Nuclear reactions primer

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1. Reactions are where the action is!

   Explains:

How elements are made, the life cycle of stars,

Fusion (in stars like our sun or on earth), Fission (in reactors, the cosmos, or bombs,

And how isotopes are made for: medicine/bio/chem/earth science/phys.

2. Reactions help us understand the quantum structure of the items reacting.

3. While reaction and structure discussions are often separated,

   it is a mistake to take this “partition” seriously.

BECAUSE

4. NP is, more and more, a study of **O**pen **Q**uantum **S**ystems (OQS) where **bound** (-ve energy) and **scattering** (+ve energy) physics must be dealt with together.

Reactions – selected topics for this lecture

I. Inseparability of reactions and structure ….. OQS

II. Reactions □□ Thermodynamics

III. Reactions – Objective: to understand the correlated medium under investigation
(best to stand back to get perspective) - *look back & look forward*

IV. Reaction Types (the 30,000 ft view).
Induced by: neutrons, charged-particles, e$^-$'s, (and photons).
fusion (CF), ICF, DIC, (multi-)nucleon transfer, inelastic (Coul-Ex), elastic
low □ ℓ-space decomposition (bird's-eye view) □ high

V. Selected examples (go as far as we can go today)

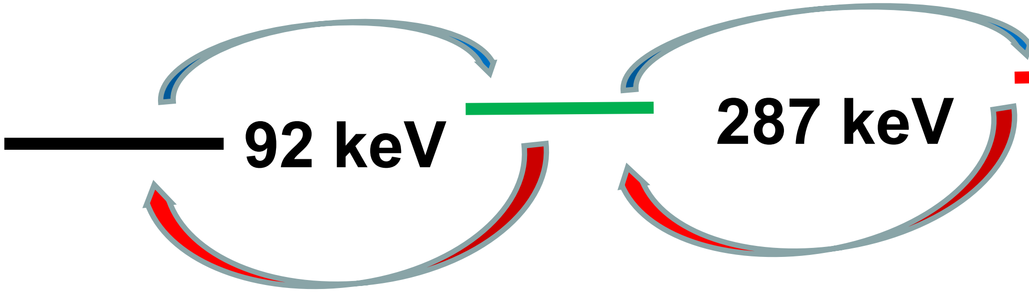
1. Elastic + □ optical potentials - *look back & look forward*
2. transfer and "matching" conditions
3. Fusion □ *look back & look forward*
4. CN decay – HF + transition-state theory & the $\mathrm{E}_{\mathrm{vaporation}}\mathrm{A}_{\mathrm{ttractor}}\mathrm{L}_{\mathrm{ine}}$
5. Angular Momentum Matching (Brink): transfer and alignment
6. Super-elastic – *up scattering* - from Hoyle

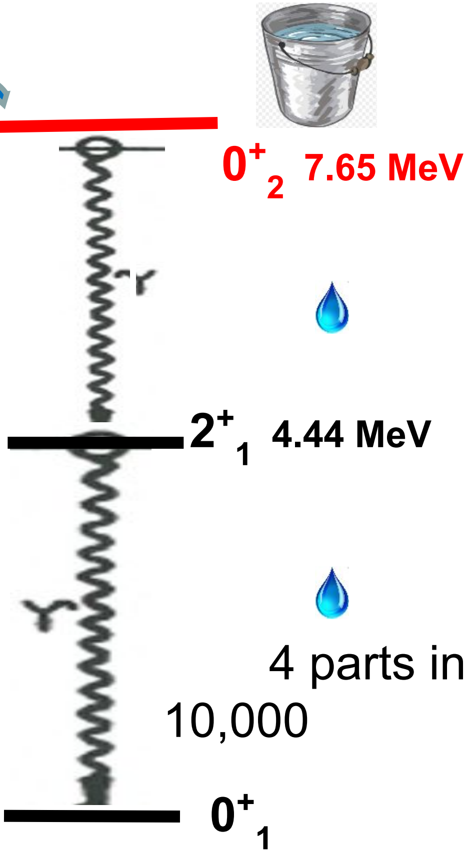
Tools, discussed by others: Gas (IC, PC…□ MUSICs, TPC's); Reverse biased diodes (Si, Ge, CZT).
Scintillators (organic and inorganic) ; Bethe-Block (dE/dx),ToF, PSD □ almost always used in combination.

I. OQS–1: Hoyle state is a “near-threshold resonance”

(α + α) + α □□ ^{8}Be + α □□ ^{12}C$_{\text{Hoyle}}$

(5.57 ev □ 118 as) (9.3 ev □ 71 as)





EM decay 4/10,000

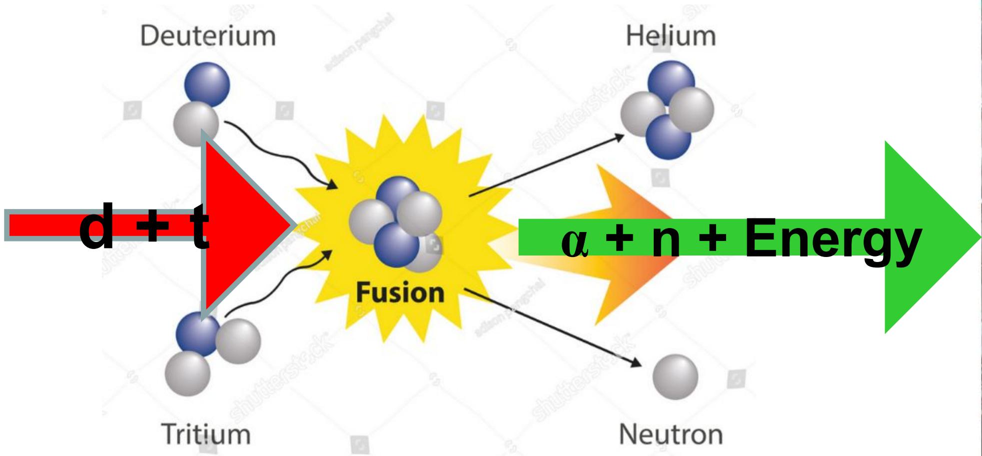
□Hoyle’s mostly just fall apart

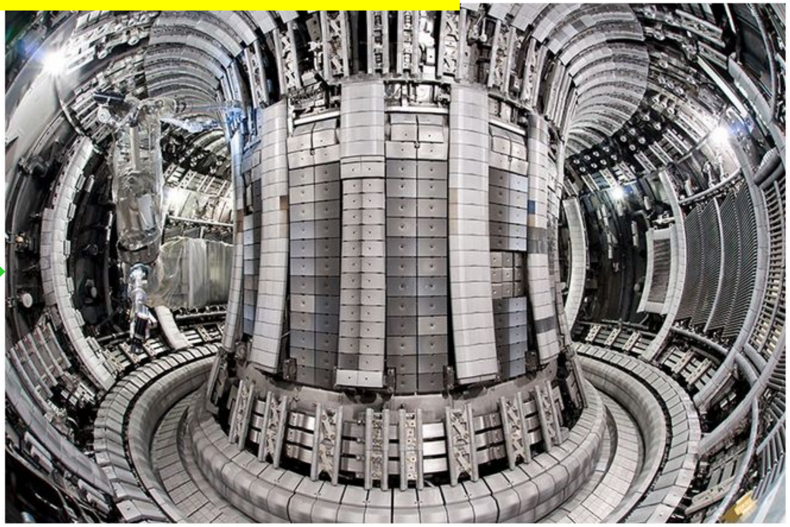
□The measly drips □ Periodic Table

Because □dynamical□ there are [eq concentrations] of [^{8}Be]$_{eq}$ & [^{12}C$_{\text{Hoyle}}$]$_{eq}$

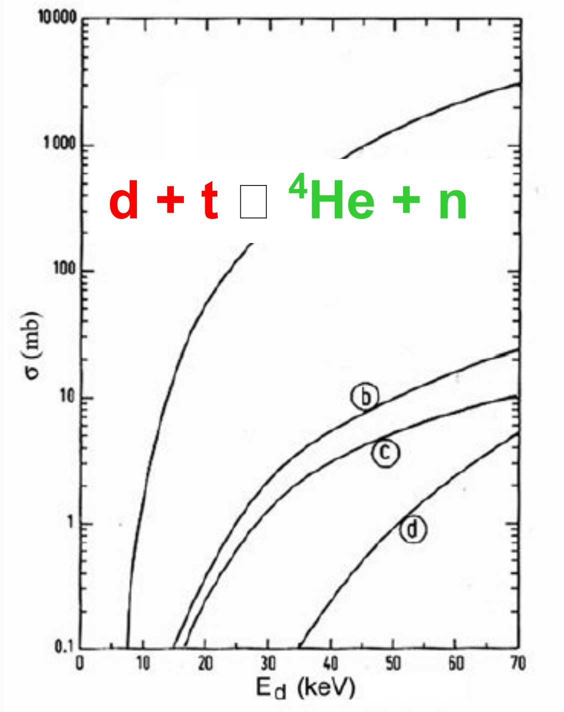
I shall return to this twice in this talk

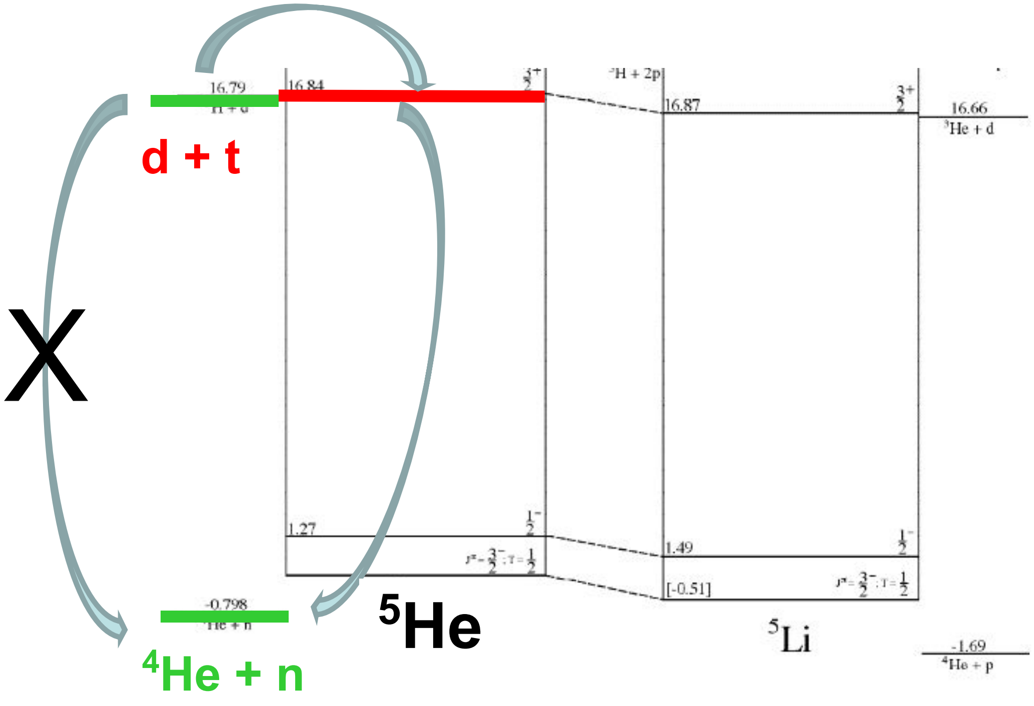
OQS-2: d + t fusion proceeds via a near-threshold resonance.





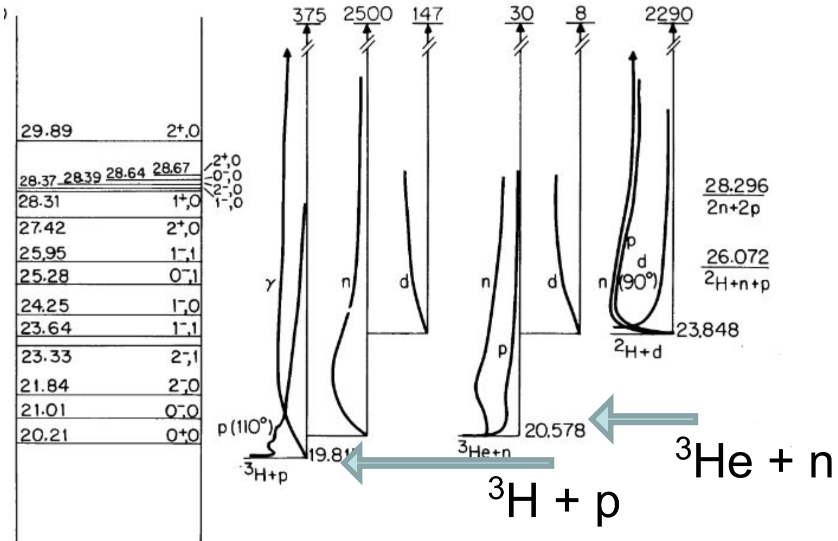
$Q = -\Delta H = 17.6\ \text{MeV} \sim 1.7 \times 10^{12}\ \text{J/mole}$





OQS-3: The exited states of the α are in the “near continuum”

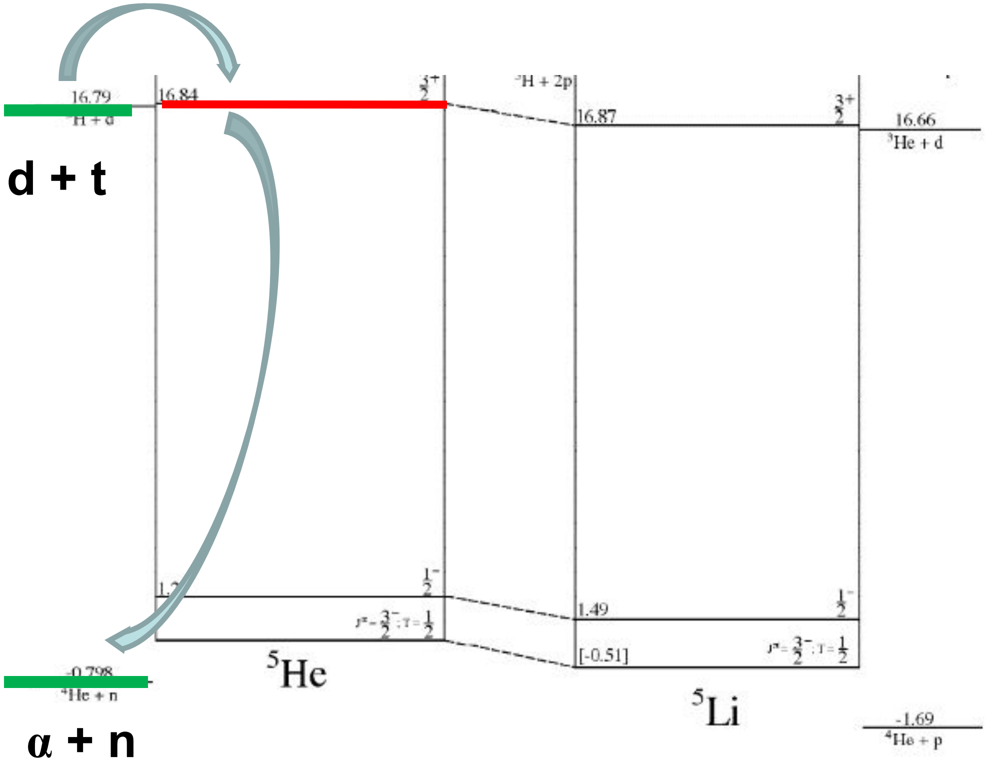
A = 4 A = 5



^{3}He + n

^{3}H + p

Longest-lived A = 5 states are just above t(3He) + d threshold

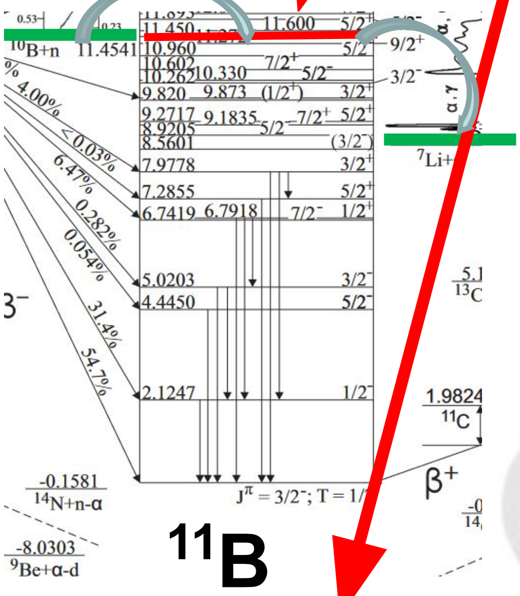


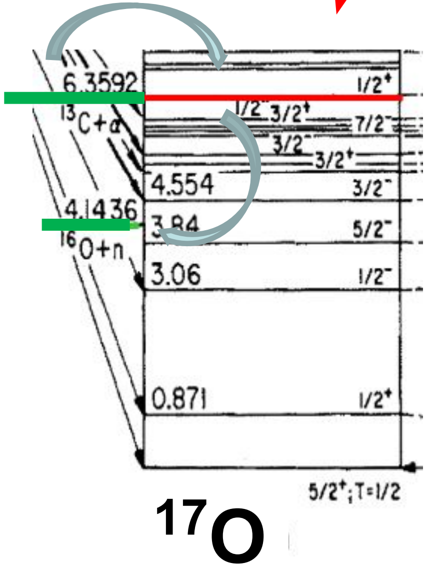
d + t

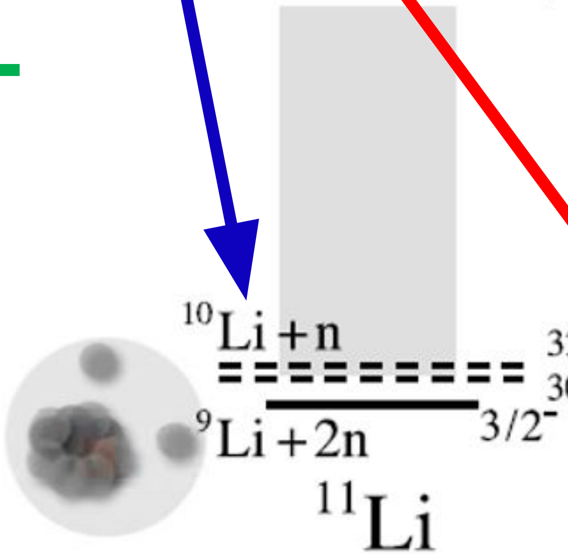
α + n

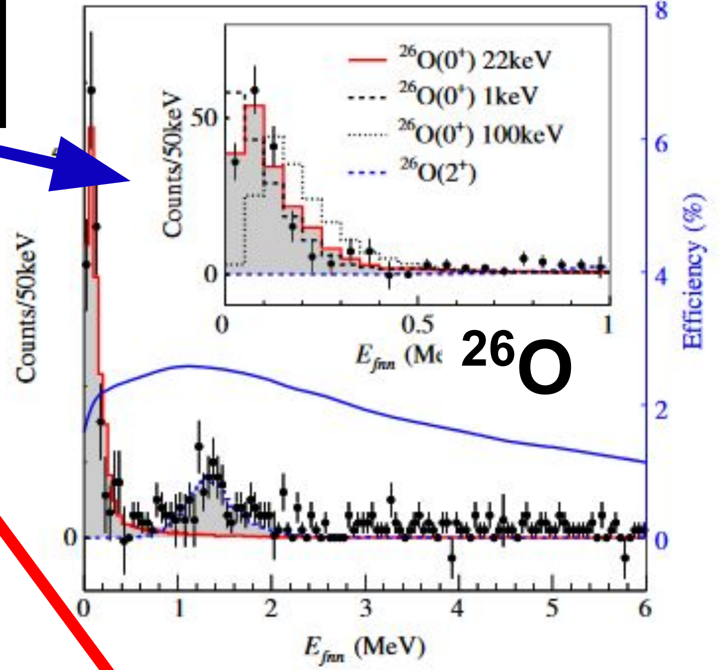
As nuclear scientists we should be able to explain cases not relevant for energy production as well as those that are.

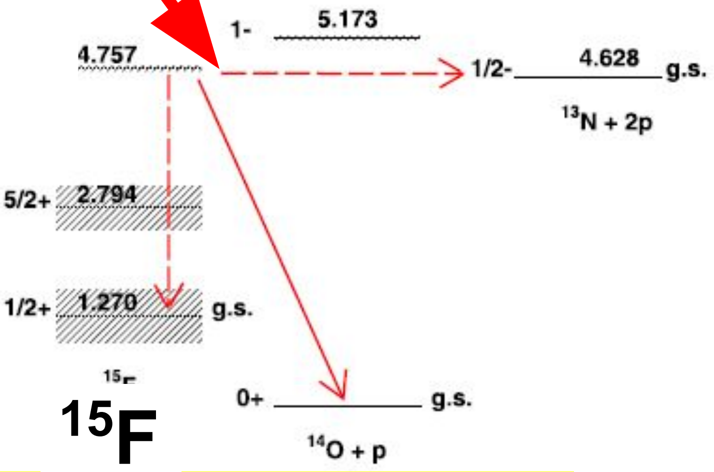
OQS-4:
A = ^{11}B, ^{17}O, ^{11}Li, ^{15}F, ^{26}O





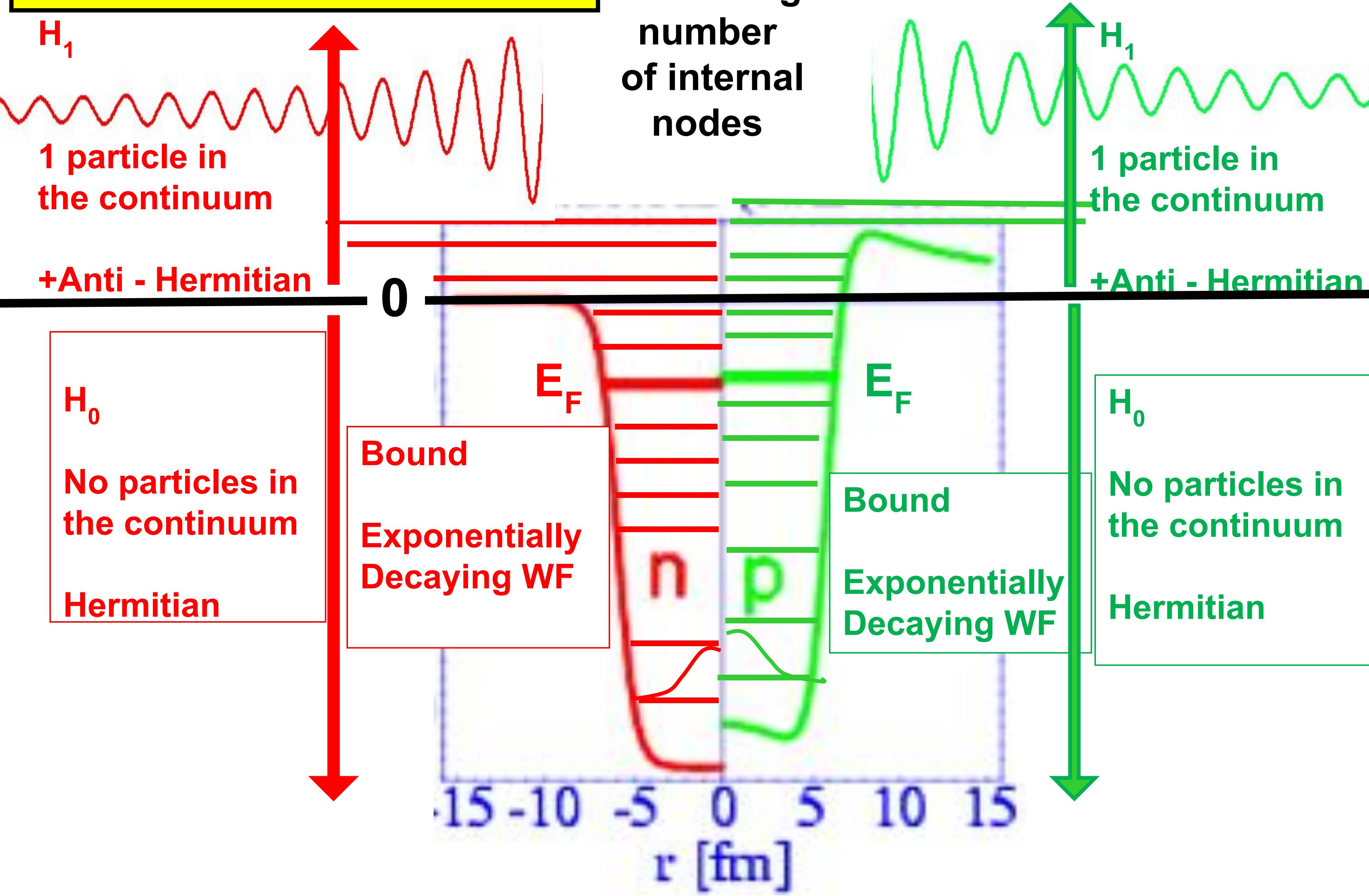






^{11}B □ Reactor control; **^{17}O** □ s-process
One should be able to explain those cases
OFF NS paths as well as those **ON** a path.

