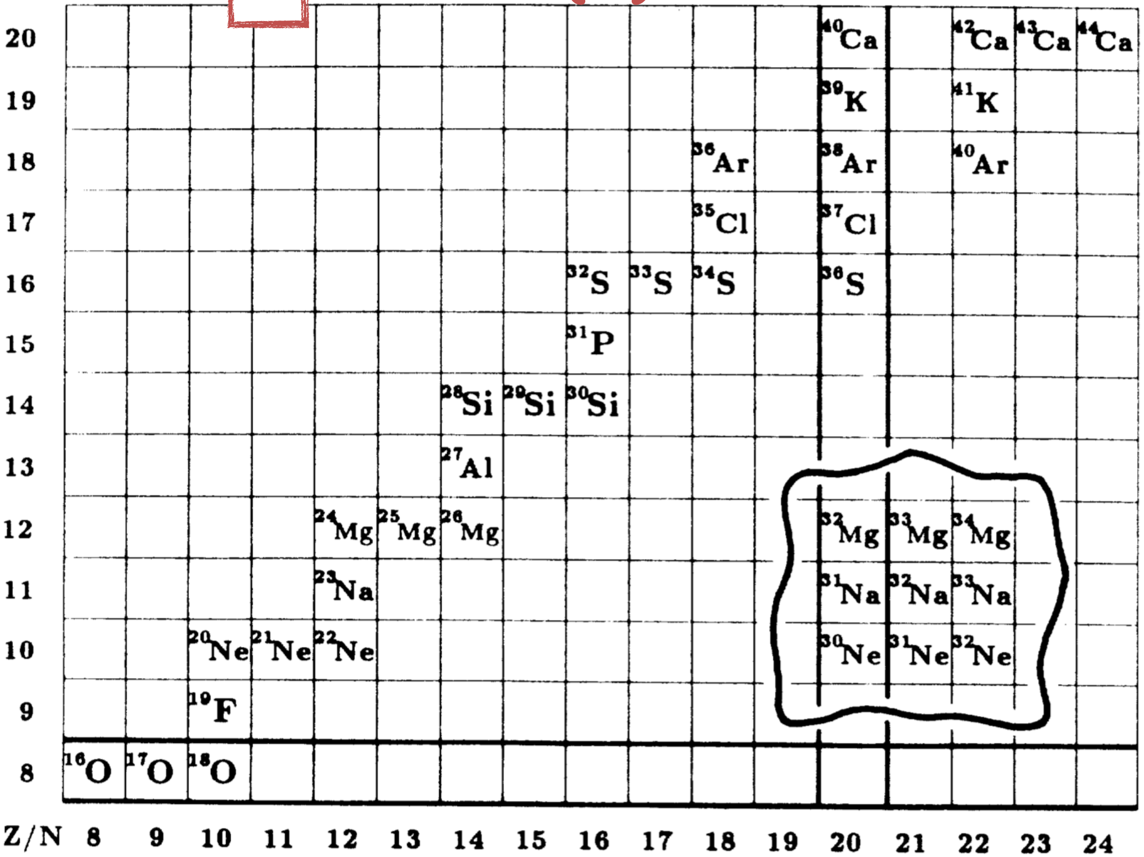


SRS+ Phys. Rev. Lett. 126 022501 (2021)

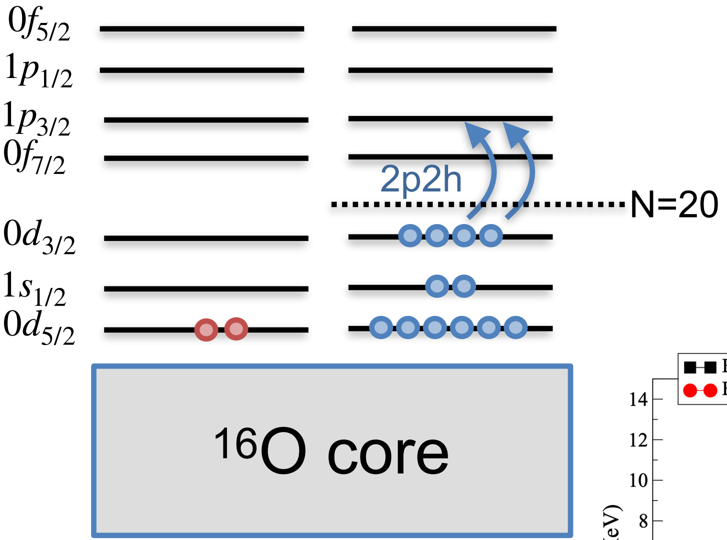

60Ca

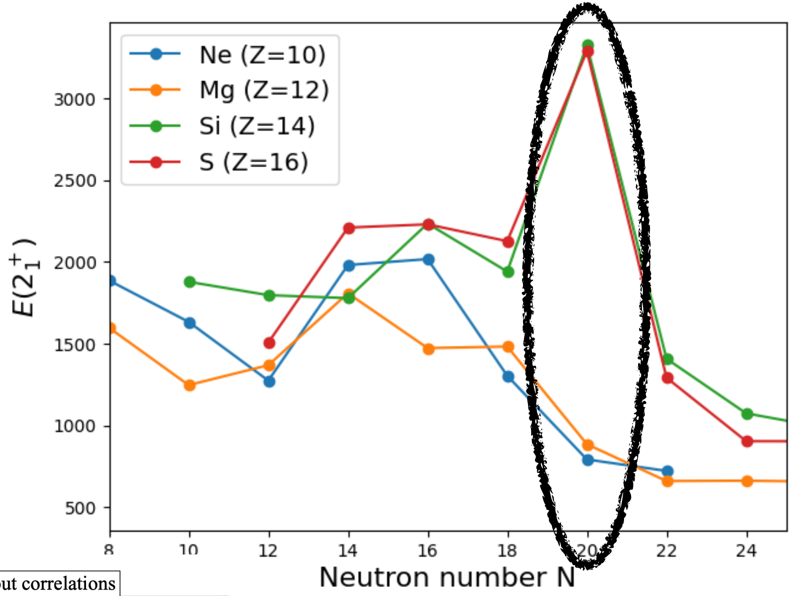
?