OQS-5: Mental view of WF



Increasing number of internal nodes

H_1

1 particle in the continuum

+Anti - Hermitian

H_0

No particles in the continuum

Hermitian

Bound

Exponentially Decaying WF

Bound

Exponentially Decaying WF

H_1

1 particle in the continuum

+Anti - Hermitian

H_0

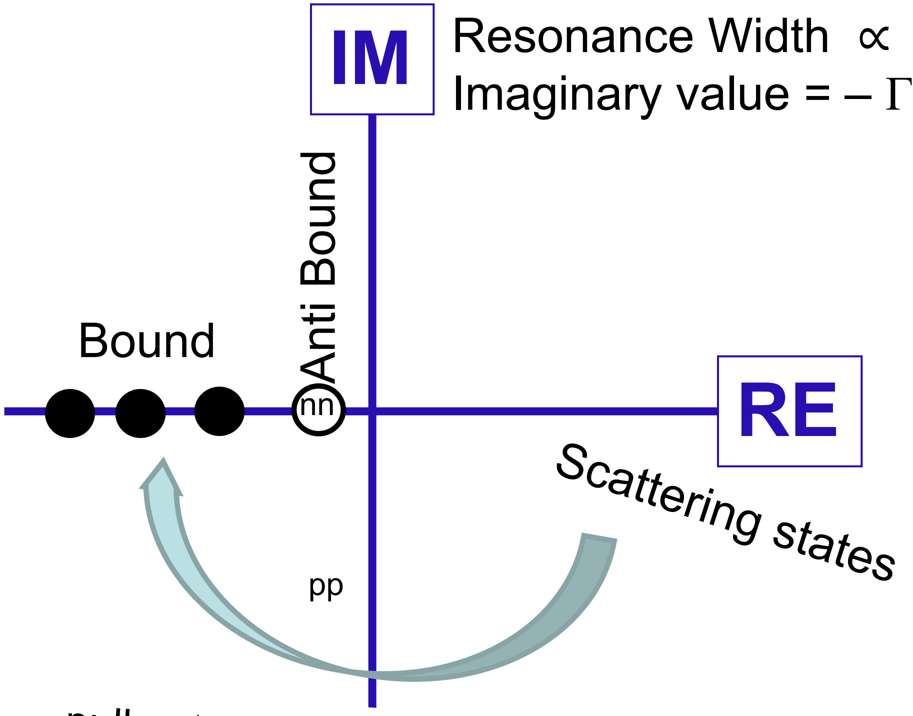
No particles in the continuum

Hermitian

OQS-6: Eigen values are Complex

FYI – peruse later

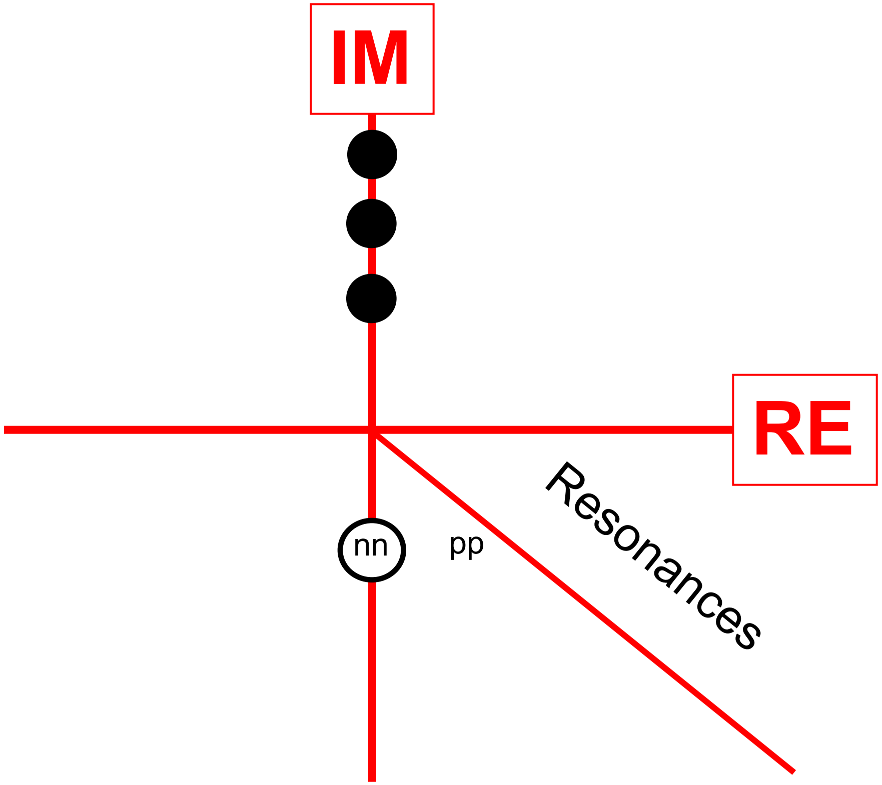
Energy



Resonance Width ∝
Imaginary value = – Γ/2

pull potential (increase depth or width)

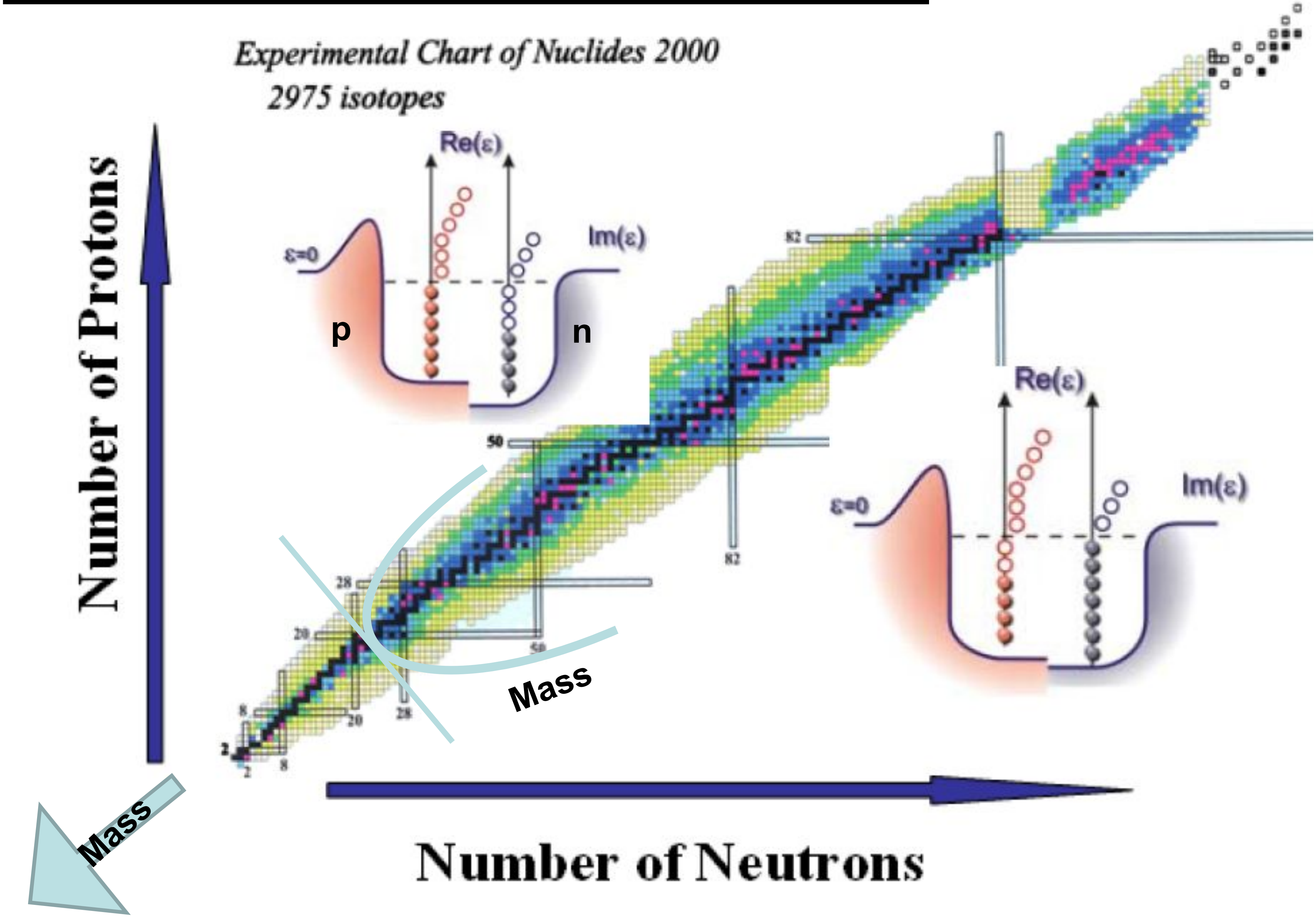
Momentum



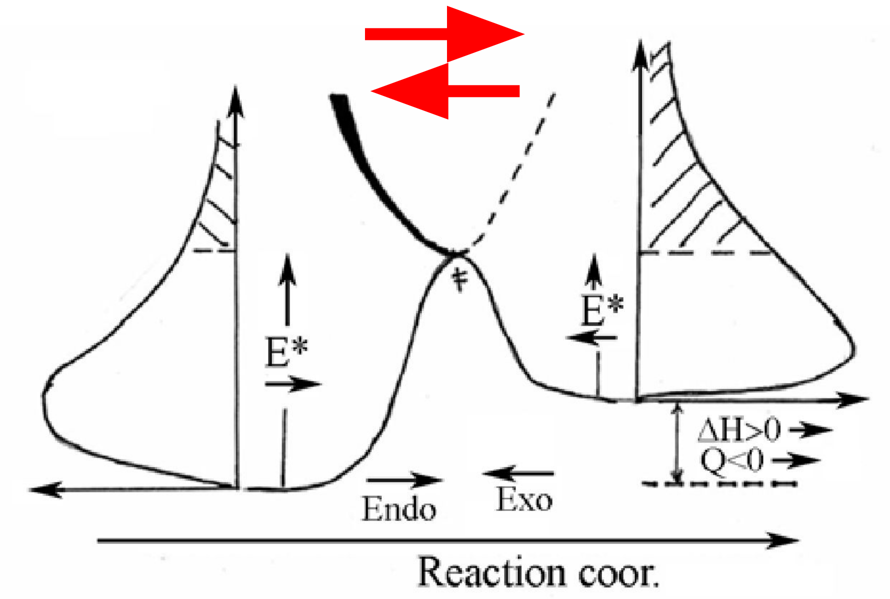
Kok, PRL 45, 427 (1980)…. ^{2}He
E_{pp} = (-140 – i 467) keV

k_{pp} = (0.0647 – i 0.0870) fm^{-1}

OQS-7: Your view of the Chart should be this ….



II. Thermodynamics-1 : **Eq** (should it exist) requires equal one-way rates.



Consider first of the two binary steps
In the “triple-alpha process”, i.e.

$$2\alpha \leftrightarrows {}^{8}\mathrm{Be}$$

Double arrows (or =) mean EQ therefore
⟶ |birth| = |death| ⟵

$$\dot{Y}_8 = \frac{1}{2} Y_4^2 \rho N_A \langle \alpha\alpha \rangle - Y_8 \frac{\Gamma_{\alpha 1}}{\hbar} = 0$$

$$K_{eq} \equiv \frac{Y_8}{{Y_4}^2} = \frac{q_8}{{q_4}^2} = \frac{\left[\frac{(2\pi m_8 kT)^{1.5}}{h^3}\right]^1}{\left[\frac{(2\pi m_4 kT)^{1.5}}{h^3}\right]^2} \{e^{-Q/kT}\} = \left[\frac{2\pi\hbar^2}{\mu kT}\right]^{3/2} \{e^{-Q/kT}\}$$

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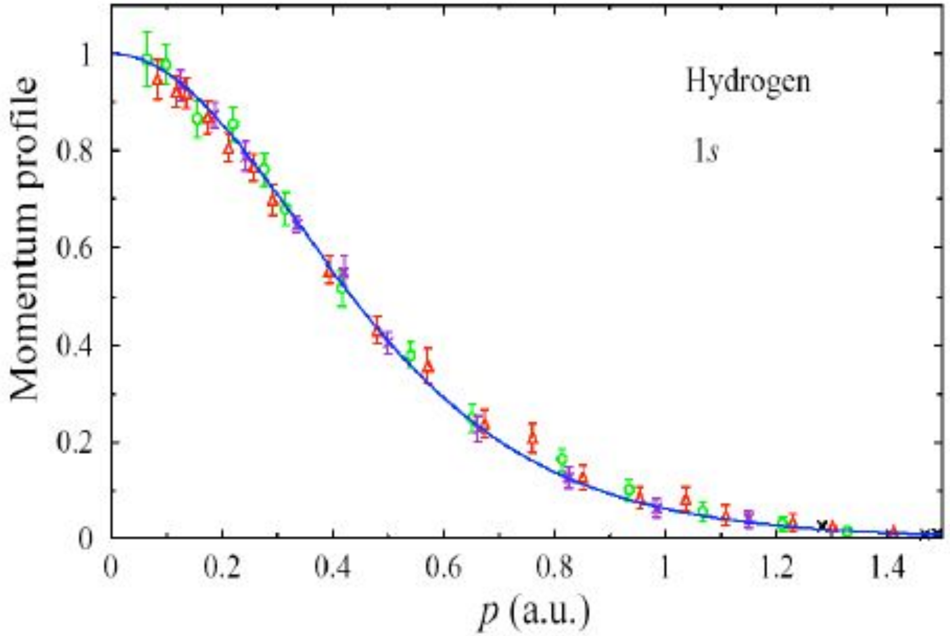
III. Reactions Objectives-1

Look to atomic physics to see what we aspire to do in nuclear physics.
Consider knock-out reactions

(e,2e) data for atoms

- Start with Hydrogen
- Ground state wave function $\phi_{1s}(\boldsymbol{p}) = \frac{2^{3/2}}{\pi} \frac{1}{(1+p^2)^2}$
- (e,2e) removal amplitude

$$\langle 0|\, a_{\boldsymbol{p}}\, |n=1, \ell=0\rangle = \langle \boldsymbol{p}\, |n=1, \ell=0\rangle = \phi_{1s}(\boldsymbol{p})$$

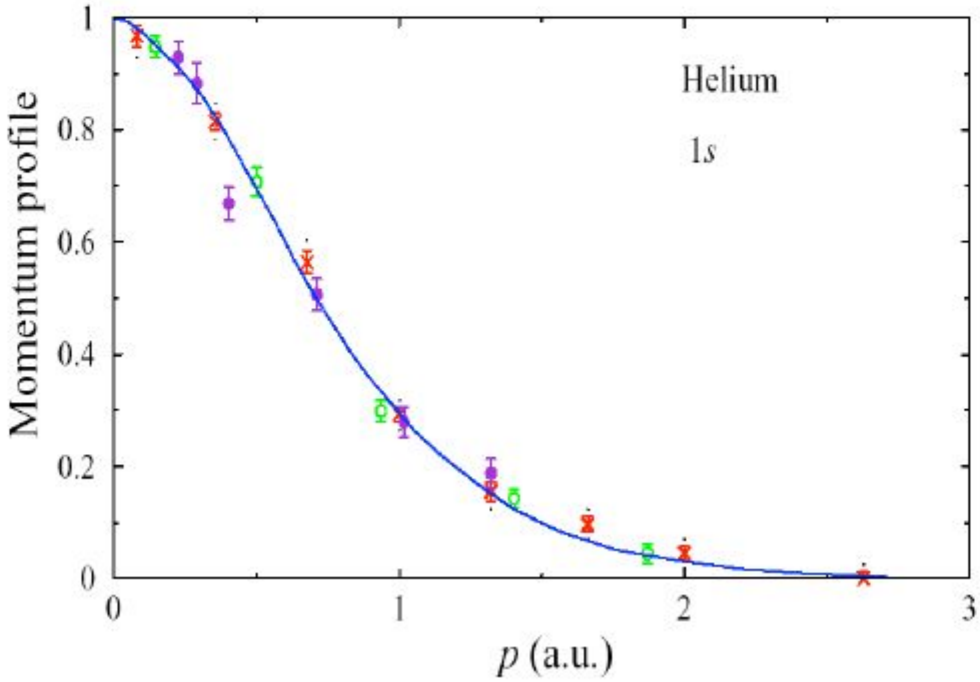


Hydrogen 1s wave function
"seen" experimentally
Phys. Lett. 86A, 139 (1981)

Reactions Objectives-2

Helium

- IPM description is very successful
- Closed-shell configuration $1s^2$
- Reaction more complicated than for Hydrogen
- DWIA (distorted wave impulse approximation)



$$S = \int dp \left| \langle \Psi_n^{N-1} | \, a_{\boldsymbol{p}} \, | \Psi_0^N \rangle \right|^2$$

agreement with IPM!

$\rightarrow 1$

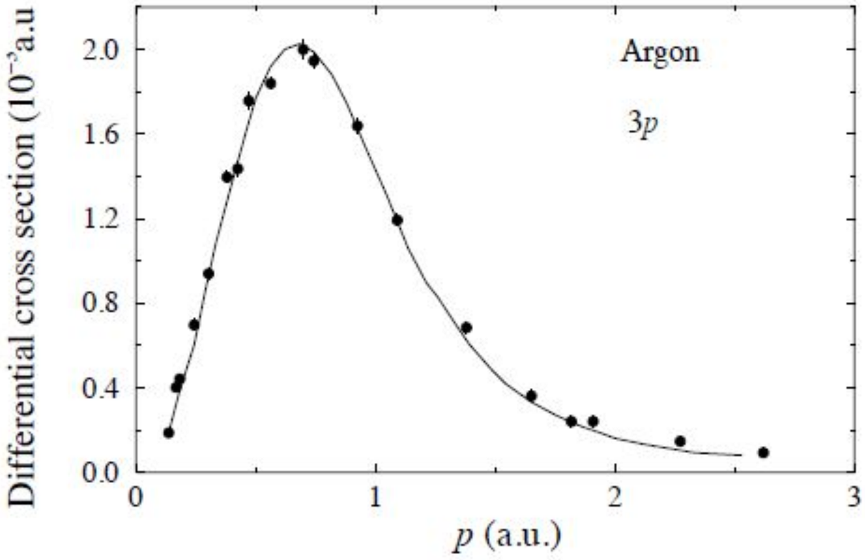
Lesson:
e^-'s are in s-orbits
□ As taught

Phys. Rev. A8, 2494 (1973)

Reactions Objectives-3

Other closed-shell atoms

- Spectroscopic factor becomes less than 1
- Neon $2p$ removal: S = 0.92 with two fragments each 0.04
- IPM not the whole story: fragmentation of sp strength
- Summed strength: like IPM
- IPM wave functions still excellent
- Example: Argon $3p$ S = 0.95
- Rest in 3 small fragments

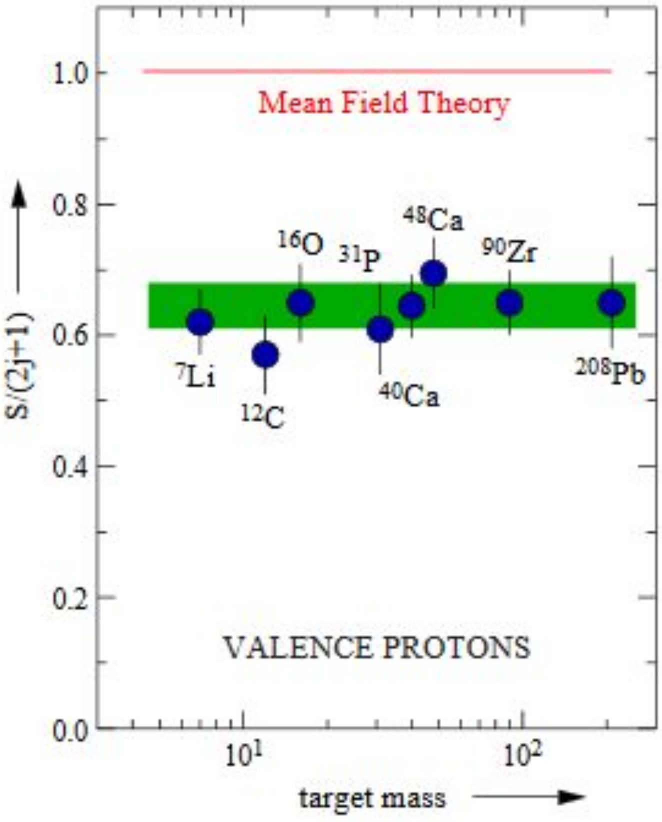


Lesson:
Some “fragmentation”
Not quite single fermions
in a one-body potential,
as taught in gen. chem/phys.

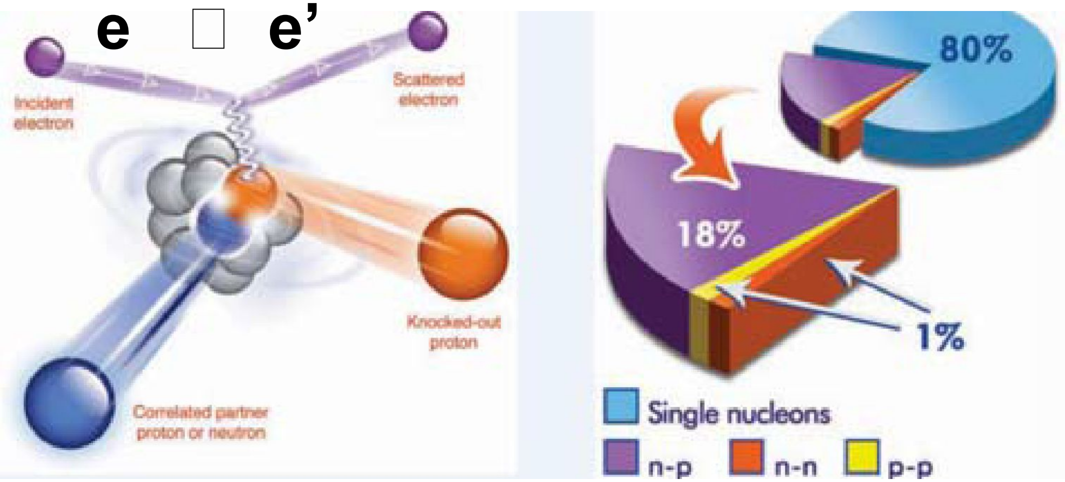
Reactions Objectives - 4

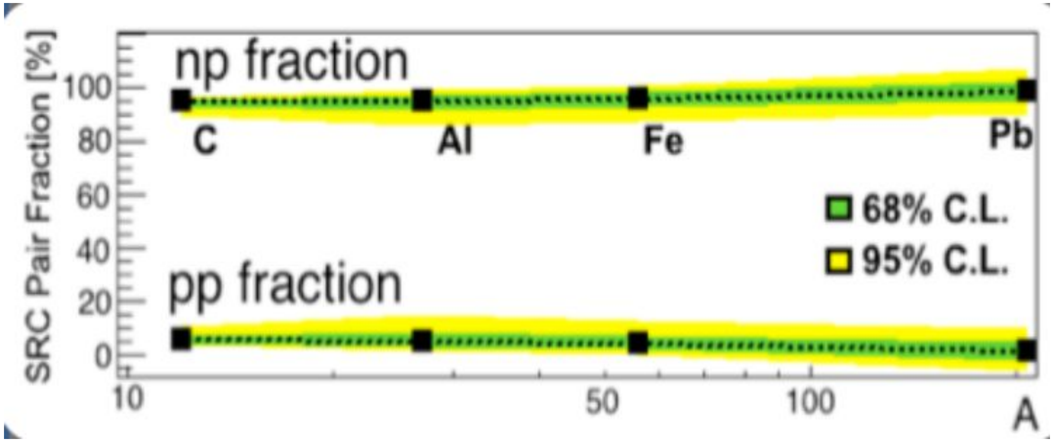
FYI – peruse later

Now look at ~ equivalent in nuclei. (e,e’p), (e,e’pn); Nikhef & Jlab experiments



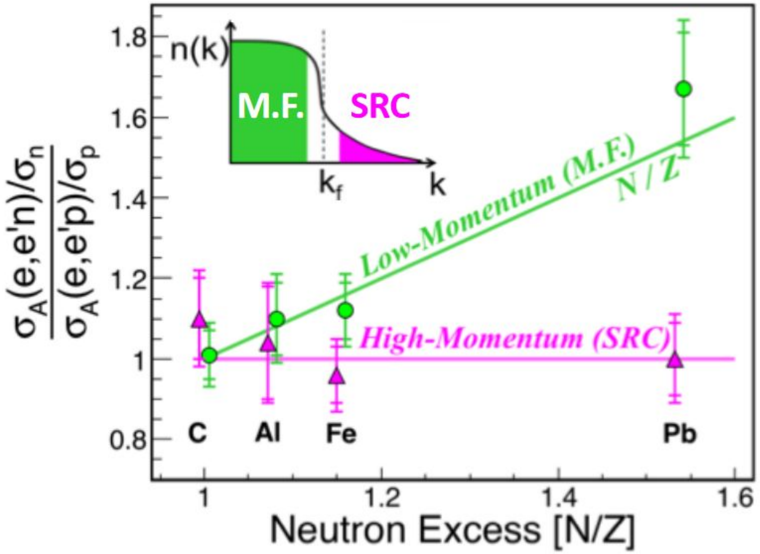
e □ e’

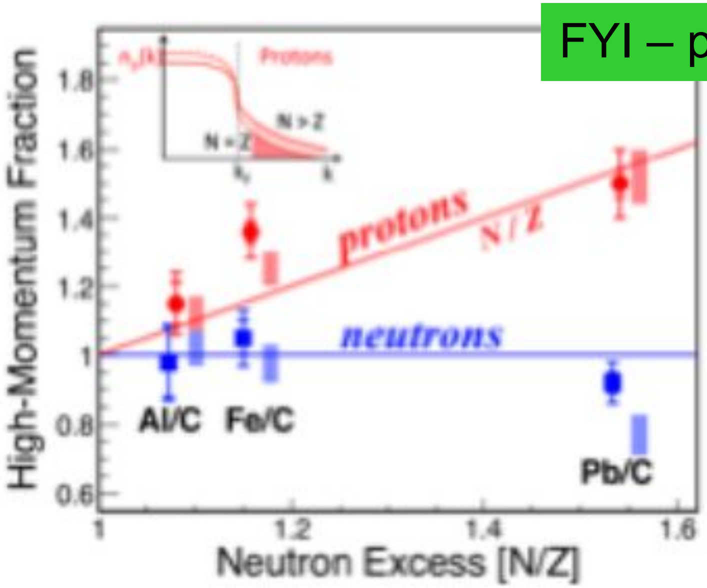


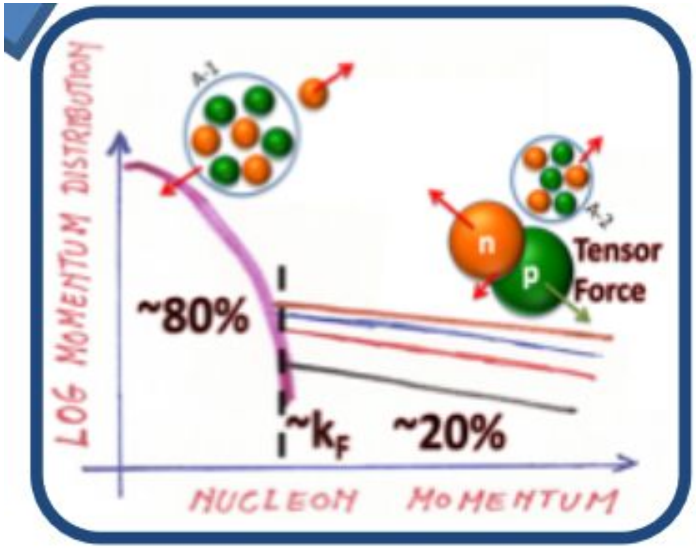


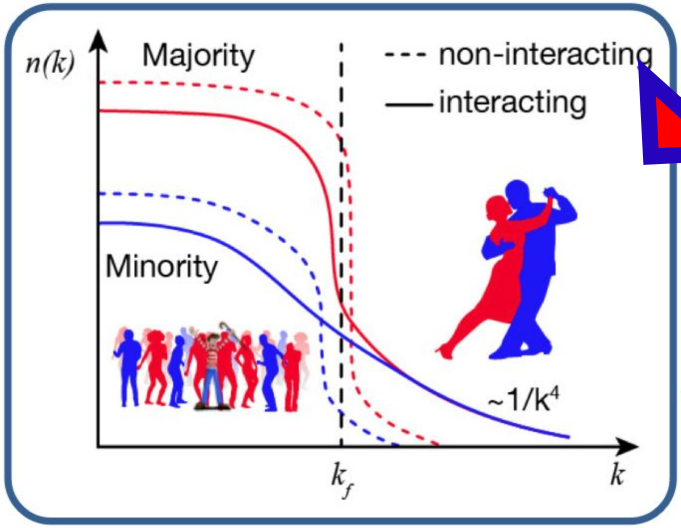
W. H. Dickhoff, C. Barbieri, *Prog. Part. Nucl. Phys.* **52**, 377 (2004).
Subedi et al., Science **320**, 1476 (2008), **O. Hen** et al., Science **364**, 614 (2014). M. Duer et al, Nature

FYI – peruse later



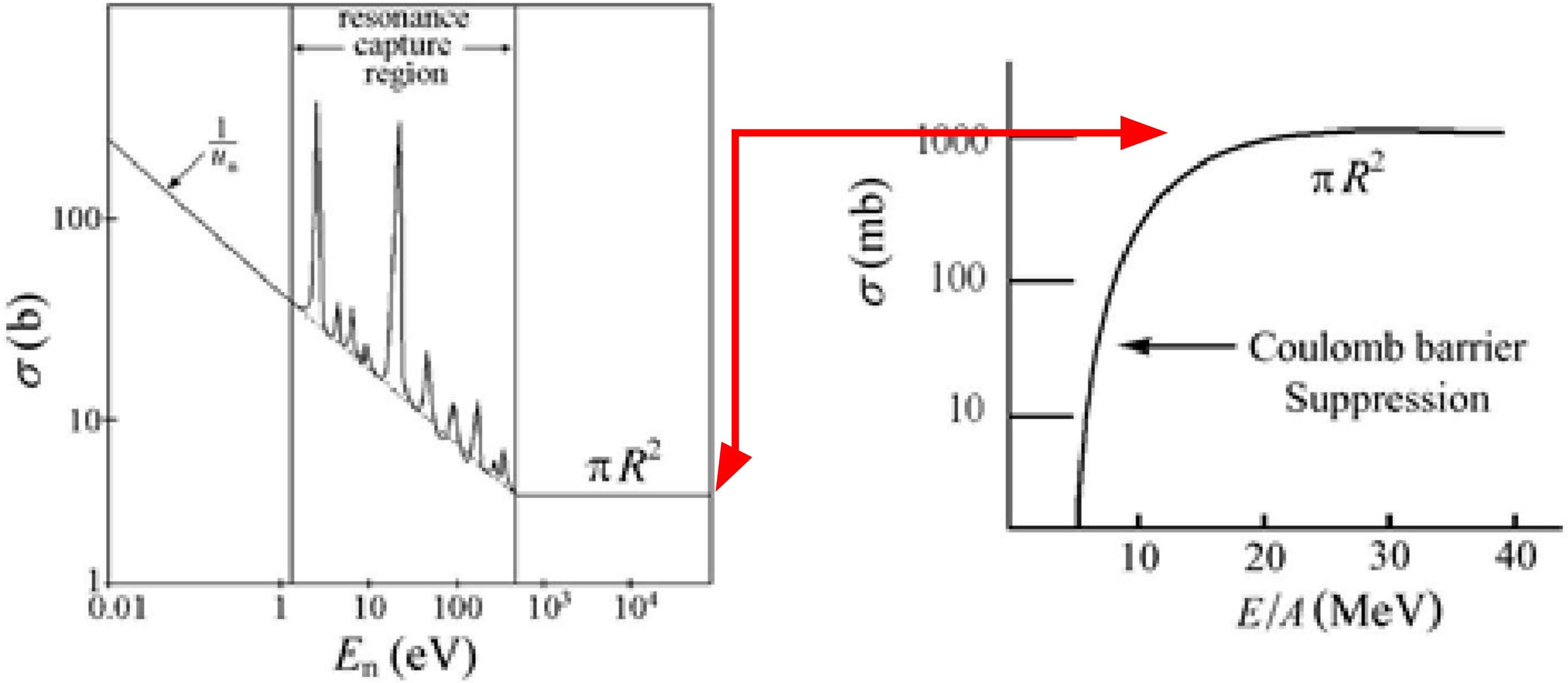






Work from the MIT group of Or Hen. Was expected from NM SCGF calculations but ….
Look forward*….. How does this picture evolve as*
the Fermi levels diverge and the continuum encroaches? Issue for your generation.

IV. Reaction types-1
induced by Neutrons* vs Charged-particles



Three regions each

sub threshold

$$K_{eq}\equiv\frac{Y_8}{{Y_4}^2}=\frac{q_8}{{q_4}^2}=\frac{\left[\frac{(2\pi m_8 kT)^{1.5}}{h^3}\right]^1}{\left[\frac{(2\pi m_4 kT)^{1.5}}{h^3}\right]^2}\{e^{-Q/kT}\}\cong\left[\frac{2\pi\hbar^2}{\mu kT}\right]^{3/2}\{e^{-Q/kT}\}$$

Resolved resonances

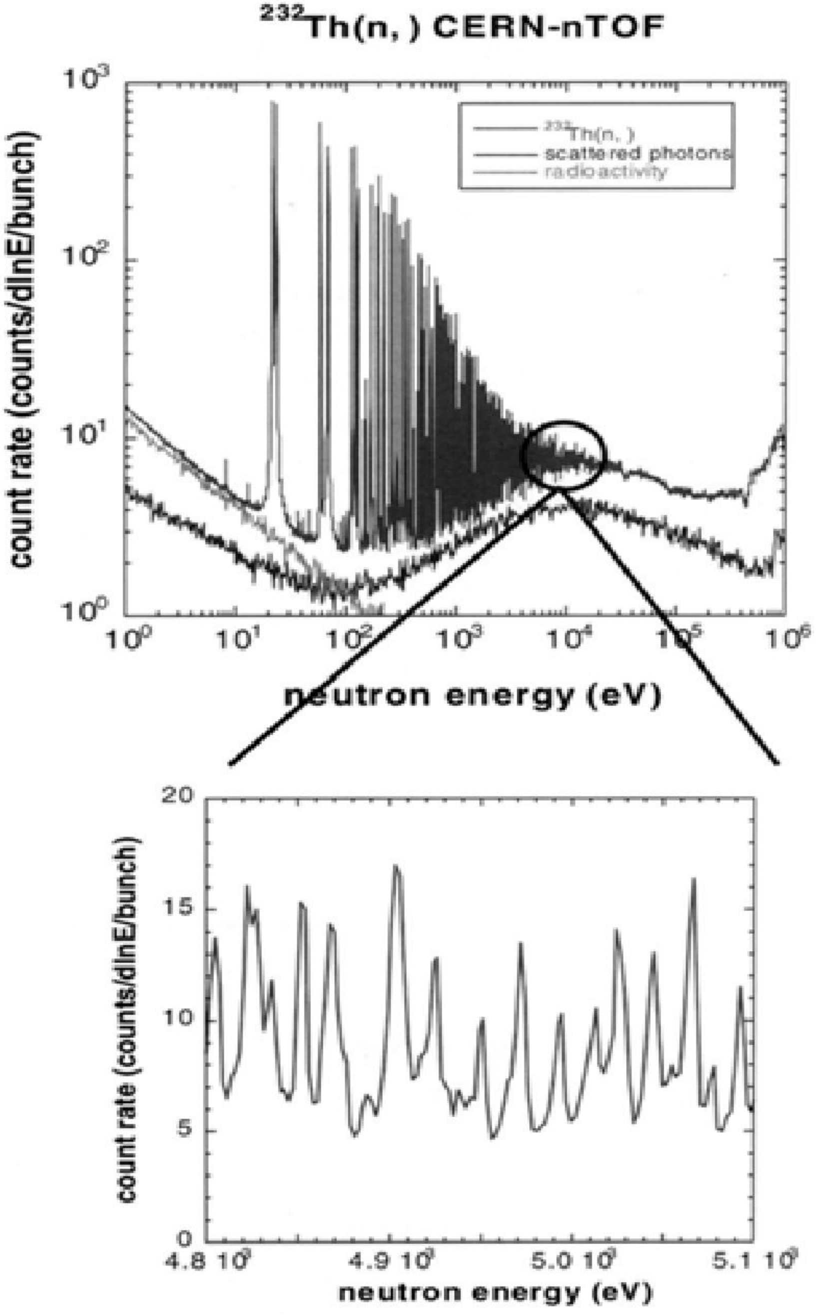
near barrier exponential increase

Geometry

Geometry

^{232}Th(n,x)

<u>**Density of states**</u> at
E* = 1 BE ….. is huge for heavy nuclei

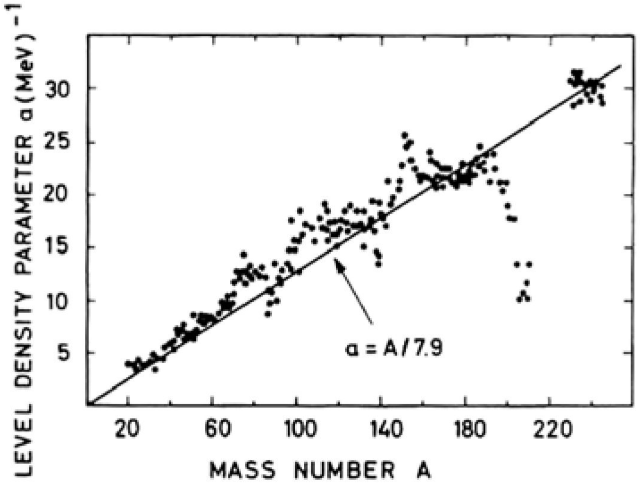


$E^* \propto aT^2$ □ $C_v \propto 2aT$
Same as $C_v^{metals}(T \to 0) \propto T$

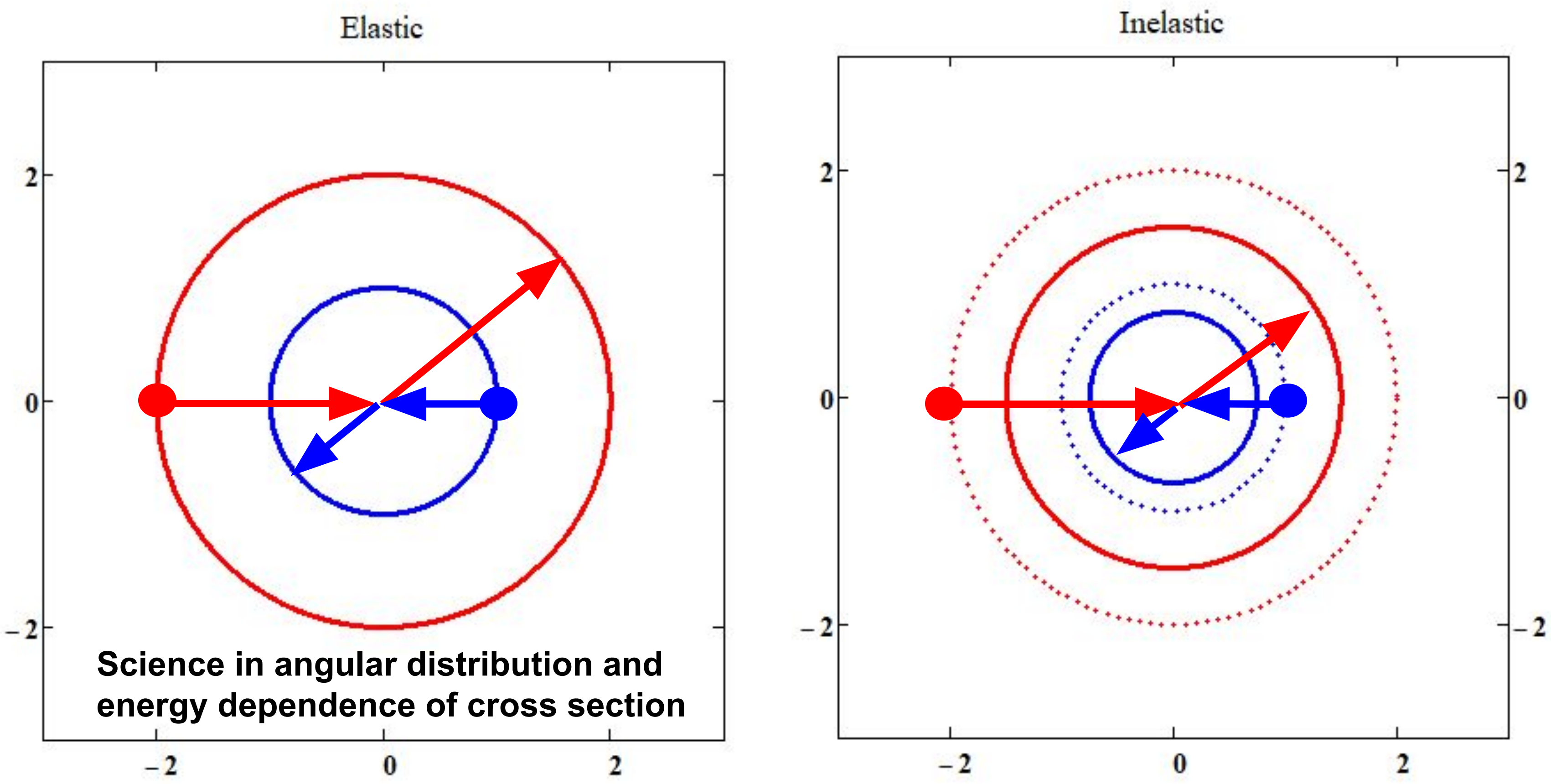
Formally: $T \equiv (\partial S/\partial E)^{-1}$ & $(\delta S/\delta T)_x = C_x/T$

□ $S^r(E^*) \propto 2aT$

Now (a' la Boltzmann) as $S = (k_B)\ln[\omega(E^*)]$
$\omega(E^*) \propto Ce^{2aT} \sim Ce^{2\sqrt{(aE^*)}}$
Level density constant a ~ A
□This is in exponential ! along with $\sqrt{E^*}$



Reaction types-2: Elastic …. Inelastic Standard 2-body kinematics in CM.

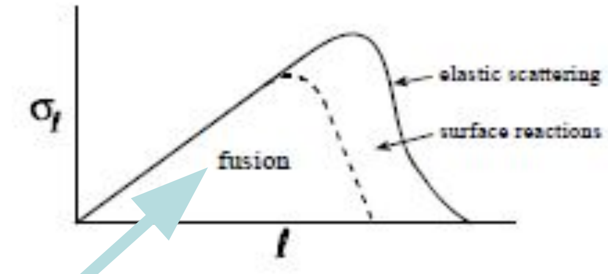


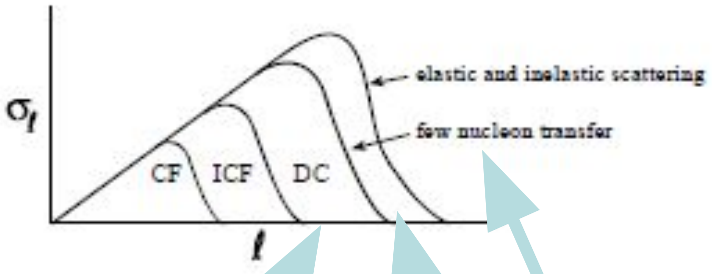
Science in angular distribution and energy dependence of cross section

Also…. in the inelastic category is **"super elastic" or "upscattering".**
Scattering only possible when reactants in excited states, e.g. isomers.
(Reminiscent of molecular anti-Stokes Raman – photon - scattering.)