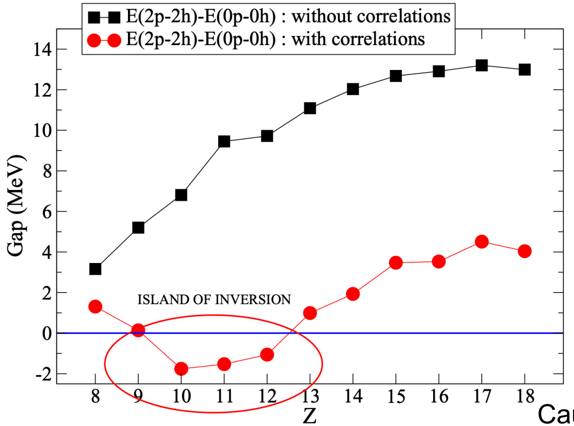
5 Island(s) of Inversion



Warburton, Becker, Brown PRC 41, 1147 (1990)

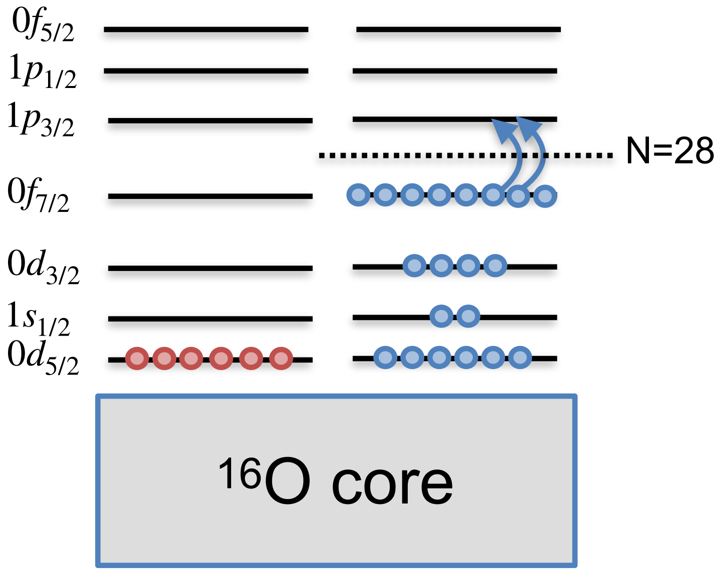


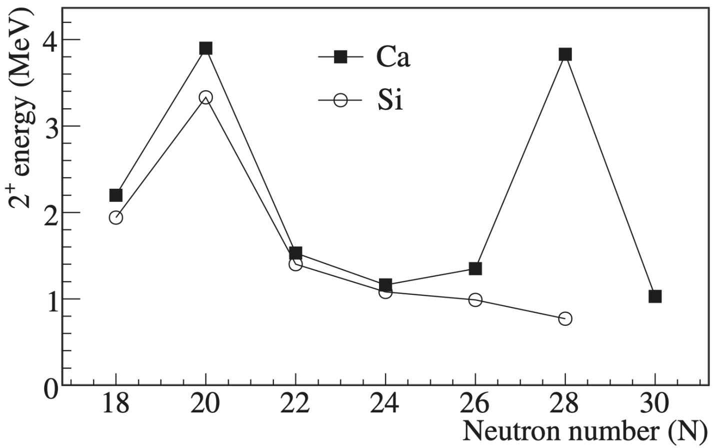




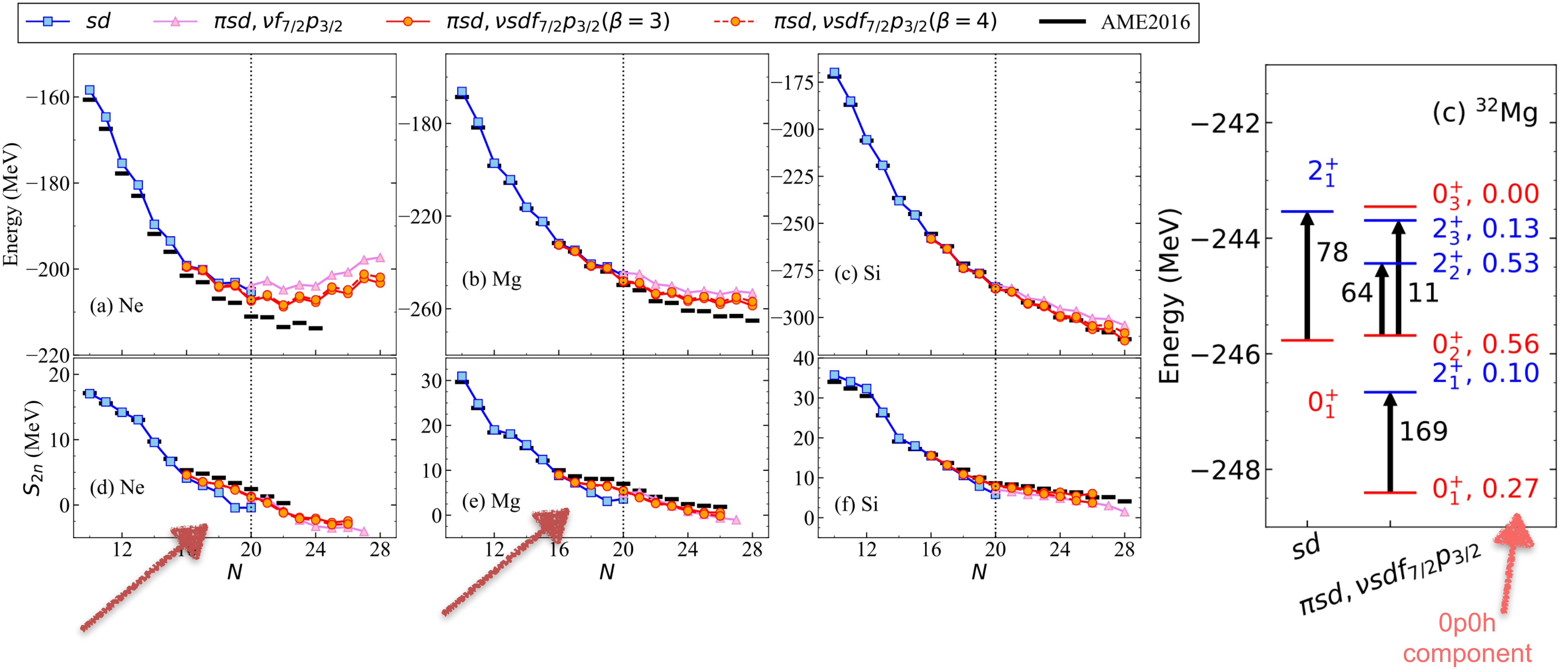
Caurier+ PRC 90 014302 (2014)