Reaction types-3: L-wave (or impact parameter b) partition of LI□HI reactions
Low-energy (reaccelerated beams at FRIB)

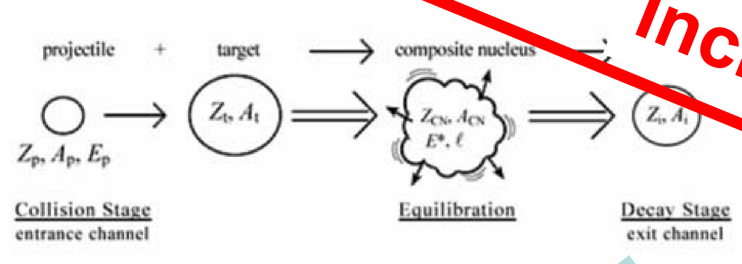
LI





HI

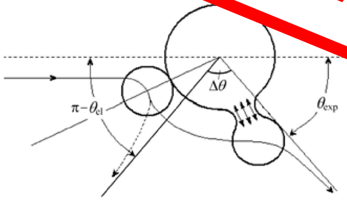
Fusion

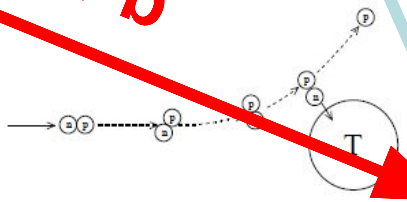


Increasing ℓ -wave or b

ply Inelastic
(DIC)
Mult
sfer

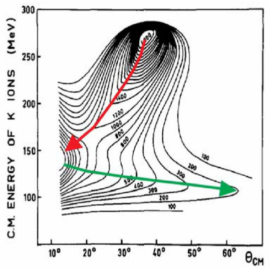
Simple Transfer



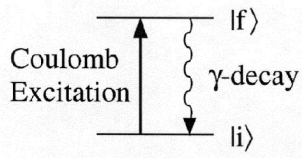


Elastic

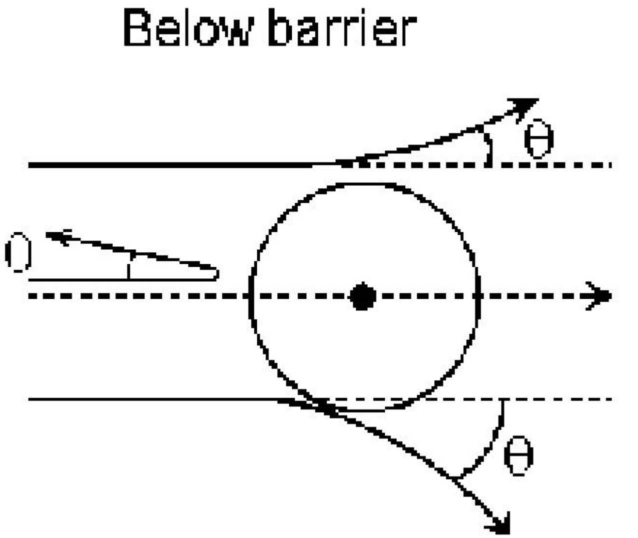
The exited CN
Will decay
“statistically”.
Learn some Stat-Model
(Hauser-Feshbach +) code
Rec: TALYS or Gemini

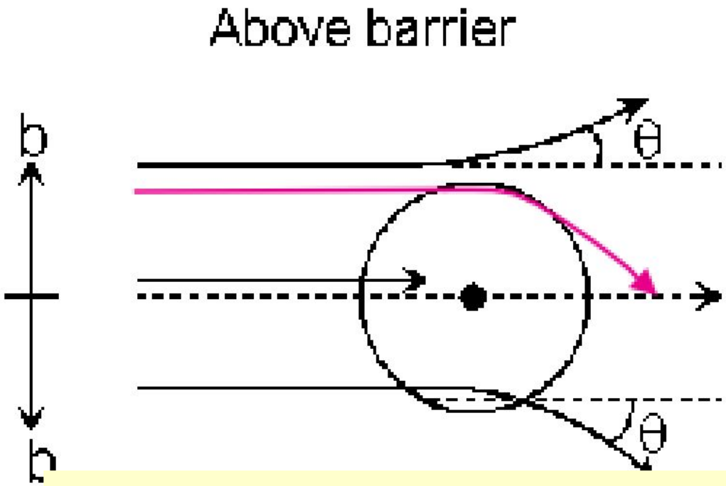


More central
More orbiting
More KE loss



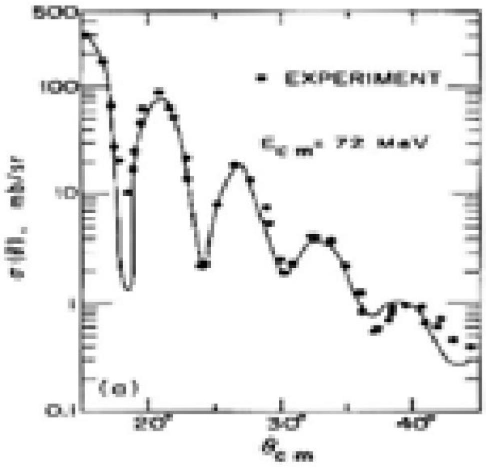
Examples
1. Elastic scattering (charged particles)

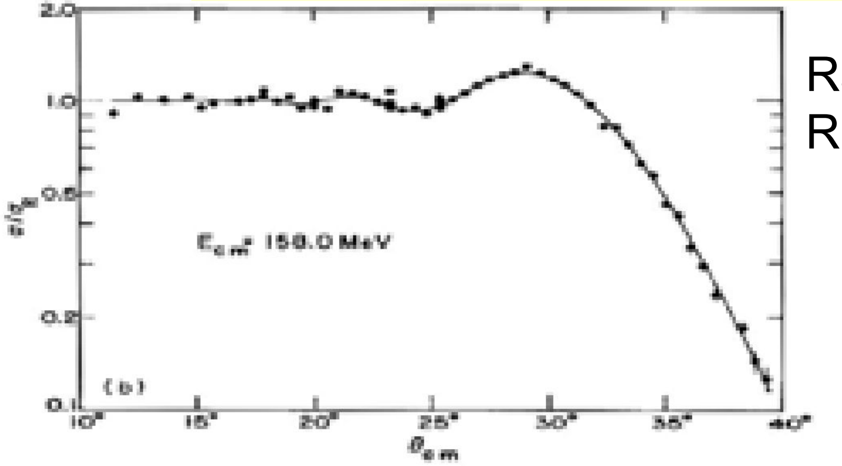




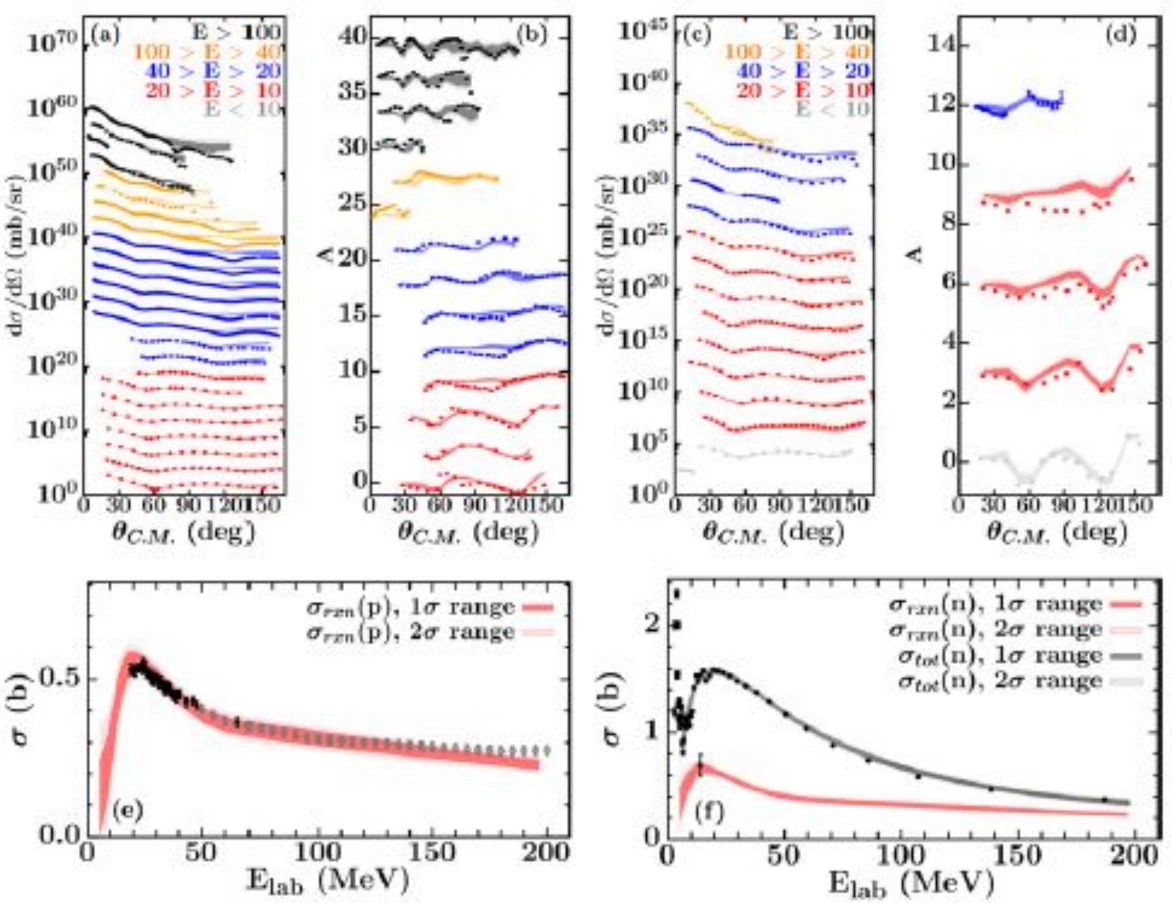
Note: opposite-side interference
AND loss of elastic flux

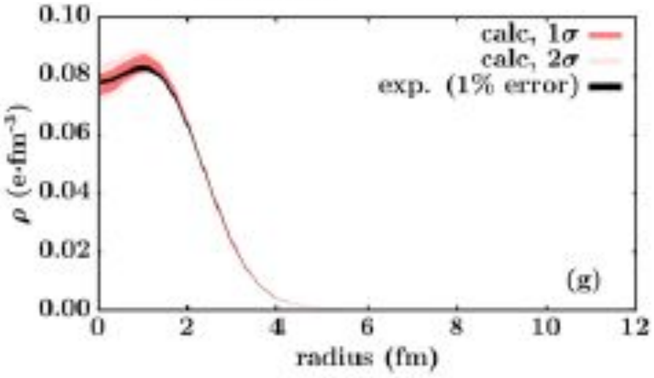
Absolute





Ratio to Rutherford





and fit to a
energy dependent & **complex potential**

Why complex and energy dependent?

First answer:
Imaginary component allows for loss from elastic channel!
But if complex, must obey dispersion relation.

Note: The elastic DATA **has been** punctuated/discretized in both energy and angle.This will **NOT** be the case in the future! Imagine elastic scattering done in an AT- TPC. Question: How to analyze such data?

-ve energy observables only for "DOM"

Fit with “Optical-Model” potential and “form factors”

$$\begin{aligned}
U(r,E) &= V(r,E) + iW(r,E),\\
V(r,E) &= -V_v(E) f_v(r) + 4a_s V_s(E)|f_s'(r) +\\
&\quad V_{so}(E) h_{SO}(r)[l \cdot s] + V_C(r),\\
W(r,E) &= -W_v(E) f_v + 4a_s W_s f_s'(r) +\\
&\quad W_{so}(E) h_{SO}(r)[l \cdot s].
\end{aligned}$$

$$\begin{aligned}
f_i(r, R_i, a_i) &= \frac{1}{1+\exp(\frac{r-R_i}{a_i})},\\
h_i(r, R_i, a_i) &= \frac{1}{2}\left(\frac{\hbar c}{m_\pi c^2}\right)^2 \frac{f'}{r} \sim 1\frac{f'}{r}.
\end{aligned}$$

What do we want from an OM potential?

1. **To predict** scattering for cases we cannot measure or cannot be bothered to measure. We cannot measure everything.
2. **To use** in, e.g. Hauser-Feshbach decay treatments, and reaction (DWBA) models
3. **To explain nuclear properties and to predict** difficult to measure nuclear quantities, e.g. A) Nuclear binding b) neutron skins

Again …
Complex to explain flux removal from elastic channel but then potential should obey dispersion relation. Dispersion links real and imaginary parts **and** –ve and +ve energy domains □ DOM.

FYI – peruse later

Kramers-Kronig relations (KKr)

See – Elements of Statistical Physics, Charles Kittel (pg 206-210)

R. de L. Kronig, J. Opt. Soc. Amer., 12, 547 (1926).
H.A. Kramers, Atti congr. Intern. Fis. Como 2, 545 (1927).

Real tells us imaginary , imaginary tells us real (after a fashion)
Frequency dependence of index of refraction □□ extinction coef.

Magnetic susceptibility, dielectric constant, impedance, conductivity

$$\chi = \chi' + i\chi''$$

$$M = \chi H$$

$$\varepsilon = \varepsilon' + i\varepsilon''$$

$$Z = R + iX$$

$$\sigma = \sigma_1 + i\sigma_2$$

And, as shown by Feshbach (58) and popularized by C. Mahaux (late 80's and 90's) the **nuclear "optical" potential (OM) should be causal** and therefore must obey KKr.

$$M(r;\varepsilon) = V(r;\varepsilon) + iW(r;\varepsilon)$$

Global OM (**GOM**) does not make use of KKr but the "dispersive" OM (**DOM**) does.

D$_{\text{ispursive}}$O$_{\text{ptical}}$M$_{\text{odel}}$ overview real- imaginary linked –ve and +ve E’s linked

$$M(r;\varepsilon) = V(r;\varepsilon) + iW(r;\varepsilon)$$

$$V(r;\varepsilon) = V_H(r;\varepsilon) + \Delta V_{dis}(r;\varepsilon)$$

Complex mean field

Real part = HF + dispersive

$$\Delta V_{dis}(r;\varepsilon) = \frac{P}{\pi}\int_{-\infty}^{\infty}\frac{W(r;\varepsilon)}{E-\varepsilon}dE$$

Dispersive part linked to imaginary by causality (Feshbach - 1958)

$$\frac{m^*(r;\varepsilon)}{m} = 1 - \frac{d}{d\varepsilon}V(r;\varepsilon)$$

Effective mass from energy dependence of real potential. What does m* mean?

$$S_{nlj} = \int_0^{\infty}[C_{nlj}u_{nlj}(r)]^2[\frac{m_H^*}{m}]_{Perey}[\frac{m}{\bar{m}}]_{dis}\,dr$$

Quasiparticle strength

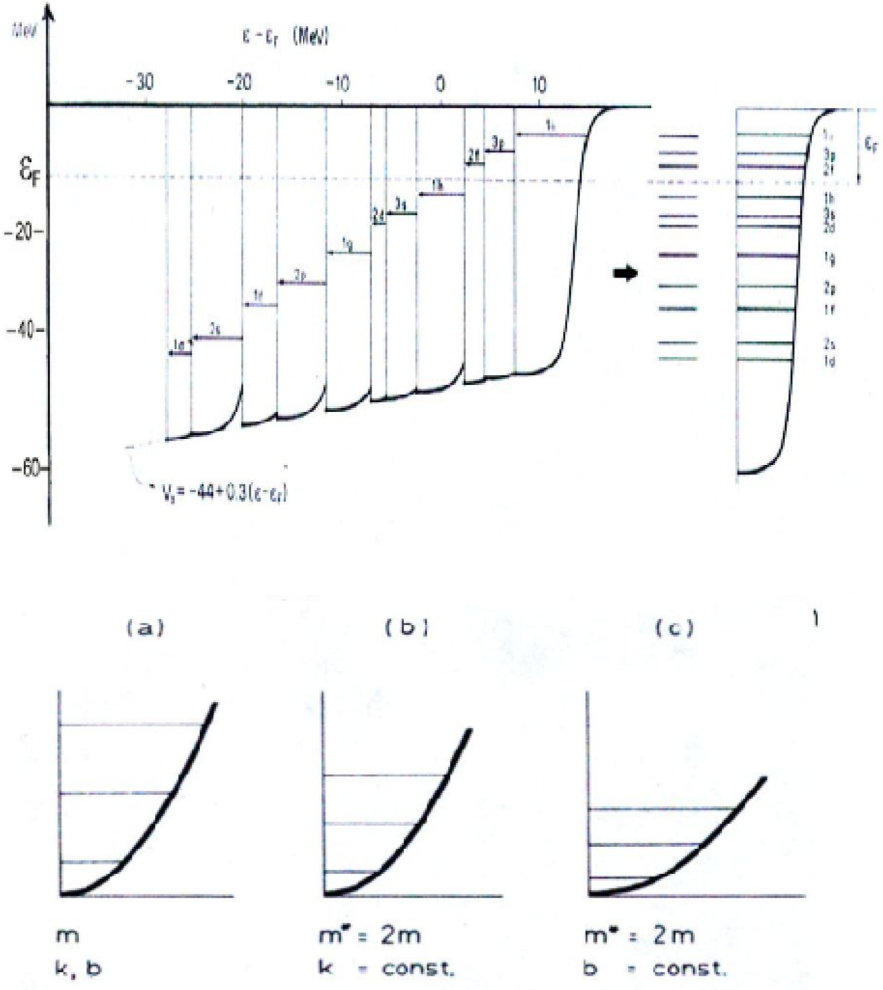
FYI – peruse later

Johnson and Mahaux, PRC 38, 2589 (1988)

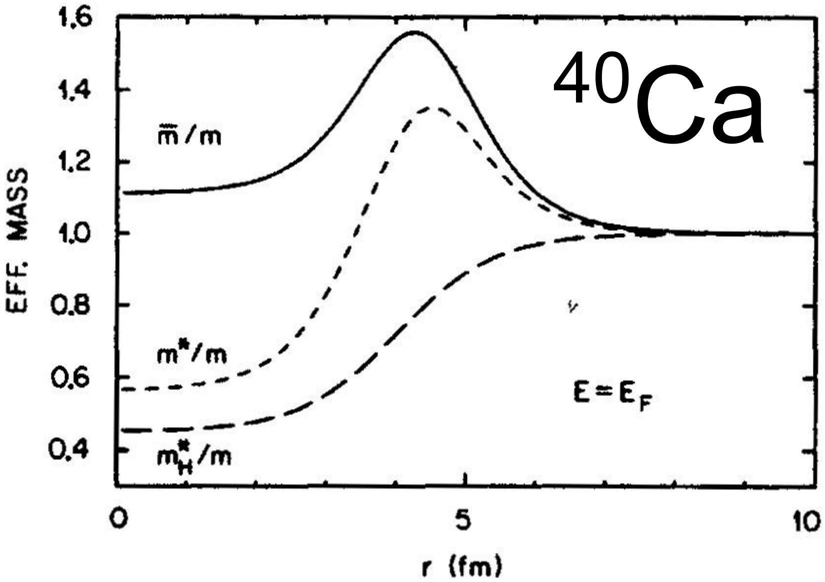
One can mock up a potential that linearly varies with energy by rescaling the mass.

<u>As in solid-state physics m* is a surrogate for the level density.</u>

(Small m* interband & high m* within a band.)



Find that – for **finite nuclei**
m* must have radially dependence.



Dispersive contribution (solid)

Total (short dashed)

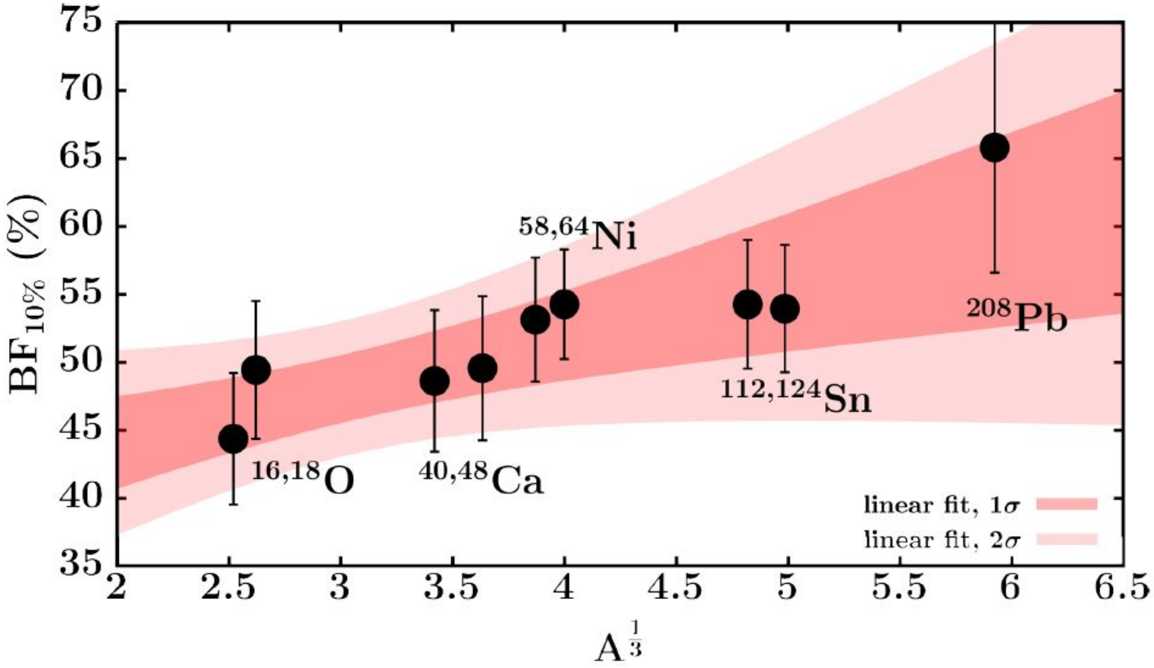
HF contribution (long dashed)

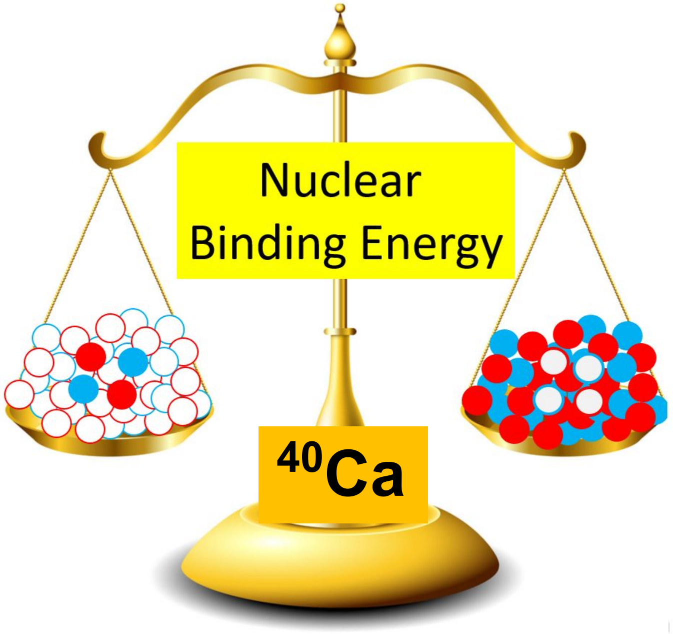
Can view surface bump as
consequence of surface vibs.

FYI – peruse later

Because –ve and –ve energies linked via dispersion A DOM informs more than on just scattering observables. Where does BE come from?

Bernie plot: 10% most bound nucleons => ~ 50% of binding!



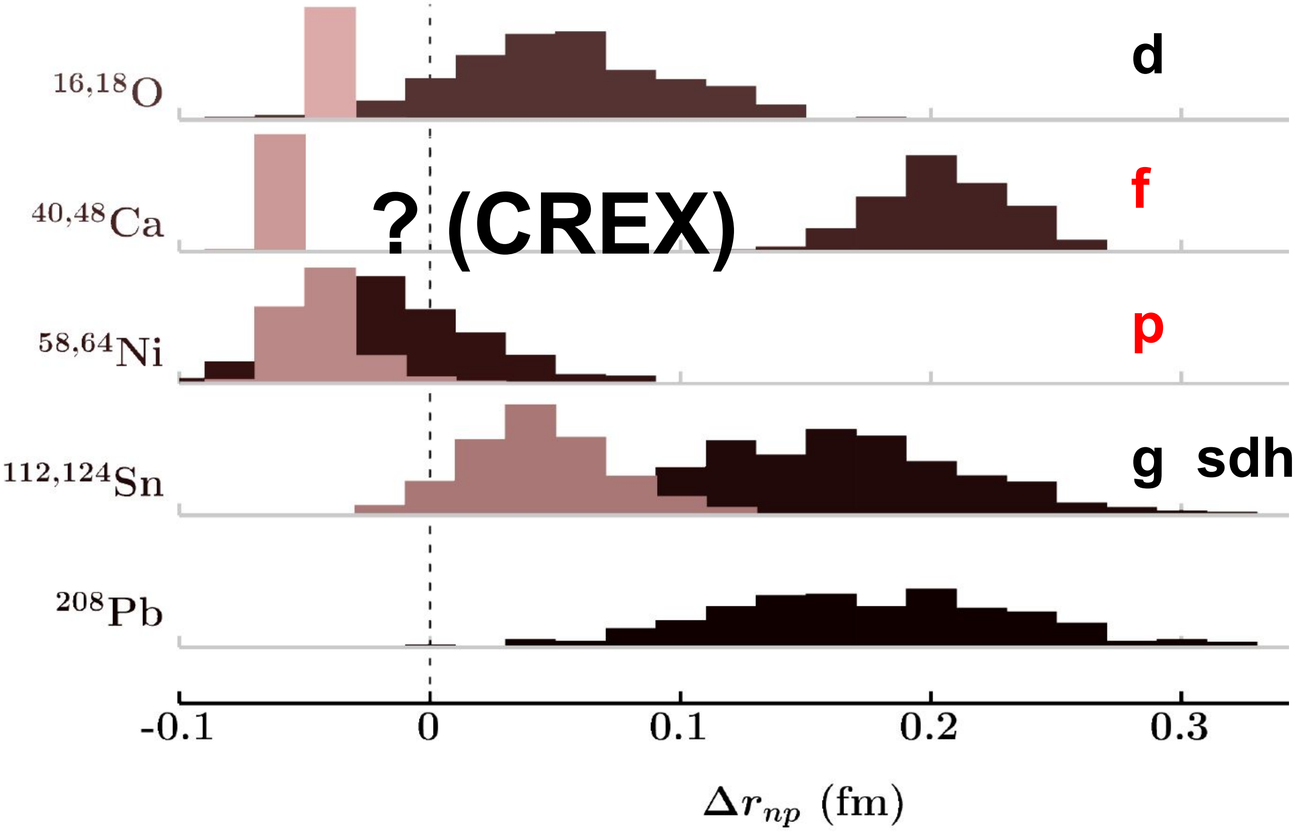


This is about 50% more BE inequality than a SP picture would predict.

Feel the Bern?

FYI – peruse later

Neutron skins □ N-star mergers □ tidal deformability (5th PNE) term



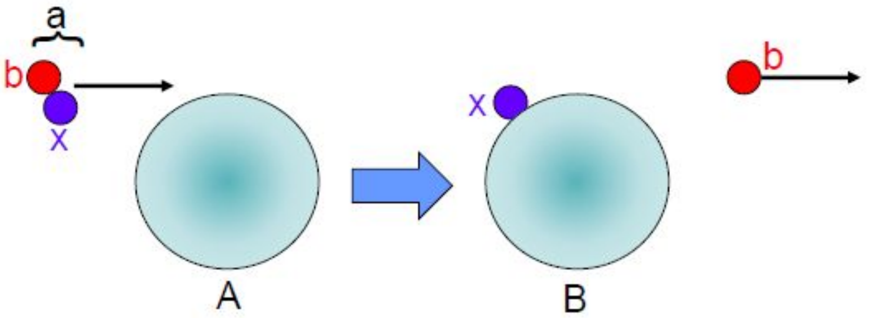
L (slope asymmetry E)
yes but also

Coulomb & **Structure**
(L-wave of added n's)

MCMC analysis In conflict with CREX
Issue for your generation to figure out.

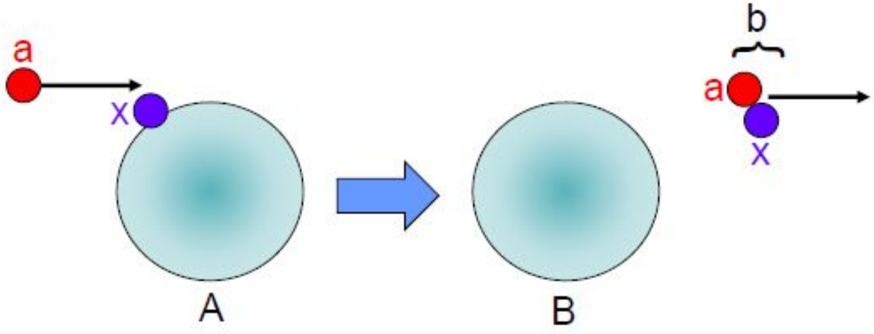
2. Direct transfer reactions

Stripping and Pickup



Adding nucleon(s) to A:
"x" is transferred from a to A, making B=A+x
and b=a-x

Known as "Stripping"
x can be one or more nucleons



Removing nucleon(s) from A:
"x" is transferred from A to b, making B=A-x
and b=a+x

Known as "Pickup"
x can be one or more nucleons

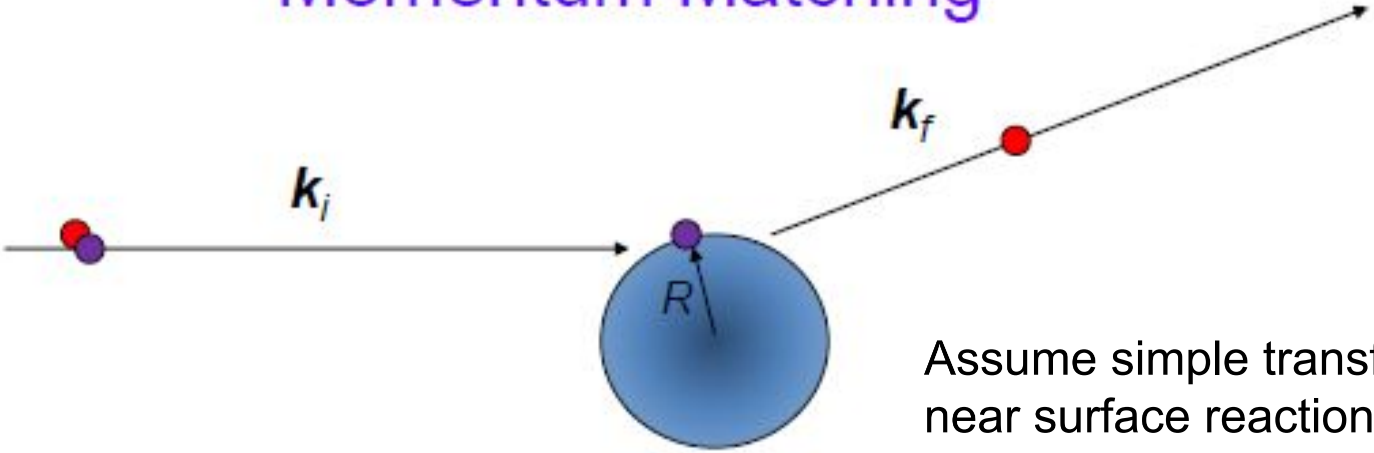
Stripping "surrogate" for neutron capture.

Single-nucleon transfer reactions preferentially populate states with strong single-particle character.

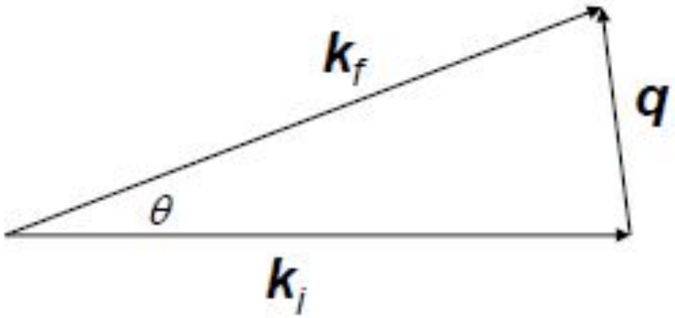
They are also subject to some simple momentum-matching conditions which inform On the ℓ of the transferred nucleon.

These figures curtesy of Alan Wuosmaa

Some simple considerations: Momentum Matching



Assume simple transfers are near surface reactions.



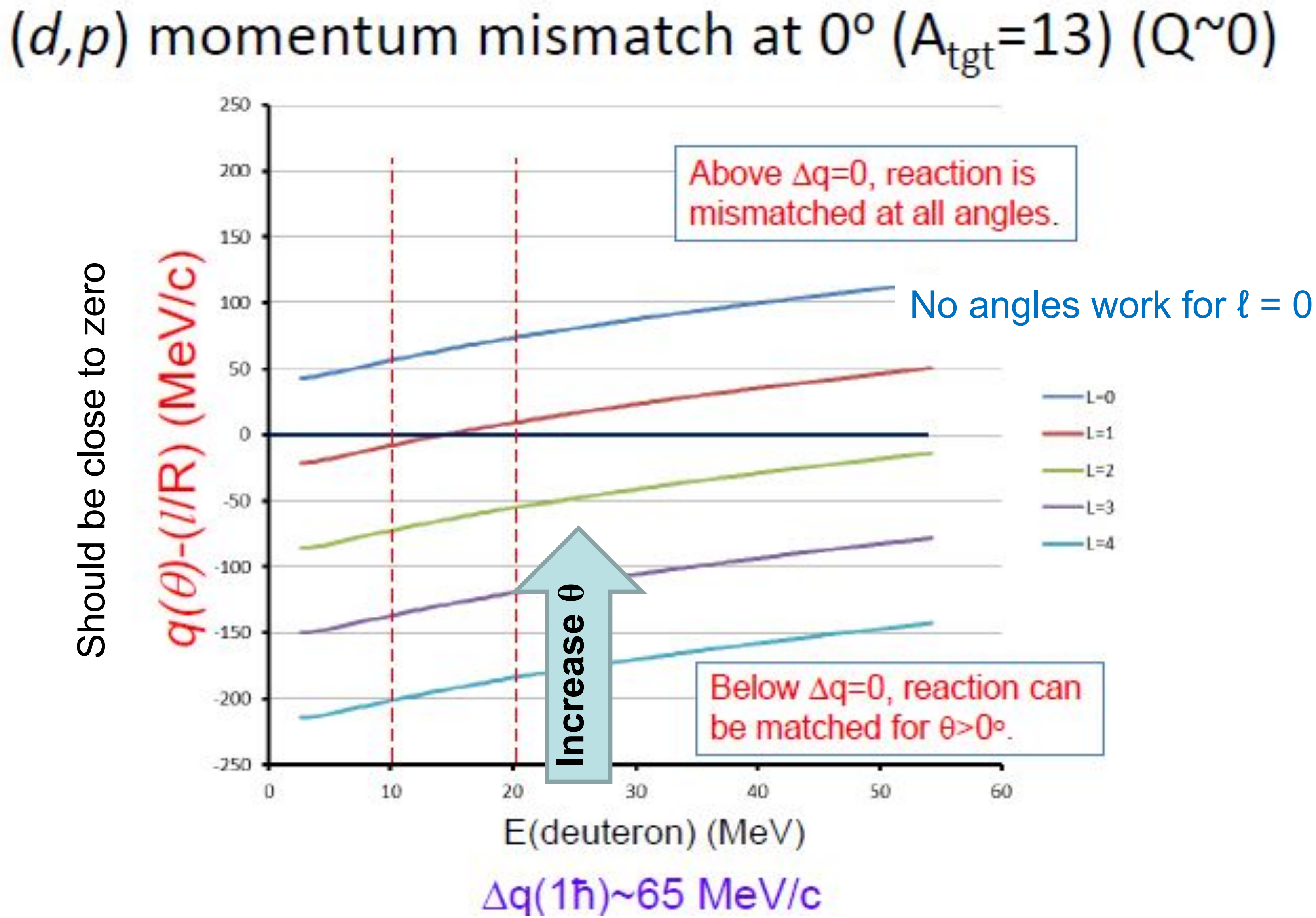
angular momentum of transferred particle = $qR = l$, or $q = l/R$
This roughly fixes the best angle for transfer:

$$q^2 = k_i^2 + k_f^2 - 2k_i k_f \cos\theta$$

$$\theta_{\max} = \cos^{-1}\left(\frac{k_f^2 + k_i^2 - (l/R)^2}{2k_f k_i}\right)$$

Small angle □ [cos(th) ~ 1] □ q small

While there is a matching condition □ general mismatching exists



(*d*,*p*) momentum mismatch at 0° (A_{tgt}=13) (Q~0)

Above Δq=0, reaction is mismatched at all angles.

No angles work for ℓ = 0

Below Δq=0, reaction can be matched for θ>0°.

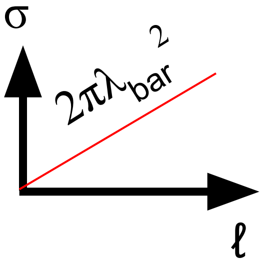
Should be close to zero

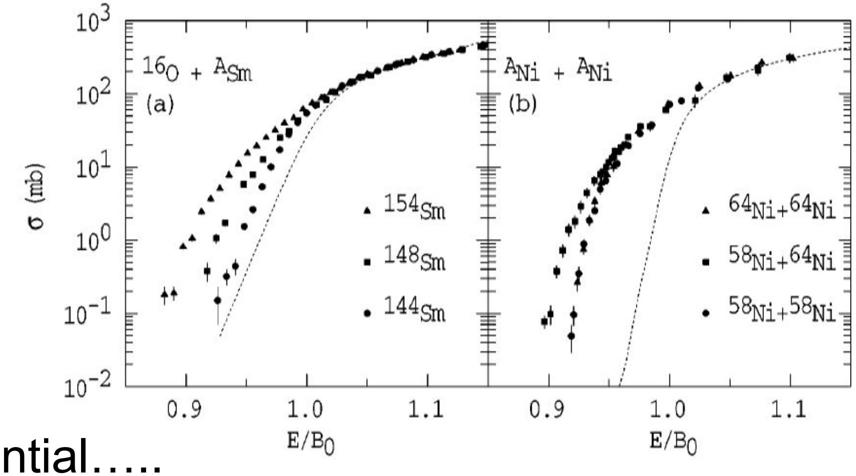
Δq(1ħ)~65 MeV/c

We shall return to a consequence of similar considerations on spin alignment.

3. HI fusion basics

$$\sigma = \frac{\pi\hbar^2}{2\mu E}\sum_{l=0}^{\infty}(2l+1)T_l(E)$$





Transmission coefs [$T_l(E)$] from OM

OR if assume a single inverted parabolic potential…..

$$T_l(E) = \frac{1}{1+\exp[(2\pi/\hbar\omega)(B-E+\hbar^2 l(l+1)/2\mu R^2)]}$$

Substantial enhancement over 1d (uncoupled) barrier penetration logic.
A logic that works reasonable well for α decay.

$$E\sigma = \frac{\hbar\omega R^2}{2}\ln(1+\exp[2\pi(E-B)/\hbar\omega]) \equiv \frac{\hbar\omega R^2}{2}\ln(1+e^x)$$

$$\frac{1}{\pi R^2}\frac{d(E\sigma)}{dE} = \frac{1}{(1+e^x)}$$

$$\frac{1}{\pi R^2}\frac{d^2(E\sigma)}{dE^2} = \frac{2\pi}{\hbar\omega}\frac{e^x}{(1+e^x)^2} \equiv G(E-B).$$

With first and second derivatives.

G is the QM resulting spreading of the 2nd der.

Assuming a distribution in Barriers D □
Then ….

$$\sigma(E) = \int \sigma(E,B)\,D(B)\,dB$$

$$\frac{1}{\pi R^2}\frac{d^2(E\sigma)}{dE^2} = \int G(E-B)\,D(B)\,dB$$

Classical result

$$\frac{1}{\pi R^2}\frac{d^2(E\sigma)}{dE^2} = D(E)$$

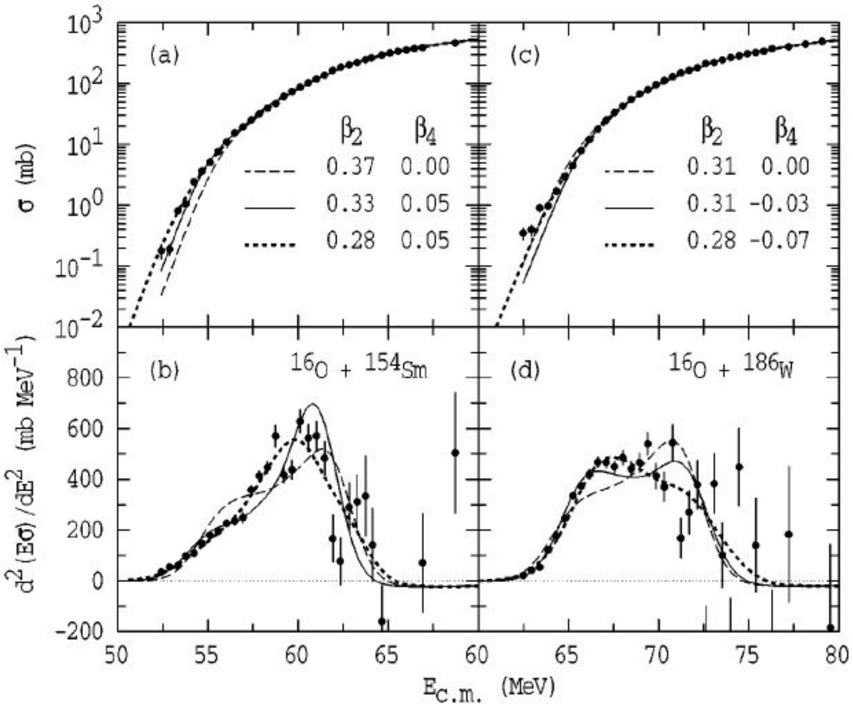
B. Balantekin and collaborators, PRC **28**,1565 (1983); **33**, 379 (1986).
ANU: Dasgupta, Hinde, **Rowley,…**Ann. Rev. Nucl. Part. Sci., **48**, 401 (1998).

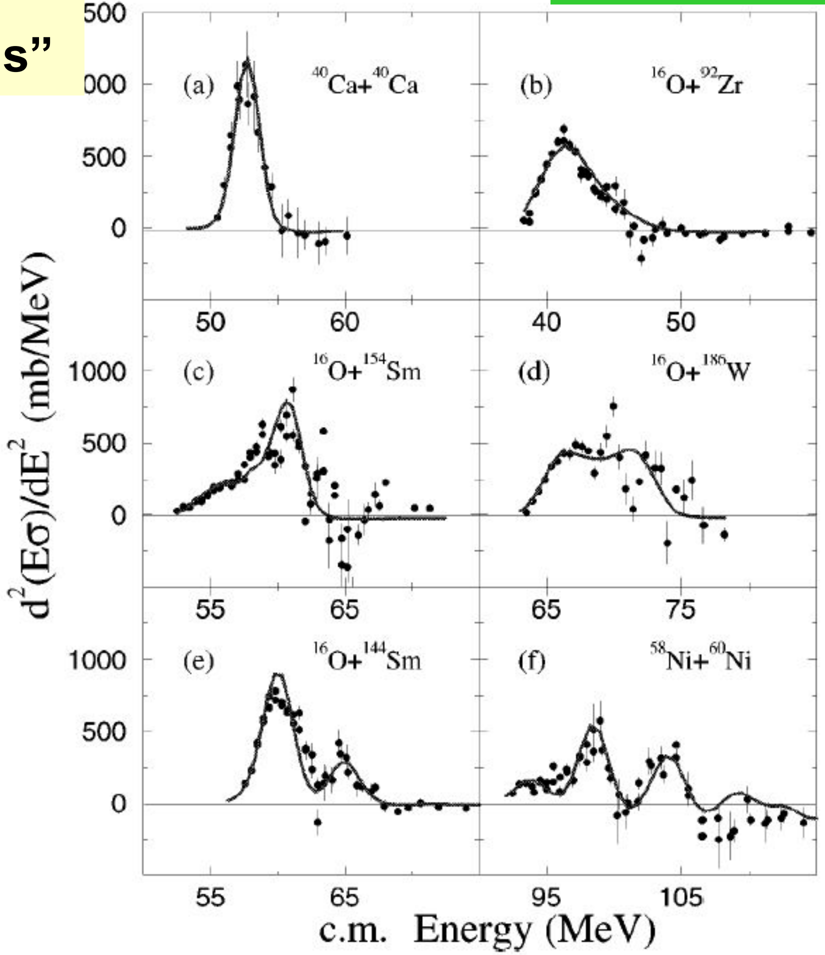
FYI – peruse later

Looking back,
Some old cases of "barrier distributions"

^{16}O +

^{154}Sm ^{186}W

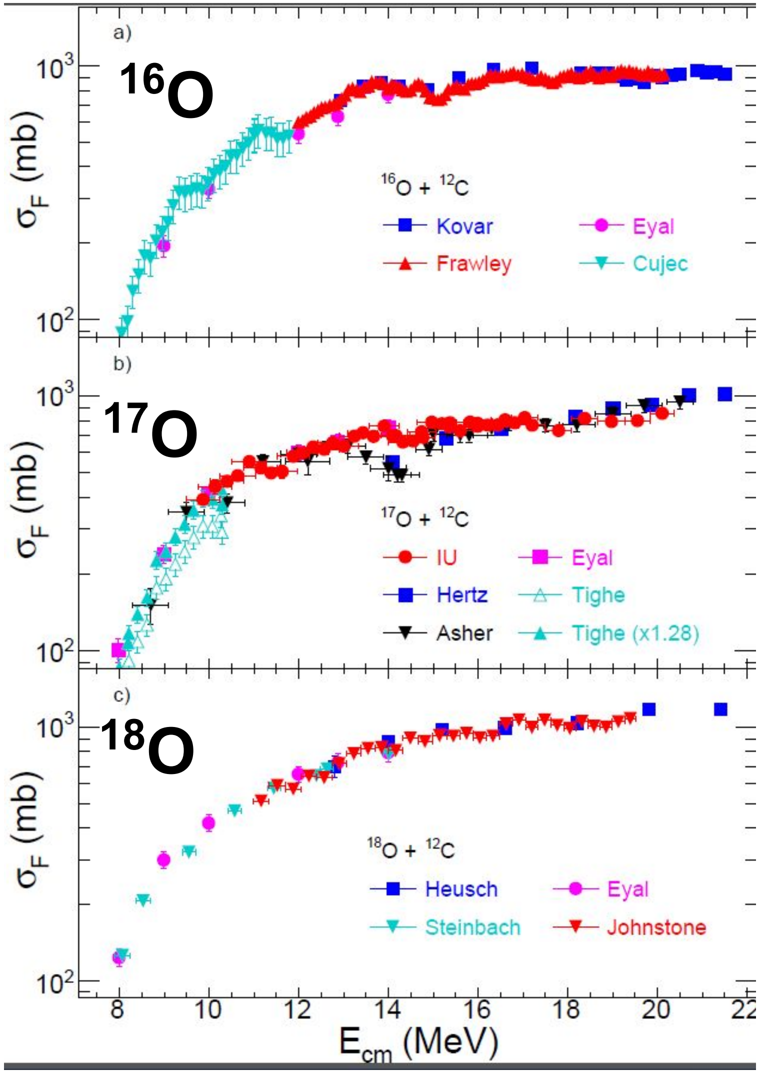


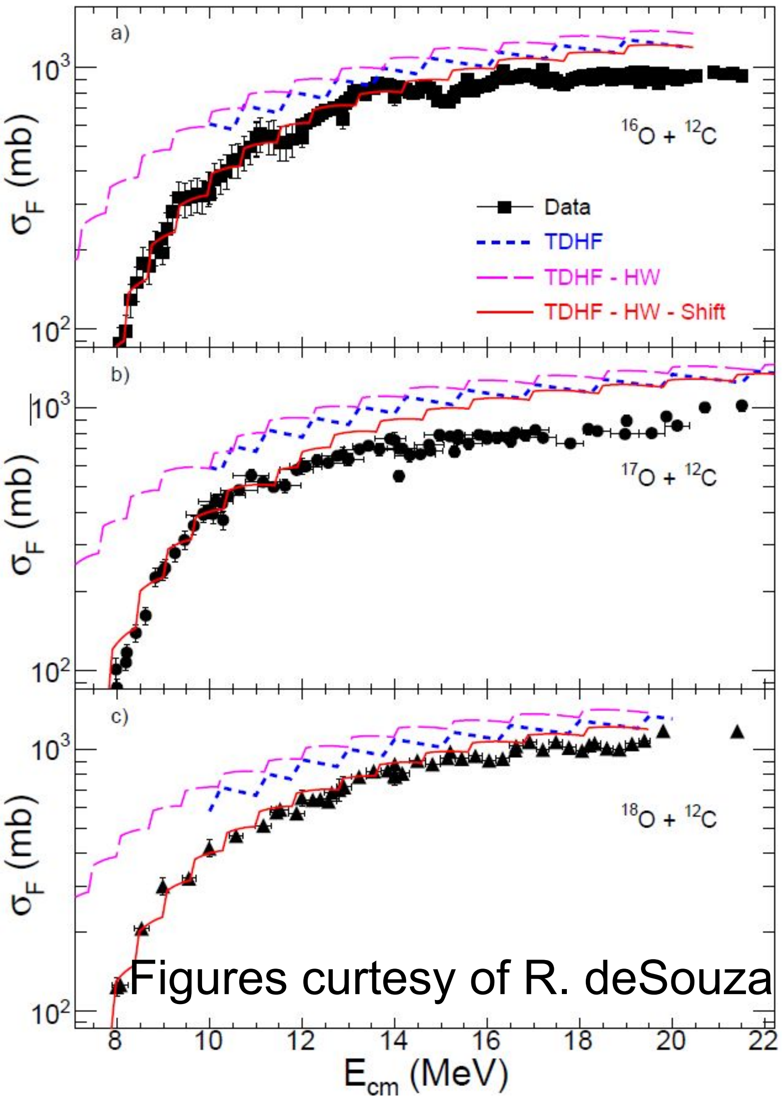


Dashed lines: quadrupole only.
Solid lines: include hexadecapole.
The two nuclei have different signs hexadecapole signs.

a) Double closed shell
b) Continuous distribution (phonon coupling)
c) continuous – deformed target (+ve β_4)
d) deformed (-ve β_4)
e) second peak … phonon coupling
f) multiple barriers…due to surface vibrations

Looking Forward: Fusion of nuclei on □ well off stability
First – some old/new data Second TDHF (TDDFT)





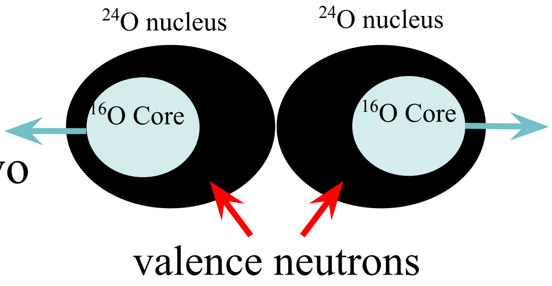
Figures curtesy of R. deSouza

Are the cross section ripples “L-wave ratcheting” ? Theory suggests …maybe.