Island of inversion at N=28



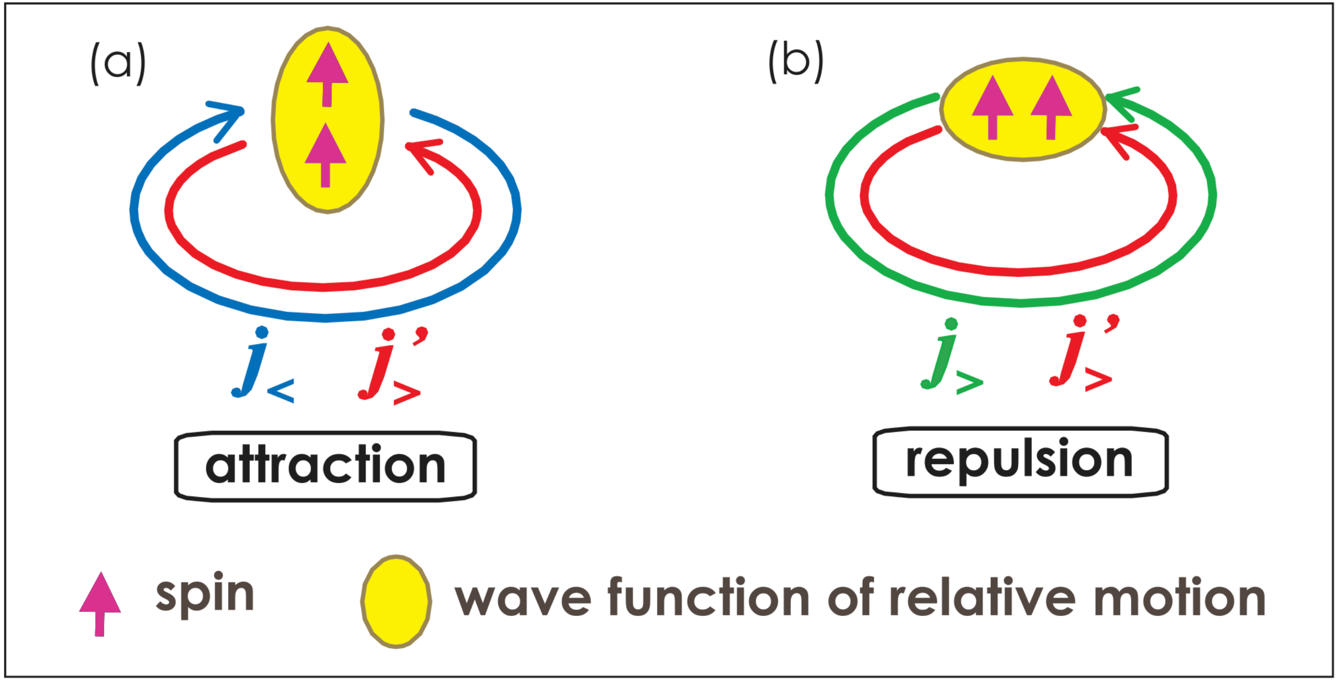


Ab initio calculations at the N=20 IOI



Miyagi+ Phys. Rev. C 102 034320 (2020)

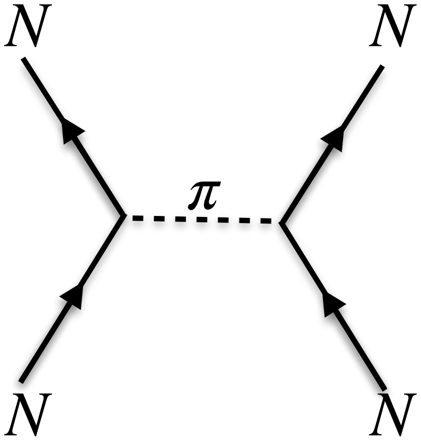
6 Shell evolution and the tensor force



Otsuka+, Phys. Rev. Lett. 95 232502 (2005)