A second issue : are polarization effects observed?

As the two nuclei approach, the barrier can change through polarization of the fusing nuclei.

Polarization in the fusion of two neutron-rich nuclei

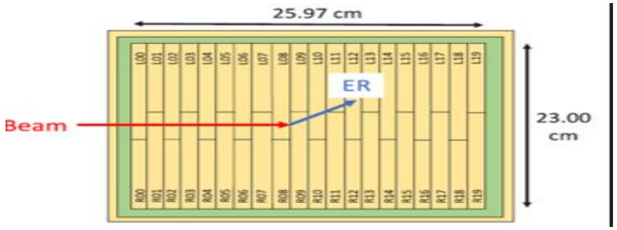


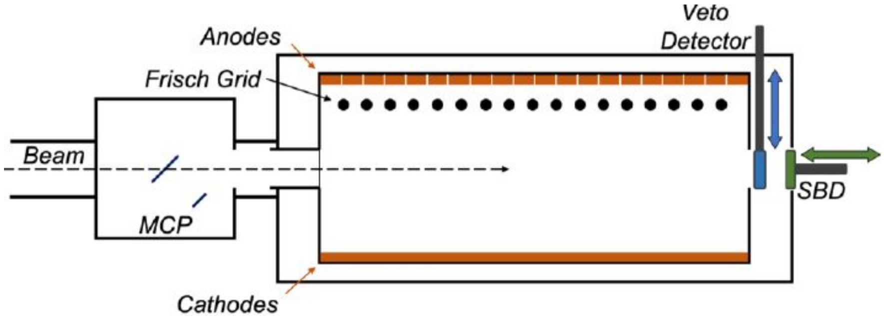
The tool:

- **Active target MUSIC’s (multi-sampling ICs).** Now with good starts and zero deg veto detectors.
- MUSICs were introduced in the 1970, but the newer generation, suitable for secondary beams are far superior.
- These are much simpler than TPC’s and The appropriate technology.
- Experimental credo: “Keep it simple stupid.”

MUSICs

First used in the 1970’s
But now much improved

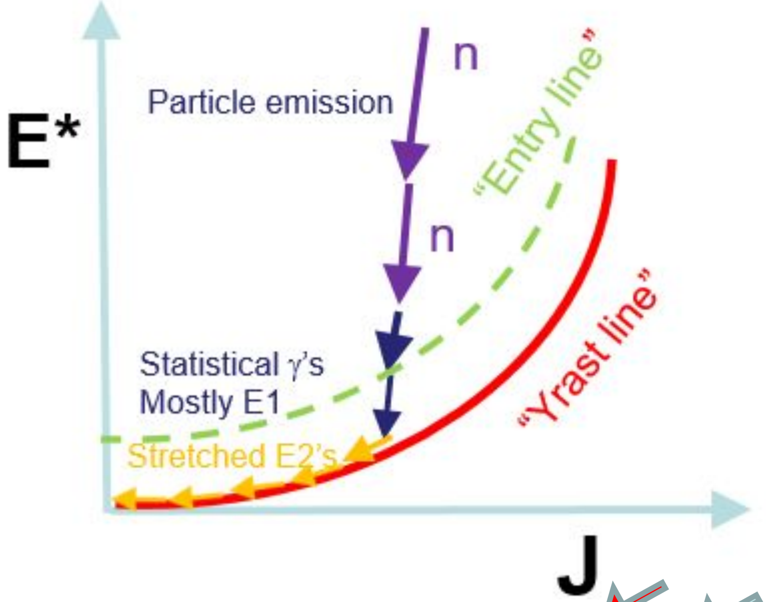




4a: CN decay: Hauser-Feshbach (HF) + Transition-State Theory (TST)
(Eyring, Polanyi, Wigner & Kramers) model
how nuclei dispose excitation energy and angular momentum.

Basic idea: at each (E*,J) point the (one-way) rates are calculated for ALL possible states. In Monte Carlo codes, the next decay step is determined by the fractional width. The decay proceeds until all E* and J are exhausted.

The ingredients are:
The **state density** **<u>ω(E*)</u>** of the daughter and (for HF) **transmission coefficients** **<u>T_ℓ (ε)</u>** of the emitted particle, the latter from an OM.



$$
\begin{aligned}
R_{i\to f}(E_i, J_i \quad &\to \quad E_f, J_f)dE = \frac{1}{\hbar\ \pi\lambda\!\!\!^{-2}}\sigma_{i\to f}\, g_s \left(\frac{w_f(E_f, J_f)}{w_i(E_i, J_i)}\right) \\
&\text{with} \\
\sigma_{i\to f} \quad &= \quad \pi\lambda\!\!\!^{-2} \frac{2J_i+1}{(2s+1)(2j+1)} \sum_{S=|j-s|}^{S=j+s} \sum_{l=|J_i-S|}^{J_i+S} T_l(\varepsilon) \\
g_s \quad &= \quad \frac{(2j+1)(2s+1)}{(2J_i+1)}, \\
w^L(E, J) \quad &\sim \quad \frac{C}{U^2} e^{\sqrt{aU}} \quad \text{with } U = E - E_{\text{col}}(J) \text{ and} \\
E_f \quad &= \quad E_i - S_\tau - \varepsilon. \qquad (13.3)
\end{aligned}
$$

$$\Gamma_f^{BW}[MeV] = \left(\frac{1}{2\pi w_{mn}^L(E^*)}\right) \int_{\varepsilon=0}^{E^*-B_f} w_{sp}^L(E^*-B_f-\varepsilon)d\varepsilon$$

$$E^* = aT^2$$

As dU=+SdT-PdV, the entropy[18] is

$$S = \left(\frac{dE^*}{dT}\right)_v = 2aT,$$

and thus, using the Third Law, S = k_B ln $\omega(E^*)$, one would expect the density of levels[19] to be roughly

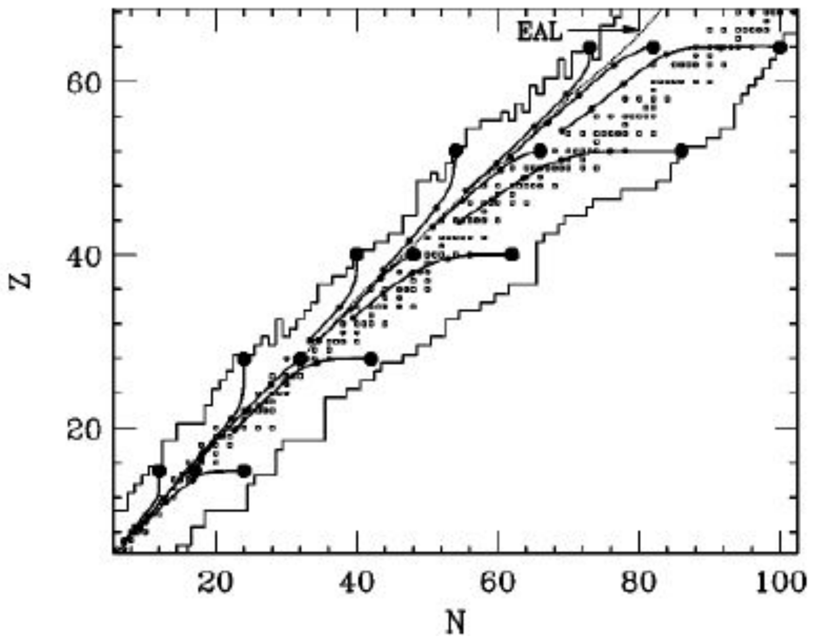
$$w^L(E^*) \sim Ce^{2aT} \sim Ce^{2\sqrt{aE^*}}.$$

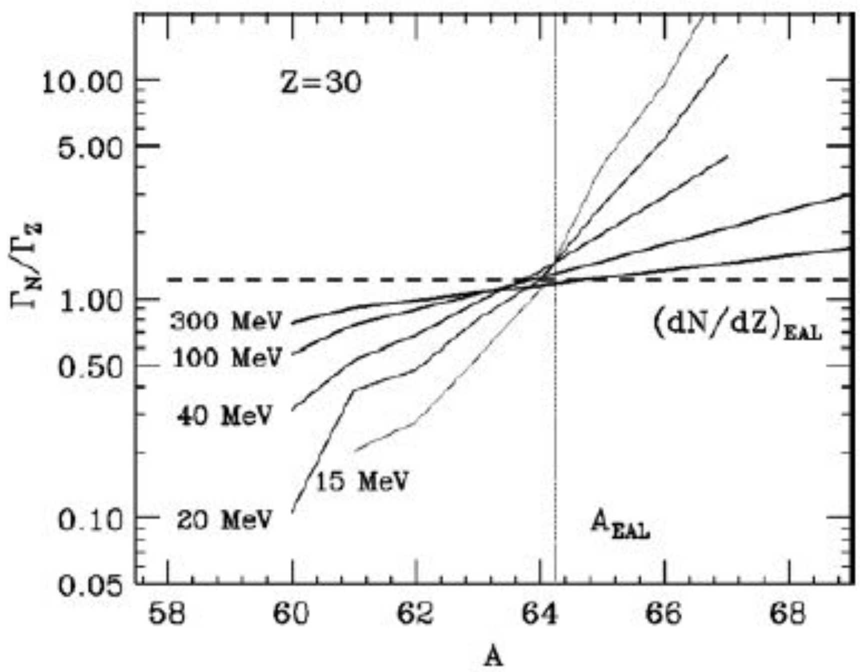
$$\Gamma_f^{Kram}[MeV] = (\sqrt{1+\gamma^2} - \gamma)\Gamma_f^{BW}$$

4b. CN decay: Evaporation attractor Line (EAL)

R. J. Charity PRC 58, 1073 (1998).

□ Evaporation corridor





The Coulomb barrier reduces the CP decay widths so the EAL is “West” of the line of stability.

The EAL determines fragment yields at FRIB.

Attractor defined by condition

$$\frac{dN}{dZ} = \left\langle \frac{\Gamma_N}{\Gamma_Z} \right\rangle$$

and ~ lies on the chart at

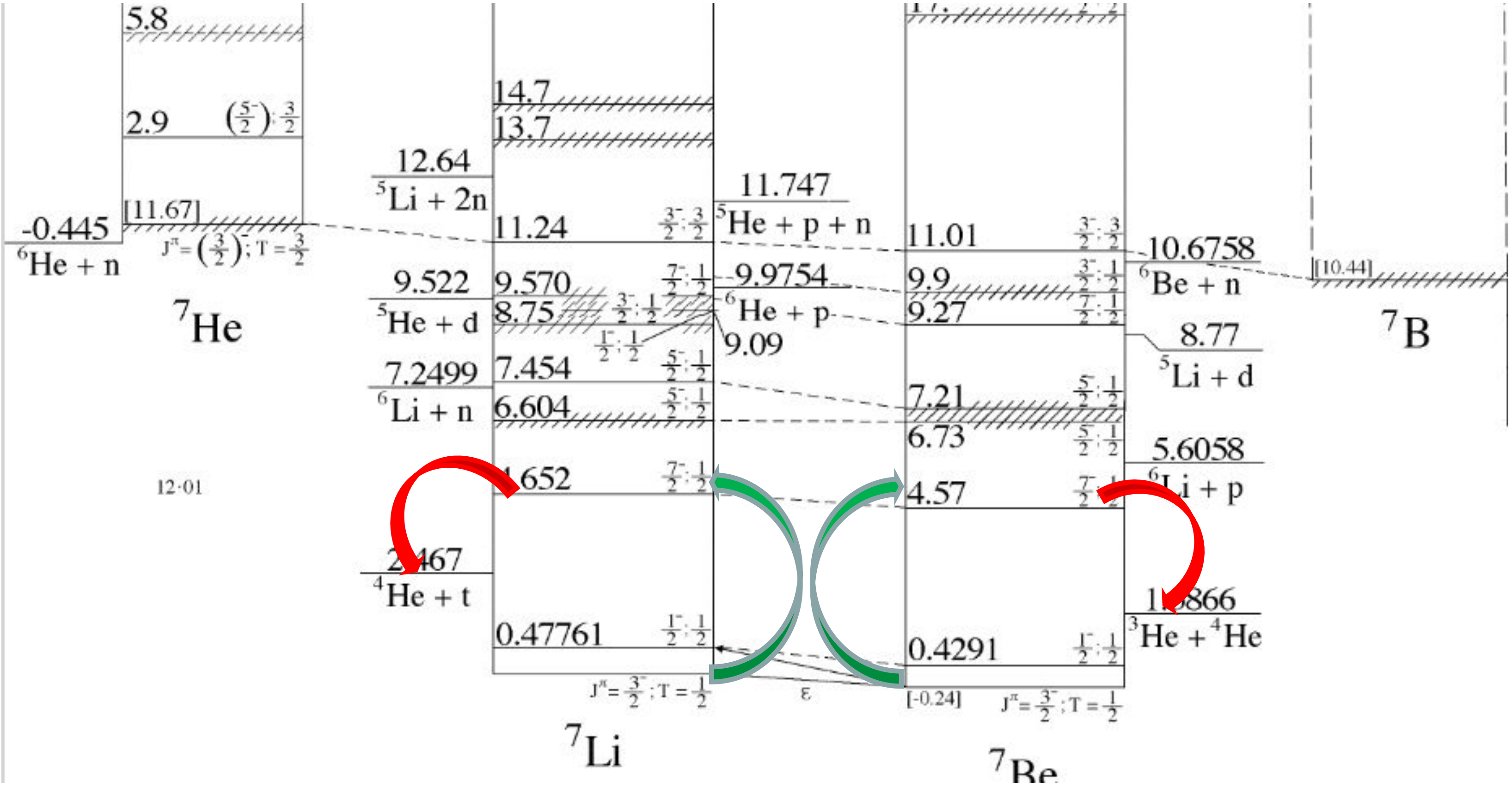
$$Z = 0.909N - 1.12\times 10^{-3}N^2$$

or

$$N = 1.072Z + 2.32\times 10^{-3}Z^2.$$

Quiz question: why do particle “evaporation” spectra look the way they do?

5. Spin alignment at intermediate energy. A consequence of E*- q matching.



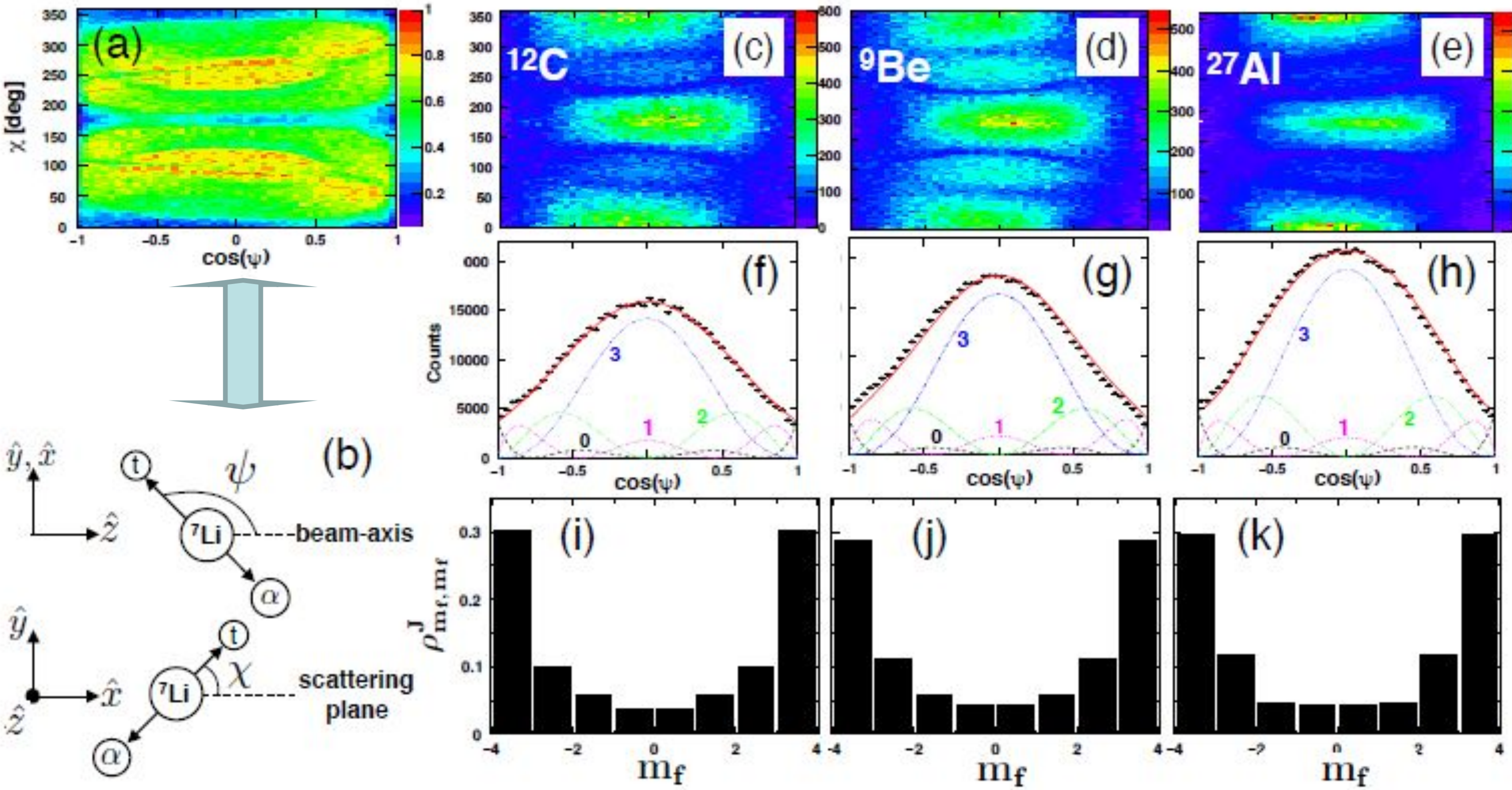
a. Inelastically excite A_{gs} = 7; $3/2^{-}_{gs}$ □ $7/2^{-}$ (acquires two units of spin).

b. Decay into α + t/^{3}He (removes two units of spin).

c. Decay is NOT isotropic (in rest frame) , i.e. m-states are NOT uniformly populated! Now observed in a ½ dozen inelastic excitations. & predicted by FRESCO.

Under fairly general conditions at NSCL energies… The decay is predominately transverse to beam

$$^{7}\mathrm{Li}_{3/2} + \mathrm{X}_{gs} \;\square\; (^{7}\mathrm{Li}^{*}_{7/2}) + \mathrm{X}_{gs} \;\square\; (t + \alpha) + \mathrm{X}_{gs}$$



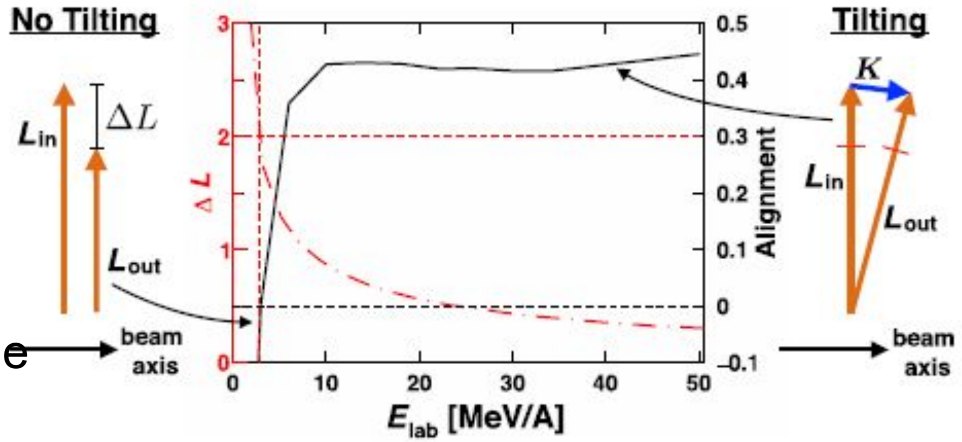
R. J. Charity et al., Phys. Rev. C. **91**, 024610 (2015).
D.E.M. Hoff et al., **Phys. Rev. Lett.** **119**, 232501 (2017); Phys Rev. C (2018).