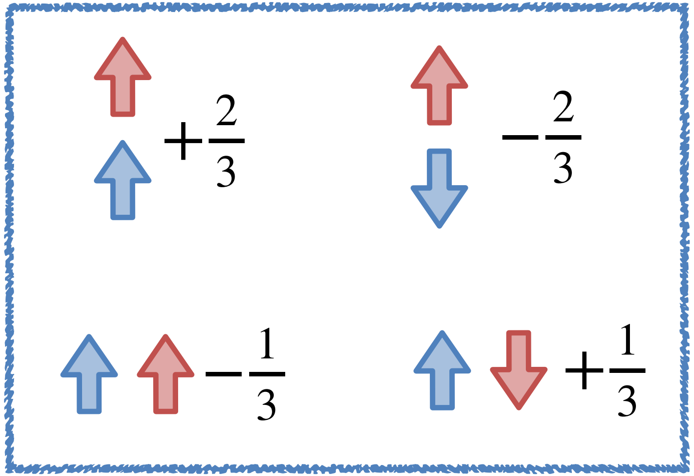
Tensor force

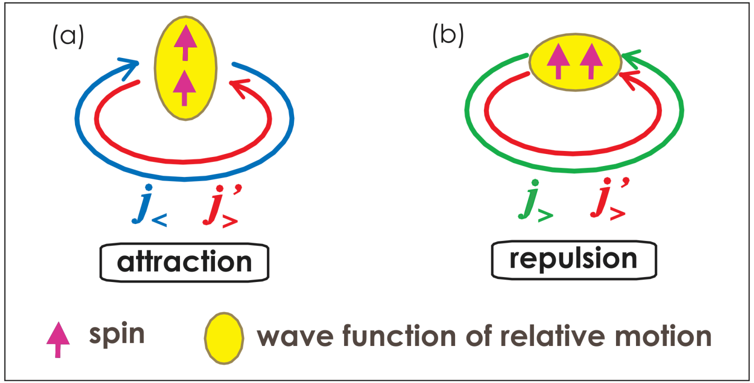
$$V_{1\pi}(r) = \left(\frac{g_A}{2f_\pi}\right)^2 \frac{\tau_1\cdot\tau_2}{12\pi} m_\pi^3 \left[S_{12}(\hat{r}) \left(\frac{3}{m^2r^2} + \frac{3}{mr} + 1 \right) + (\sigma_1\cdot\sigma_2) \right] \frac{e^{-mr}}{mr}$$



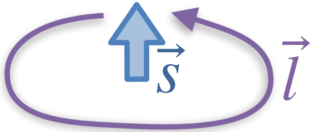
$$\tau_1\cdot\tau_2 = \begin{cases} -3, & T=0 \\ +1, & T=1 \end{cases}$$

$$S_{12}(\hat{r}) \equiv (\sigma_1\cdot\hat{r})(\sigma_2\cdot\hat{r}) - \frac{1}{3}\sigma_1\cdot\sigma_2$$

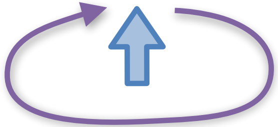


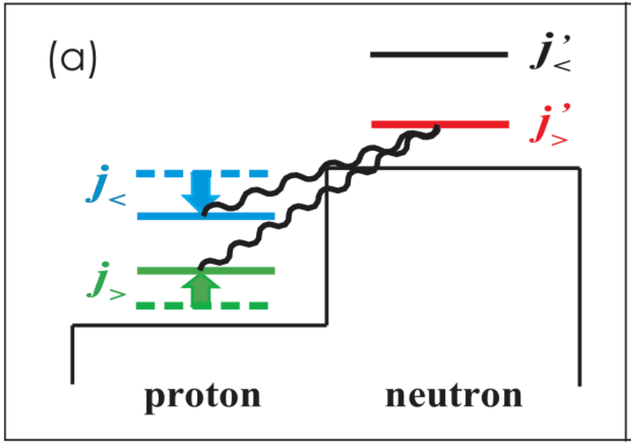


$$j_> = l + s$$



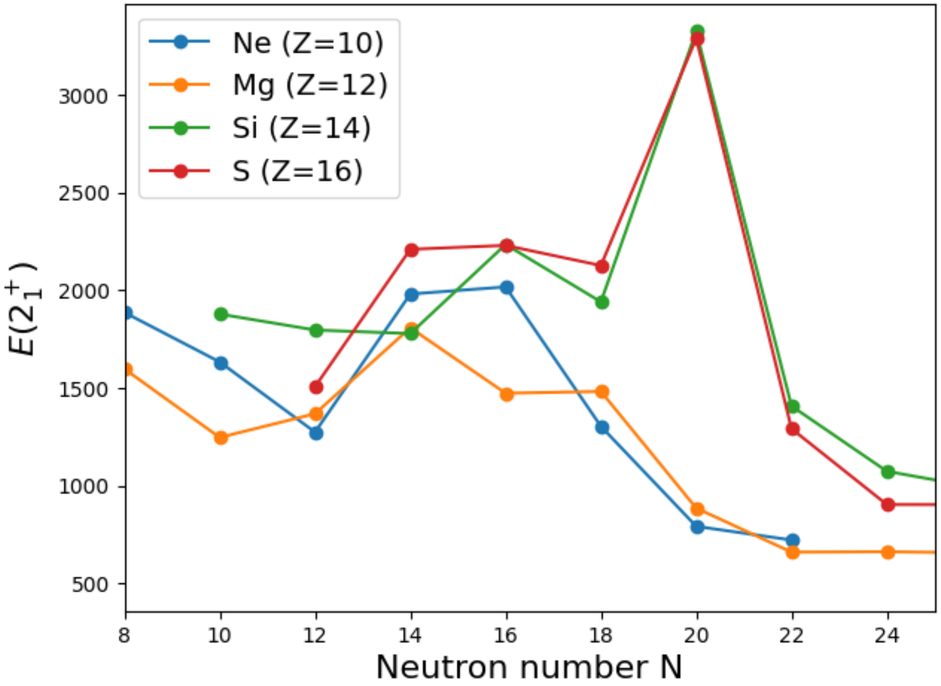
$$j_< = l - s$$

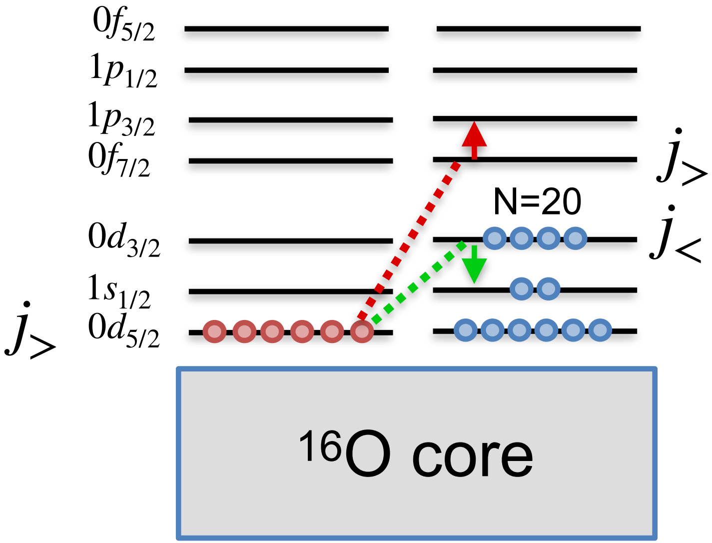




Otsuka+, Phys. Rev. Lett. 95 232502 (2005)

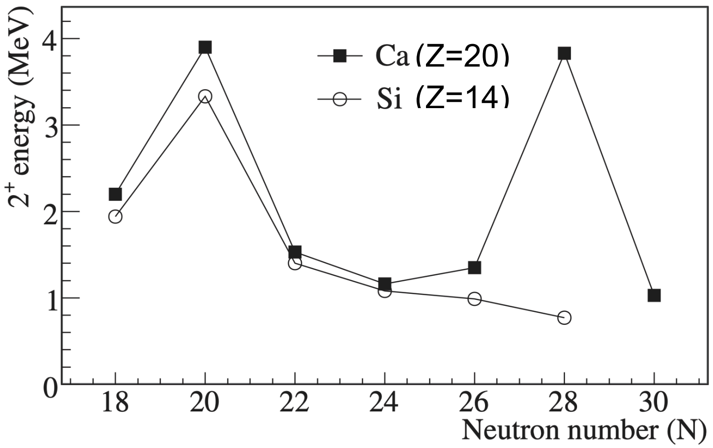
Island of inversion around N=20



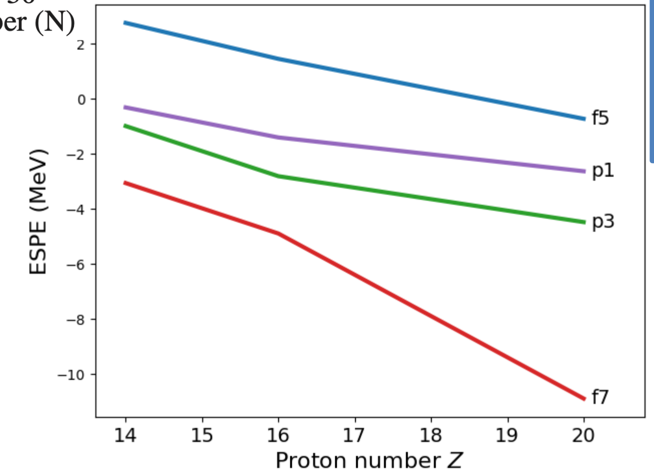


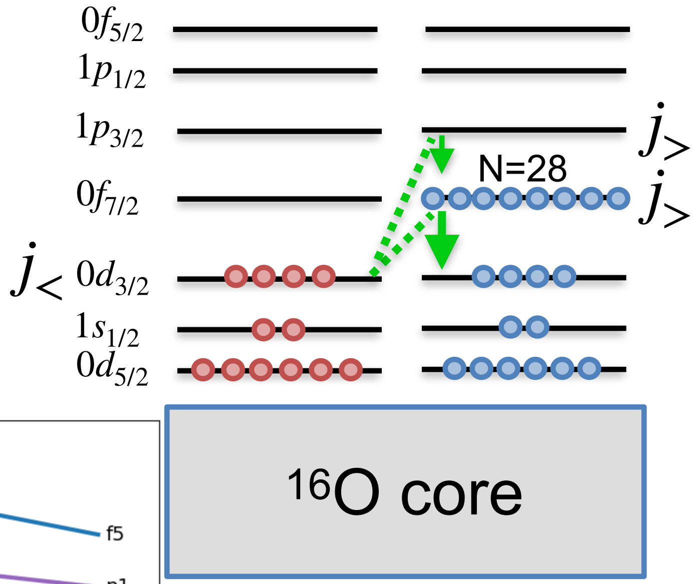
Protons in the $d_{5/2}$ support the N=20 shell gap via the tensor force.
Remove protons ⇒ reduce gap.