Alignment has not been fully exploited

The “ go of it”

FYI – peruse later

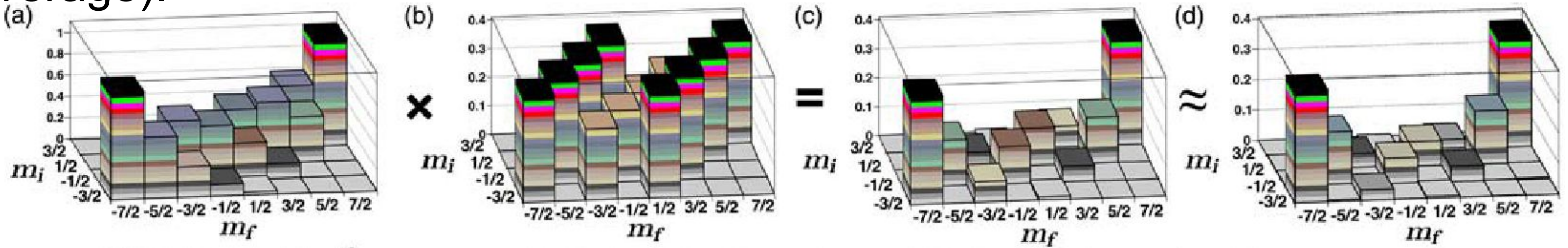
Step 1: An angular-momentum – excitation energy-mismatch
This compels “tilting” above a certain beam energy.



Step 2: The transition matrix (defining the m-state distribution) can ~ be written as the sum of an “internal” and “external” CG coeffs.

$$T^L_{m_i,m_f} \propto \langle J_i, m_i; K, M | J_f, m_f\rangle \langle L, 0; K, M | L, M\rangle.$$

Step 3: Details (averaging, interference, angle coverage).

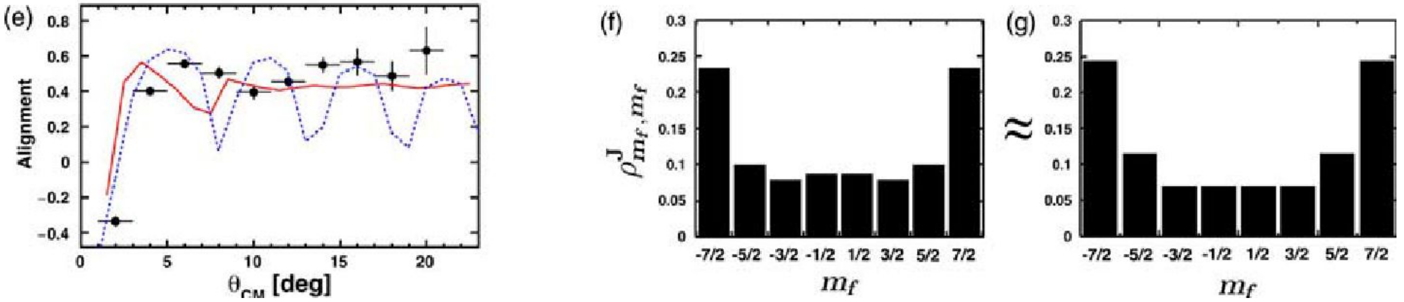


$\langle 3/2, m_i\,;\,\mathbf{2}, M\,|\,7/2, m_f\rangle^2$

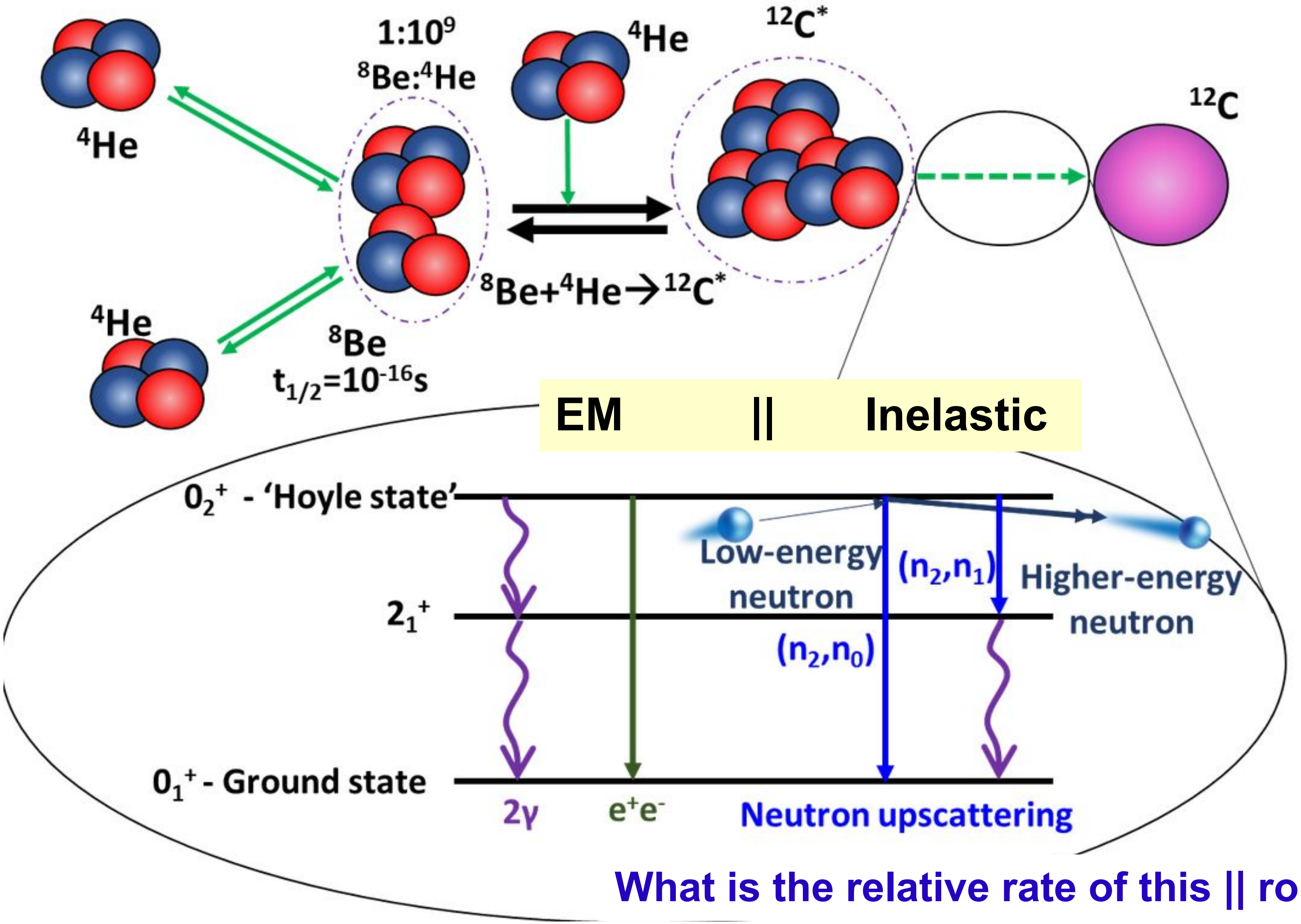
$\langle 35, 0\,;\,\mathbf{2}, M\,|\,35, M\rangle^2$

$\langle 3/2, m_i\,;\,\mathbf{2}, M\,|\,7/2, m_f\rangle^2\,\langle 35, 0\,;\,\mathbf{2}, M\,|\,35, M\rangle^2$

$$\mathcal{A} = \sum_{m_f} \frac{3m_f^2 - J(J+1)}{J(2J-1)} \rho^J_{m_f,m_f}.$$



6. Can introduce both upscattering and TPC's with ….
Is there a || path to ^{12}C (nucleosynthetic seeds), via "upscattering"?



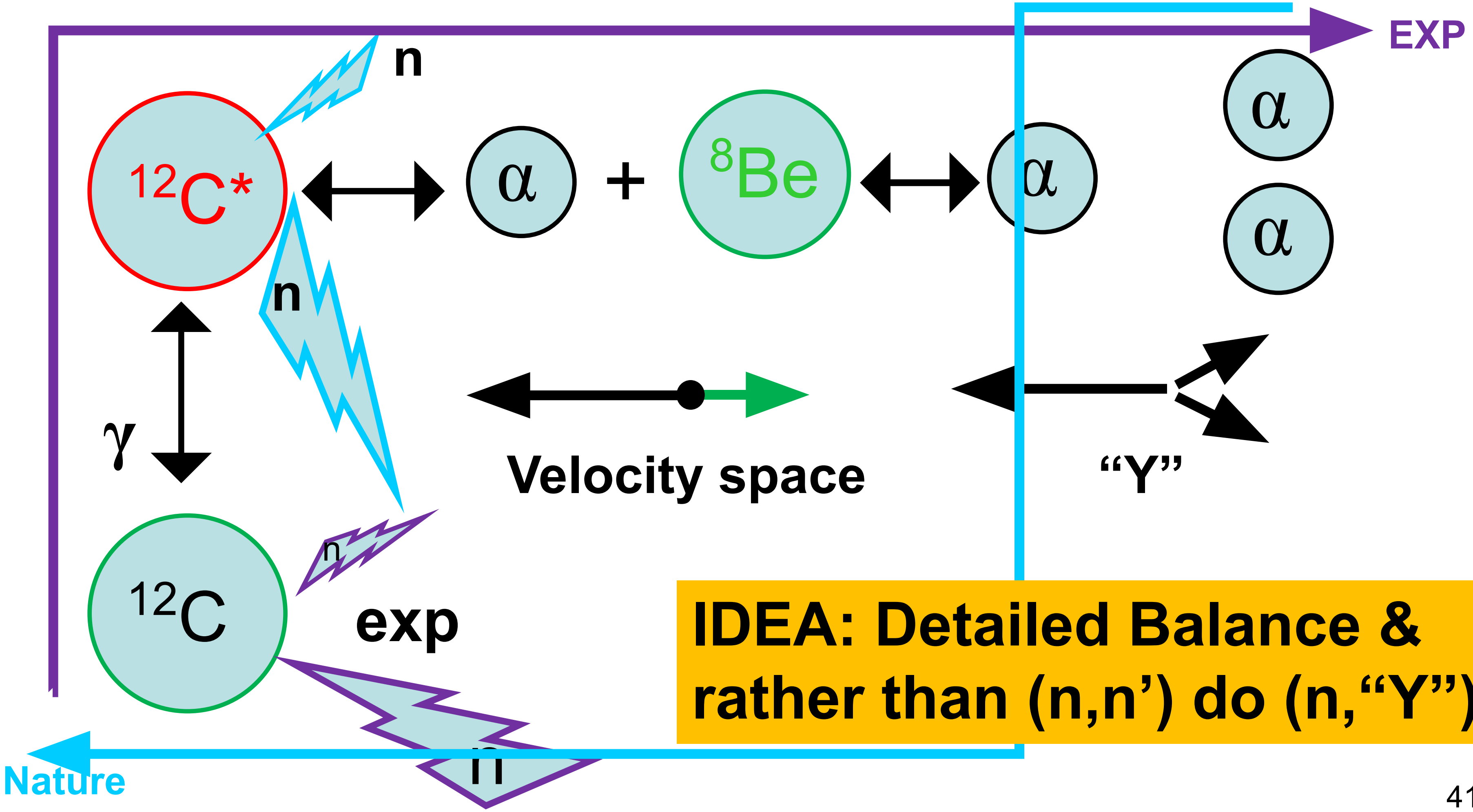
EM || Inelastic

What is the relative rate of this || route?

J. W. Truran & B.-Z. Kozlovsky, Ap. J. 158,1021 (1969).

Idea: microscopic reversibility & detect “Y”

$^{12}C^*$ □□ α + ^{8}Be □□ α + 2 α



IDEA: Detailed Balance & rather than (n,n’) do (n,“Y”)

Detailed Balance

In equilibrium each elementary process is in equilibrium with its reverse process

1. At equilibrium the one-way rates must be equal □ = □

$$R_{\rightarrow}[1/cm^3 s] = N_n N_{12C} <\sigma_{\rightarrow} v>_{MB} = N_{n'} N_{12C*} <\sigma_{\leftarrow} v>_{MB} = R_{\leftarrow}[1/cm^3 s]$$

2. The forward/backward Maxwellian averaged cross section ratio is just equal to the number ratio (or K_{eq}) and thus equal to a partition function ratio.

□The neutron partition functions drop out as T & m are the same and all that remains are the spin degeneracy ratio and the difference in energies.

$$\frac{<\sigma_{\leftarrow} v>_{MB}}{<\sigma_{\rightarrow} v>_{MB}} = \frac{N_n N_{12C}}{N_{n'} N_{12C*}} = \frac{q_n q_{12C}}{q_{n'} q_{12C*}} = \left(\frac{q_n}{q_{n'}}\right)\left[\frac{q_{12C}}{q_{12C*}}\right] = (1)\left[\frac{2I+1}{2I'+1} e^{-\Delta E/kT}\right]$$

3. BTW, the Maxwellian averaged cross sections are just…..

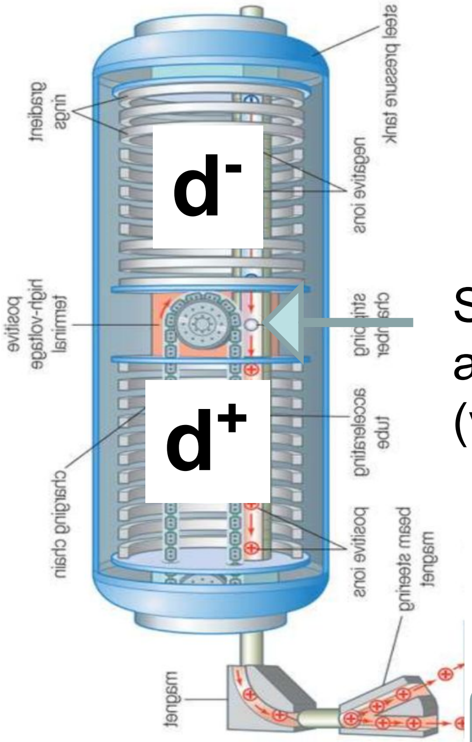
$$<\sigma v>_{MB} = \left(\frac{8}{\pi\mu}\right)^{1/2} \left(\frac{1}{kT}\right)^{3/2} \int_0^{\infty} E\sigma(E) e^{-E/kT} dE$$

TPC @ Ohio U. (E_{dwards} A_{cc} $L_{aboratory}$) to detect "Y" 's in $^{12}C_{gs}$(n,"Y") $^{12}C_{Hoyle}$

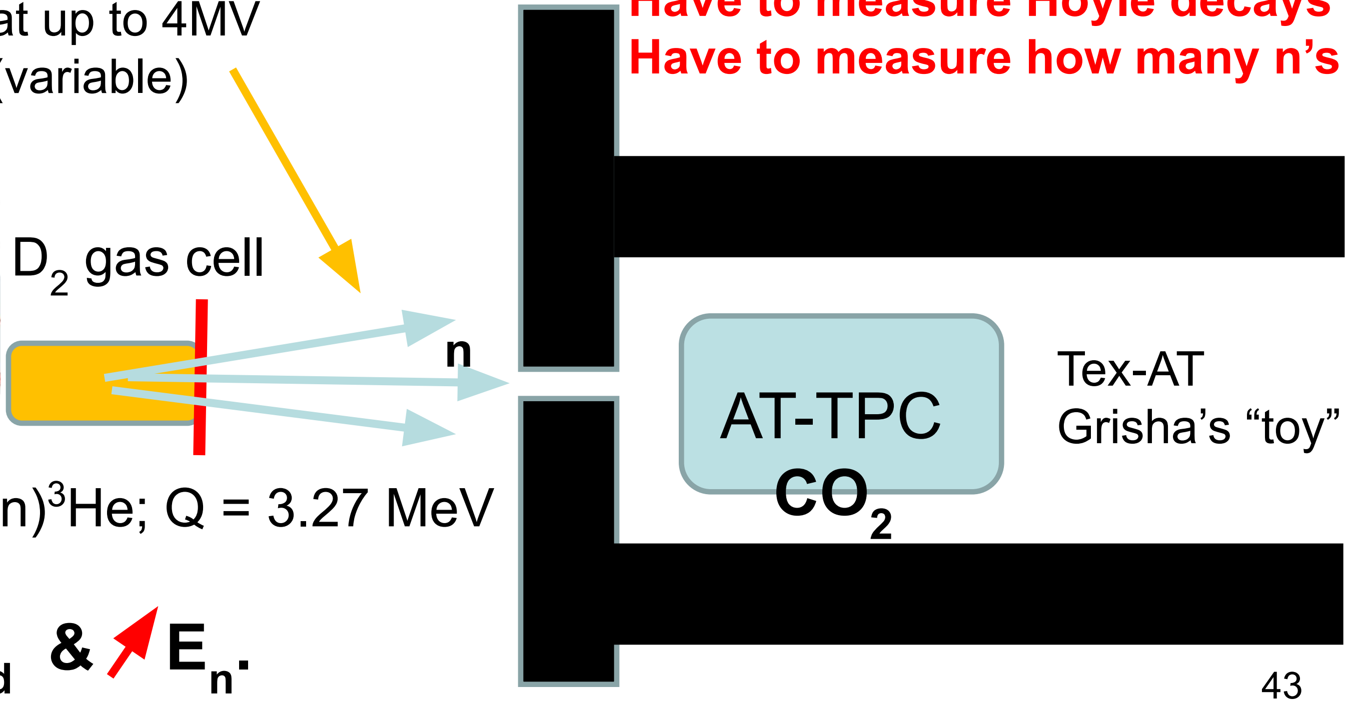
Ion source

d$^-$

n beams are the prototypical nuclear "exotic beam"
TPC are a tool indispensable for exotic-beam work.



Stripper foil
at up to 4MV
(variable)

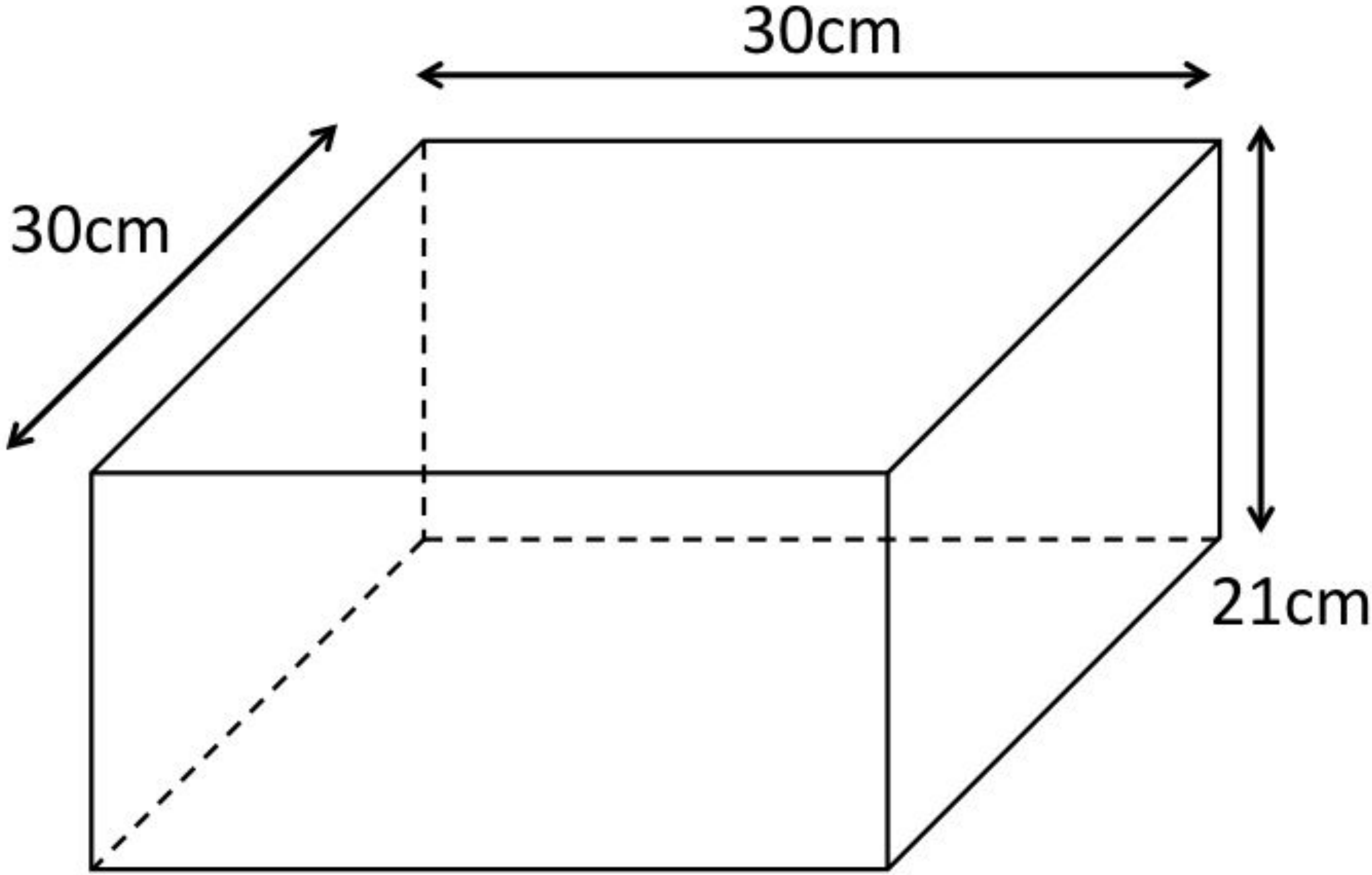


Have to measure Hoyle decays
Have to measure how many n's

Tex-AT
Grisha's "toy"