Collapse of the N=28 shell gap

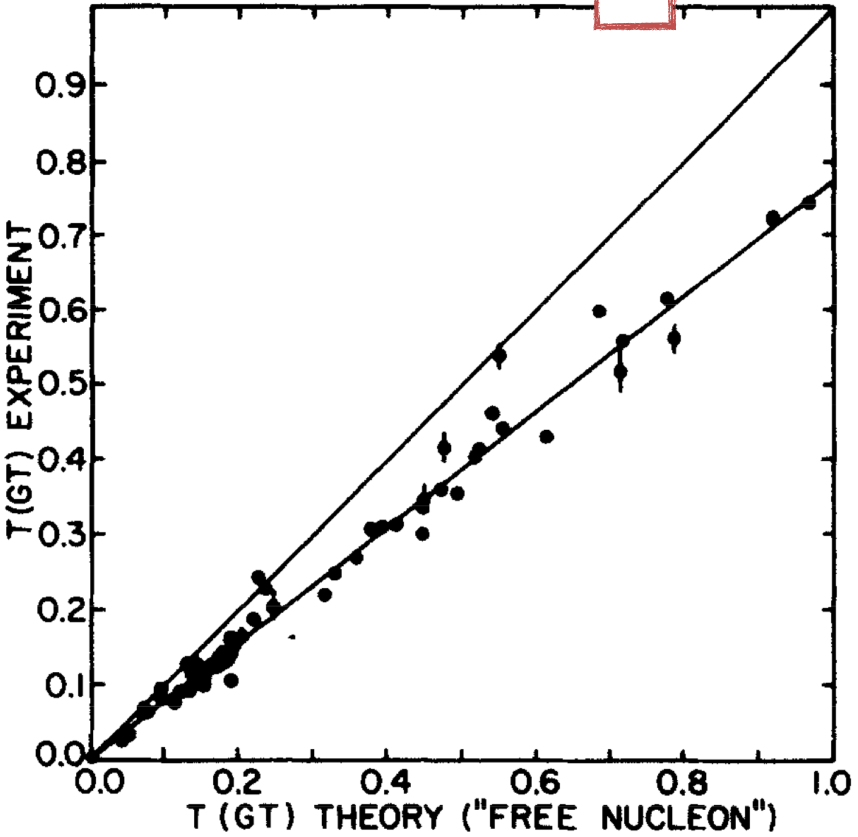


Bastin+, Phys. Rev. Lett. 99, 022503 (2007)

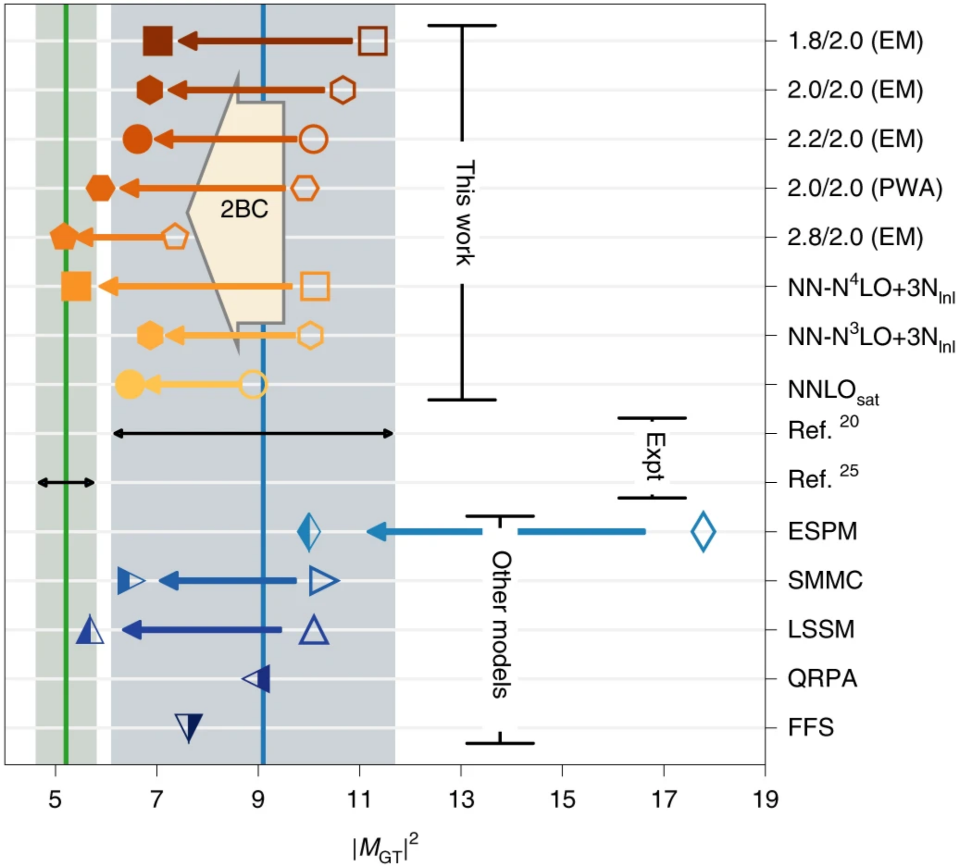




7 Quenching of g_A



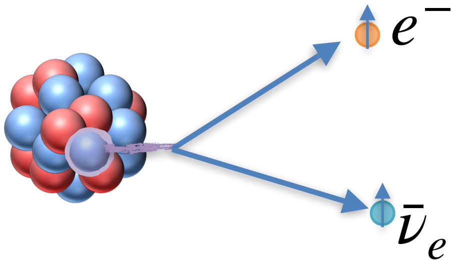
Brown & Wildenthal, At. Dat. Nucl. Dat. Tab. 33, 347 (1985)

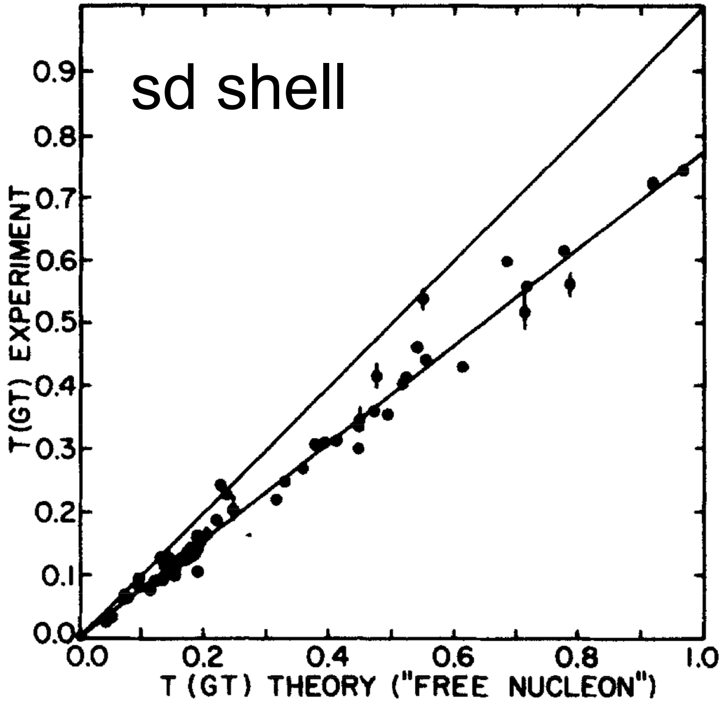


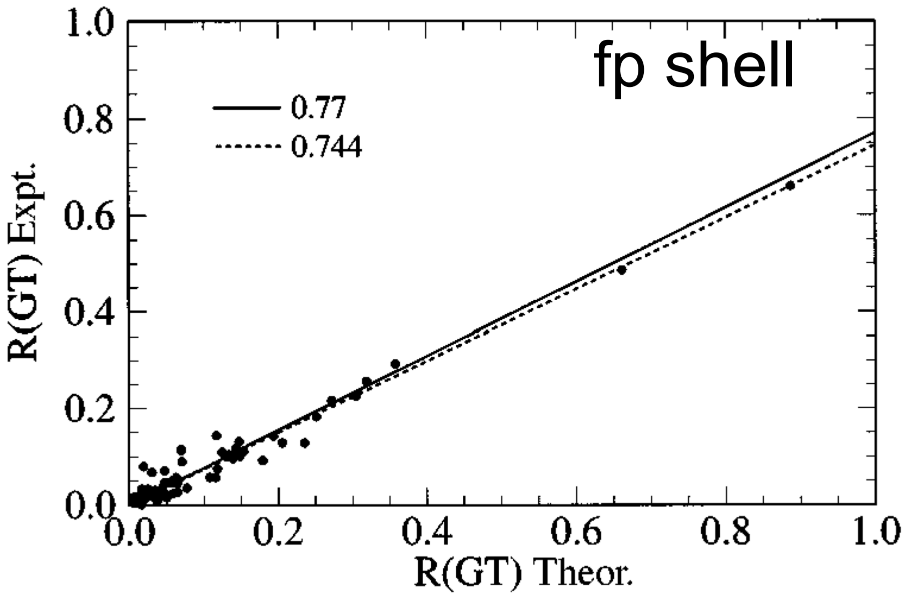
Gysbers+ Nat. Phys. 15, 428 (2019)

Gamow-Teller beta decay

$$\mathscr{M}_{GT} = \langle \Psi_f \| \sigma\tau \| \Psi_i \rangle$$

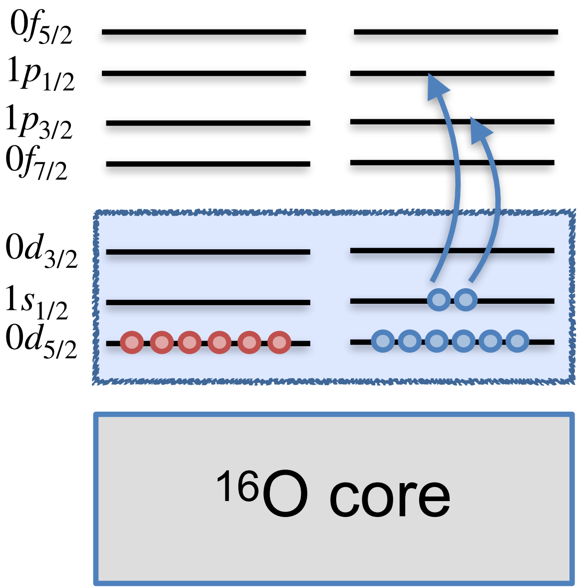




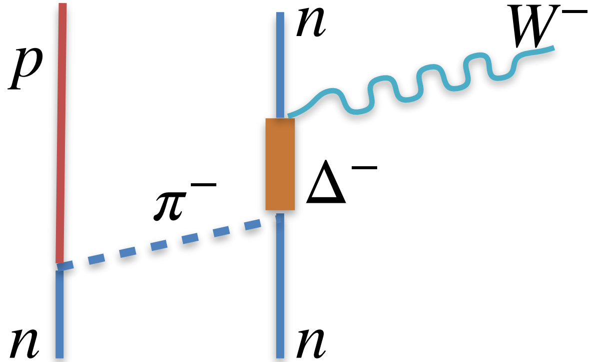


Sources of discrepancy

Excitations out of the valence space

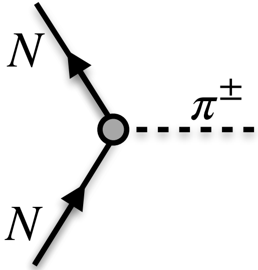


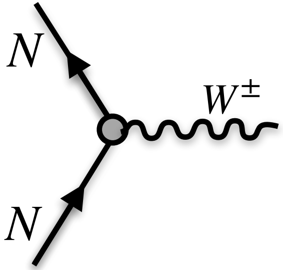
Non-nucleonic degrees of freedom



Axial currents are not conserved by the strong interaction.

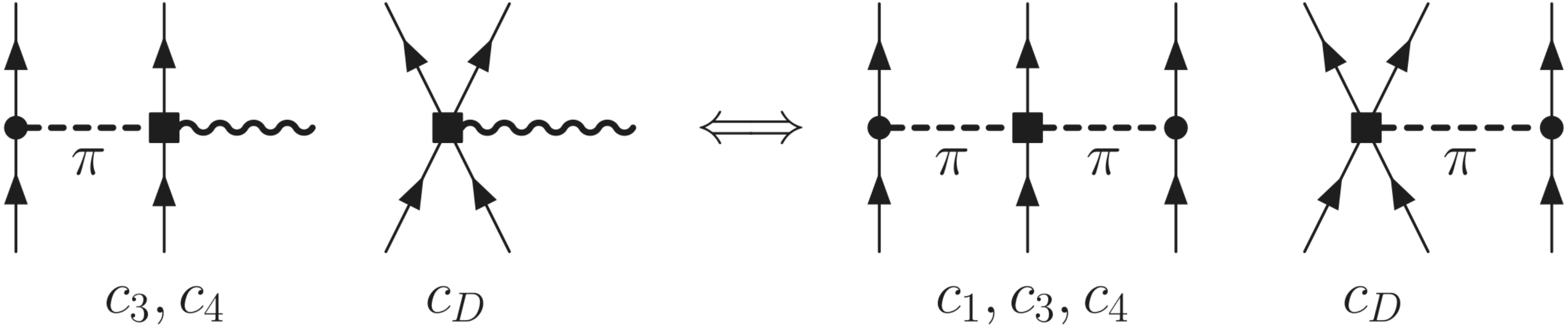
Chiral effective field theory predicts how EM and weak currents couple to nucleons





$$\mathcal{L}_{\pi NN} = \frac{g_A}{2f_\pi}\bar{N}\left(\vec{\sigma}\cdot\vec{\nabla}\tau^{\mp}\right)N\pi^{\pm}$$

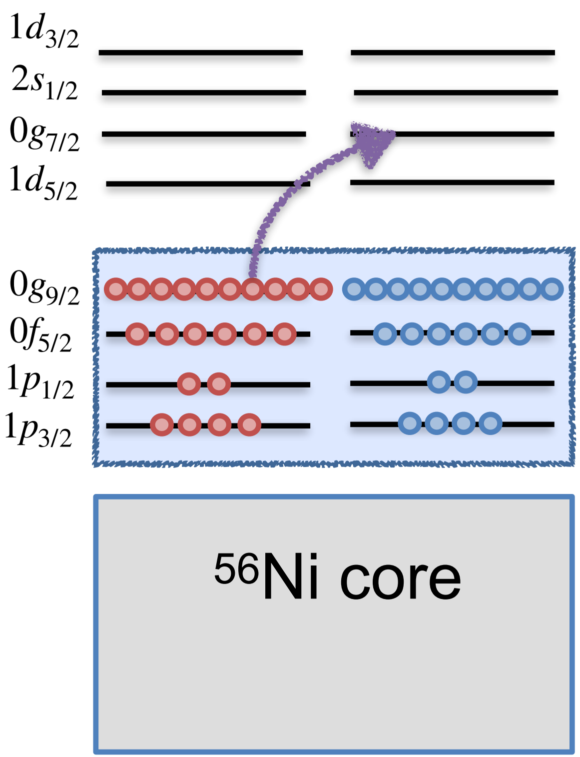
$$\mathcal{L}_{WNN} = g_A\bar{N}\left(\vec{\sigma}\tau^{\mp}\right)N\vec{J}_A^{\pm}$$

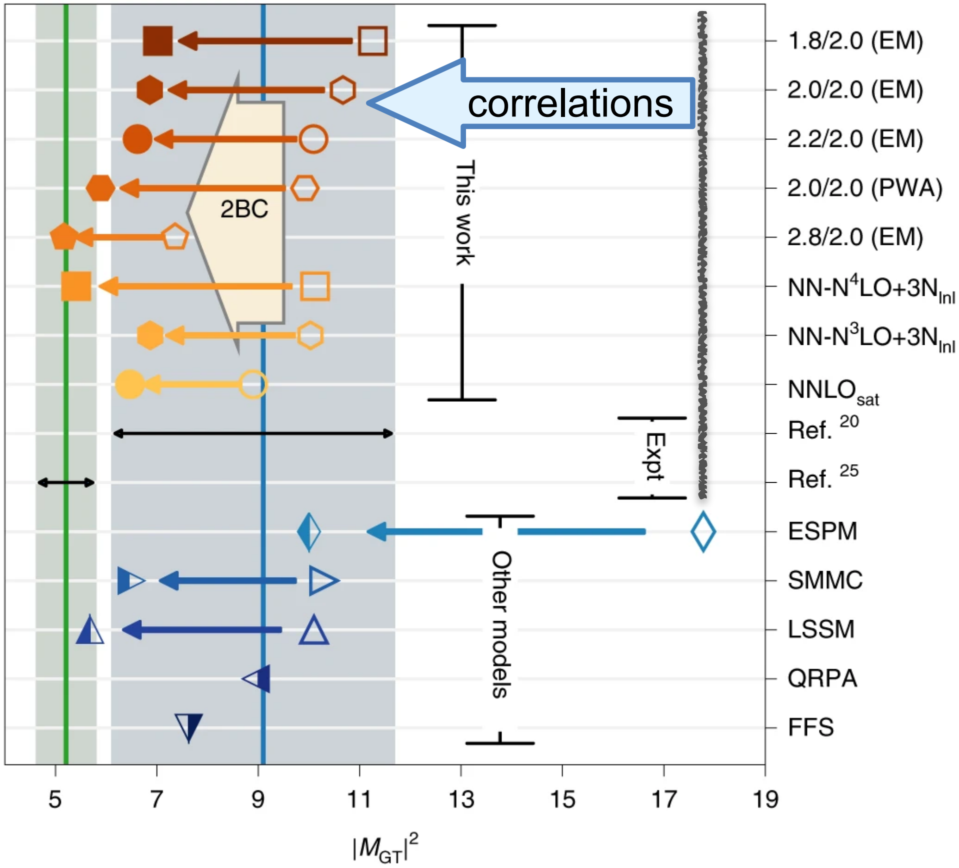


No free parameters!

Menendez+ Phys. Rev. Lett. 107, 062501 (2011)
Gardestig & Phillips, Phys. Rev. Lett. 96, 232301 (2006)

$$\sigma\tau \left| {}^{100}\mathrm{Sn}(0^+) \right\rangle \rightarrow \left| {}^{100}\mathrm{In}(1^+) \right\rangle$$





Thanks!