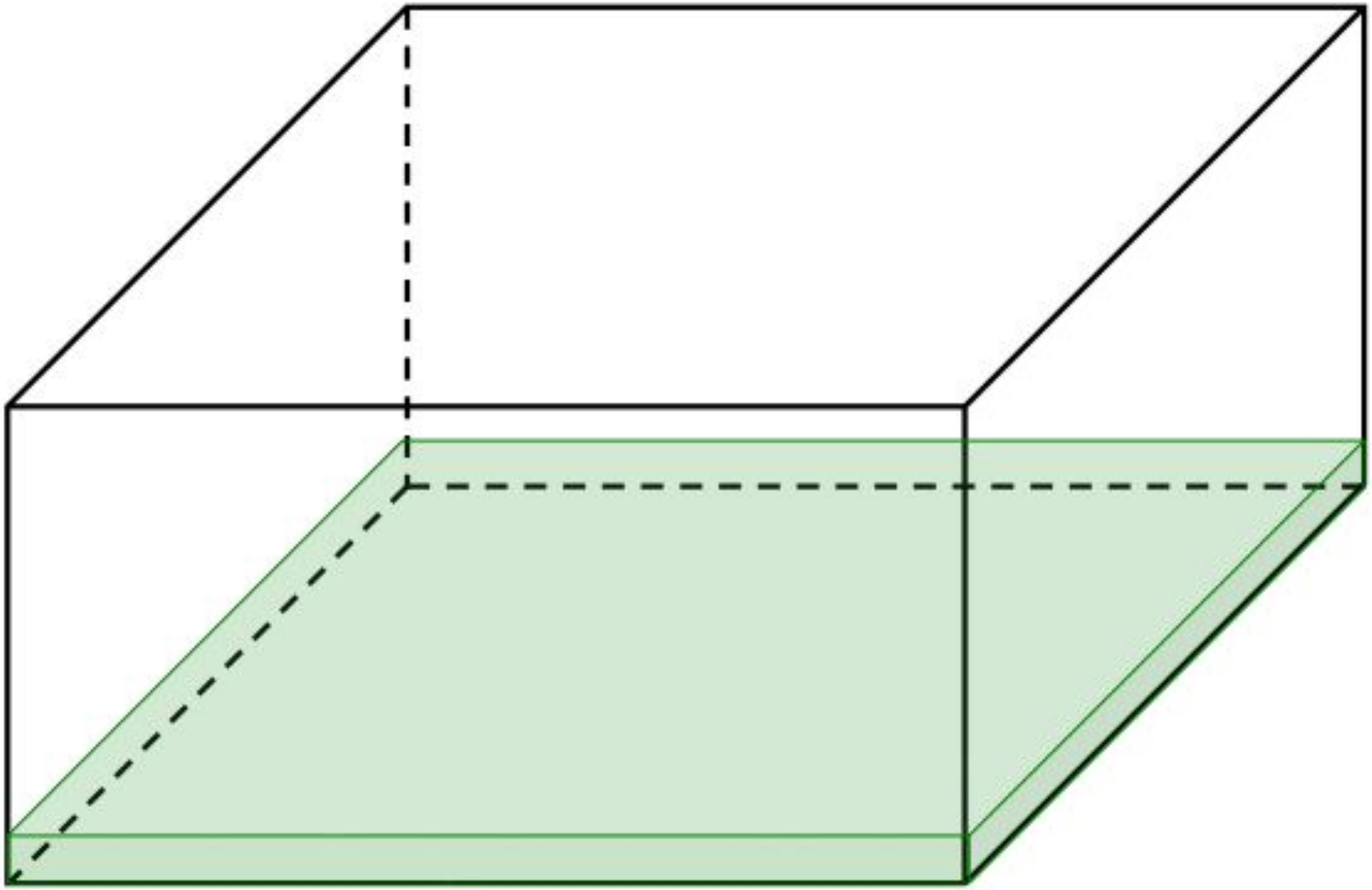
d(d,n)^{3}He; Q = 3.27 MeV

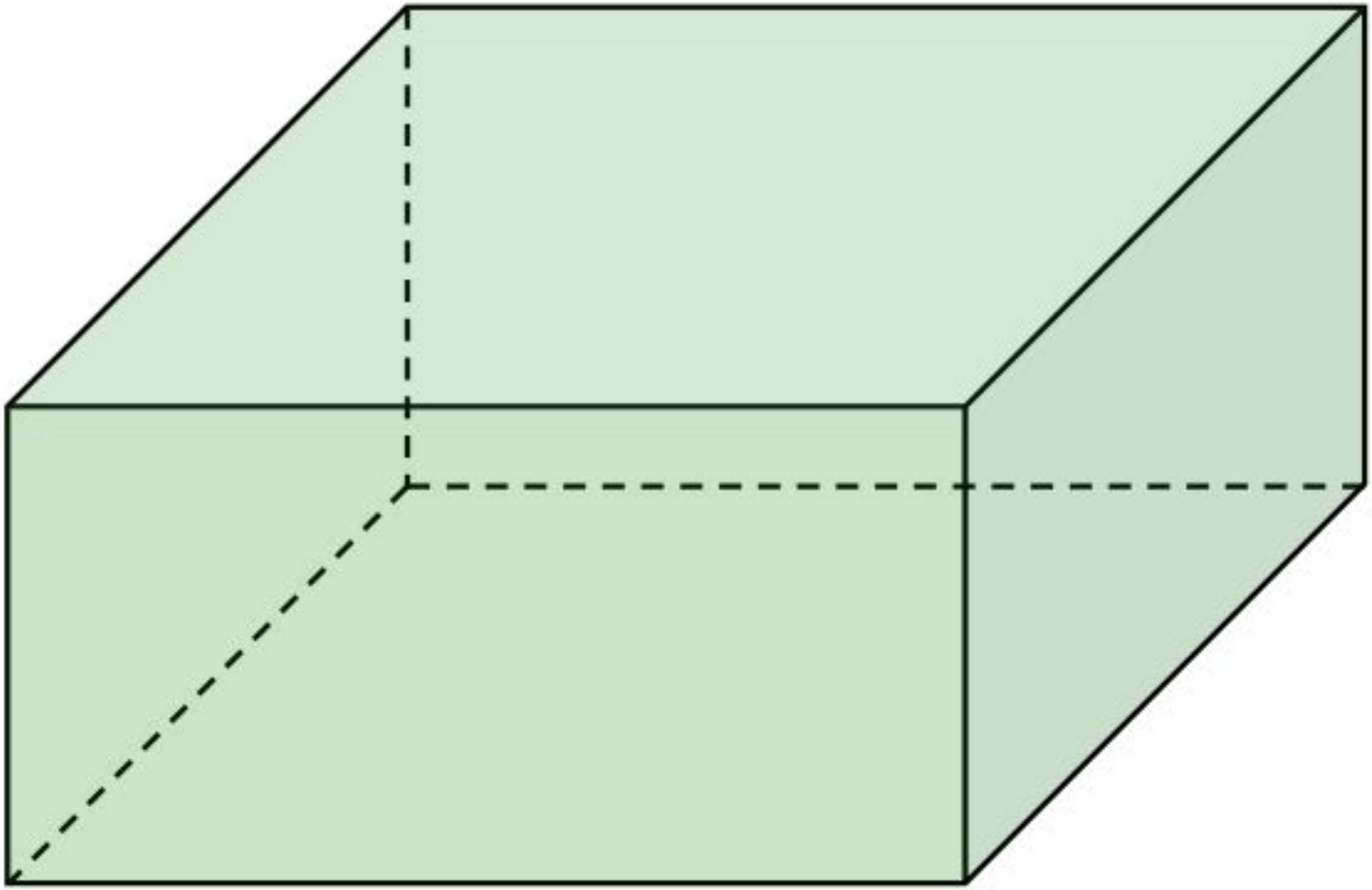
↗terminal V: ↗E_d & ↗E_n.



Time Projection Chamber

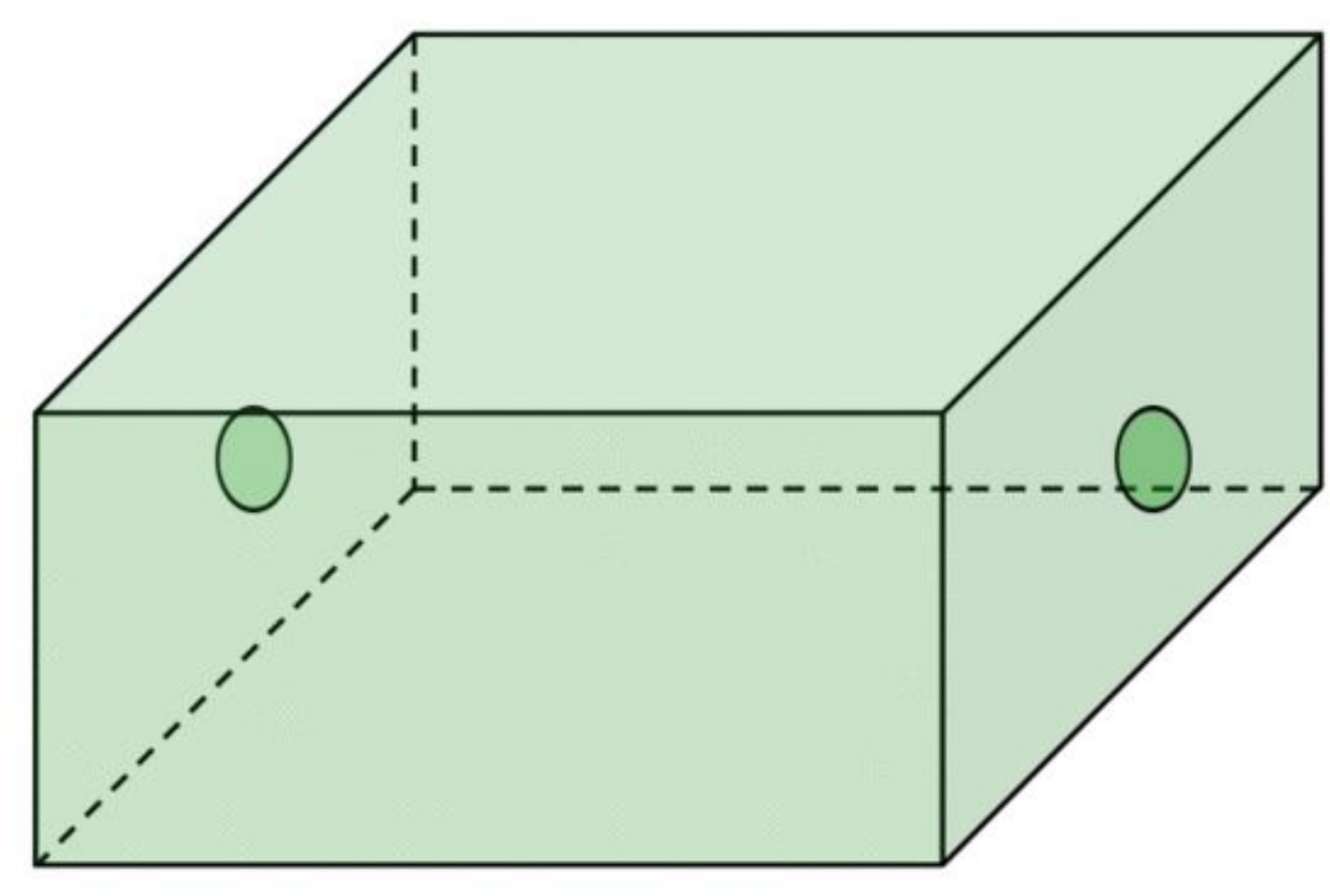
CO_2



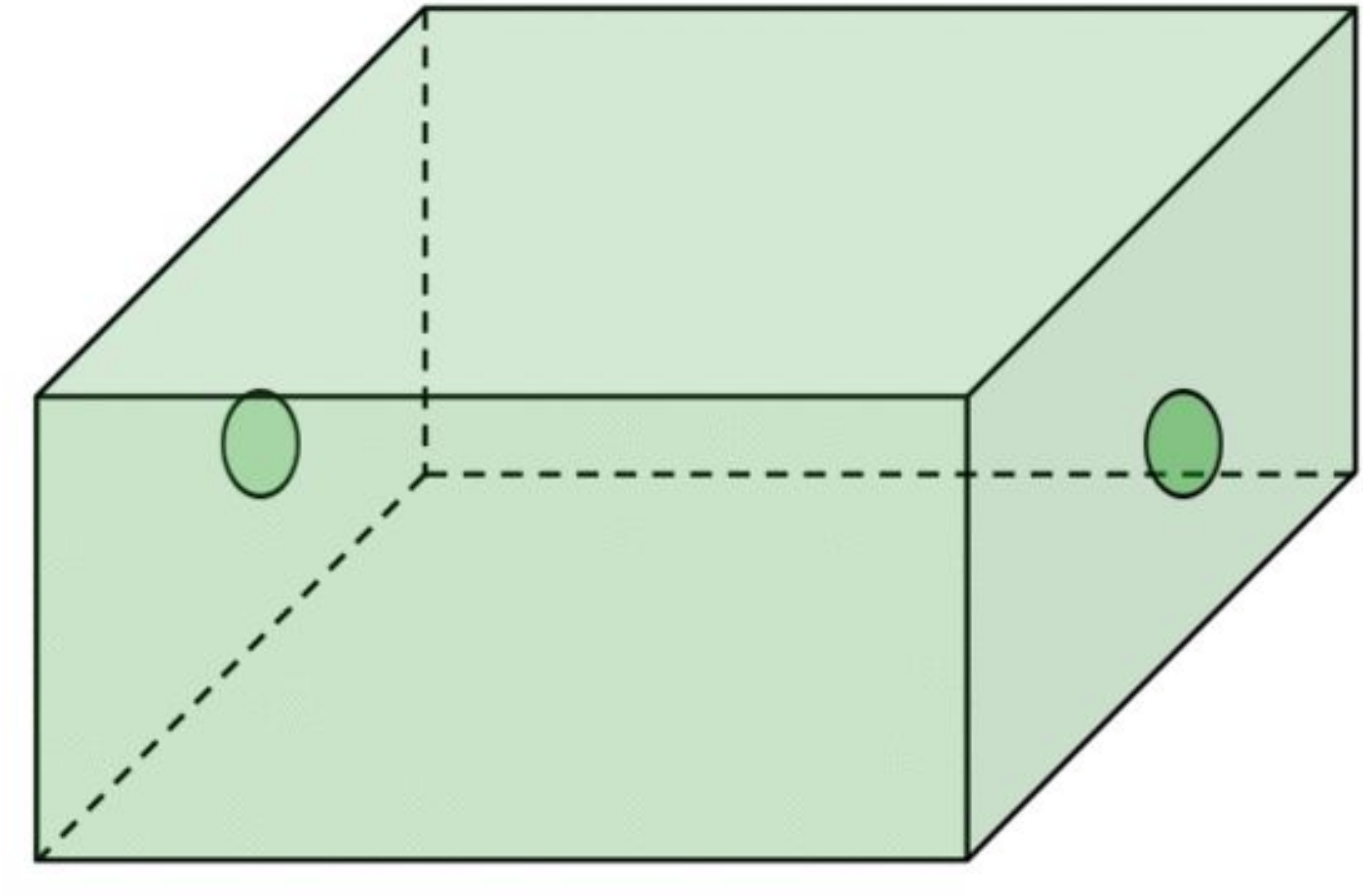


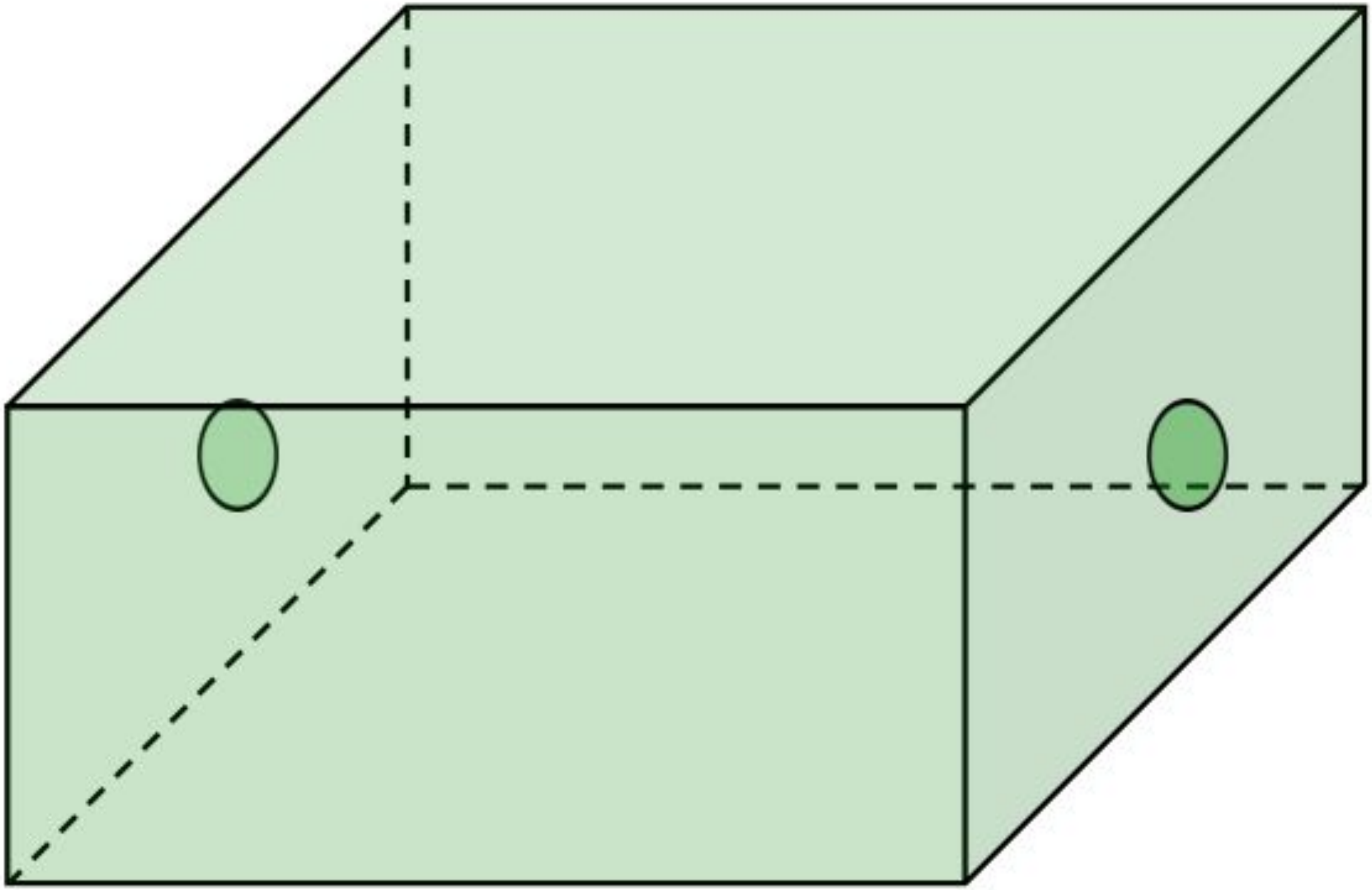
10^8 γ/s or 10^6 n/s

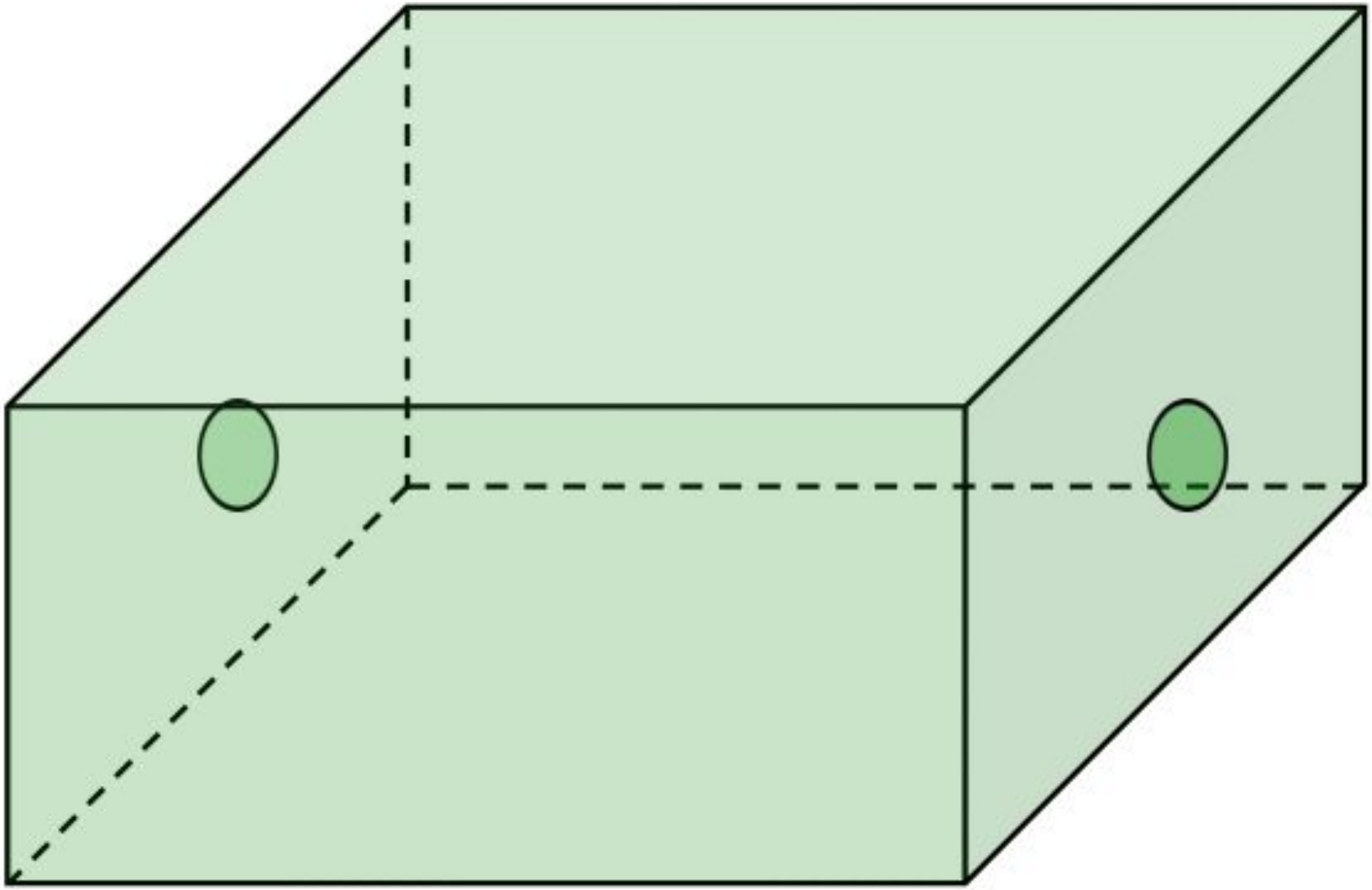
γ's circularly polarised

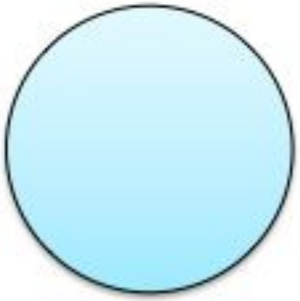


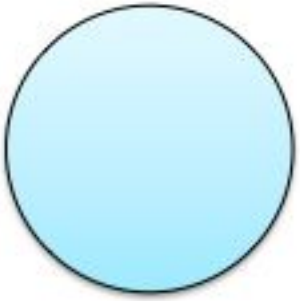
$\sigma_\gamma \approx 130$ keV, $\sigma_n \approx 300$ keV,



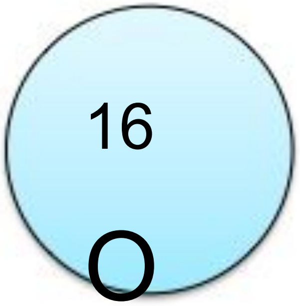


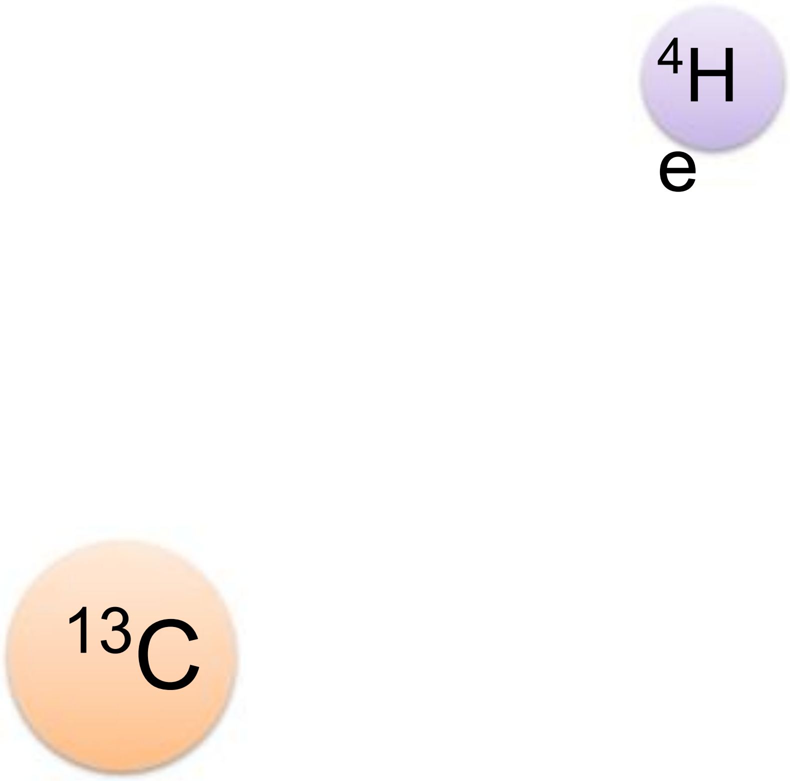


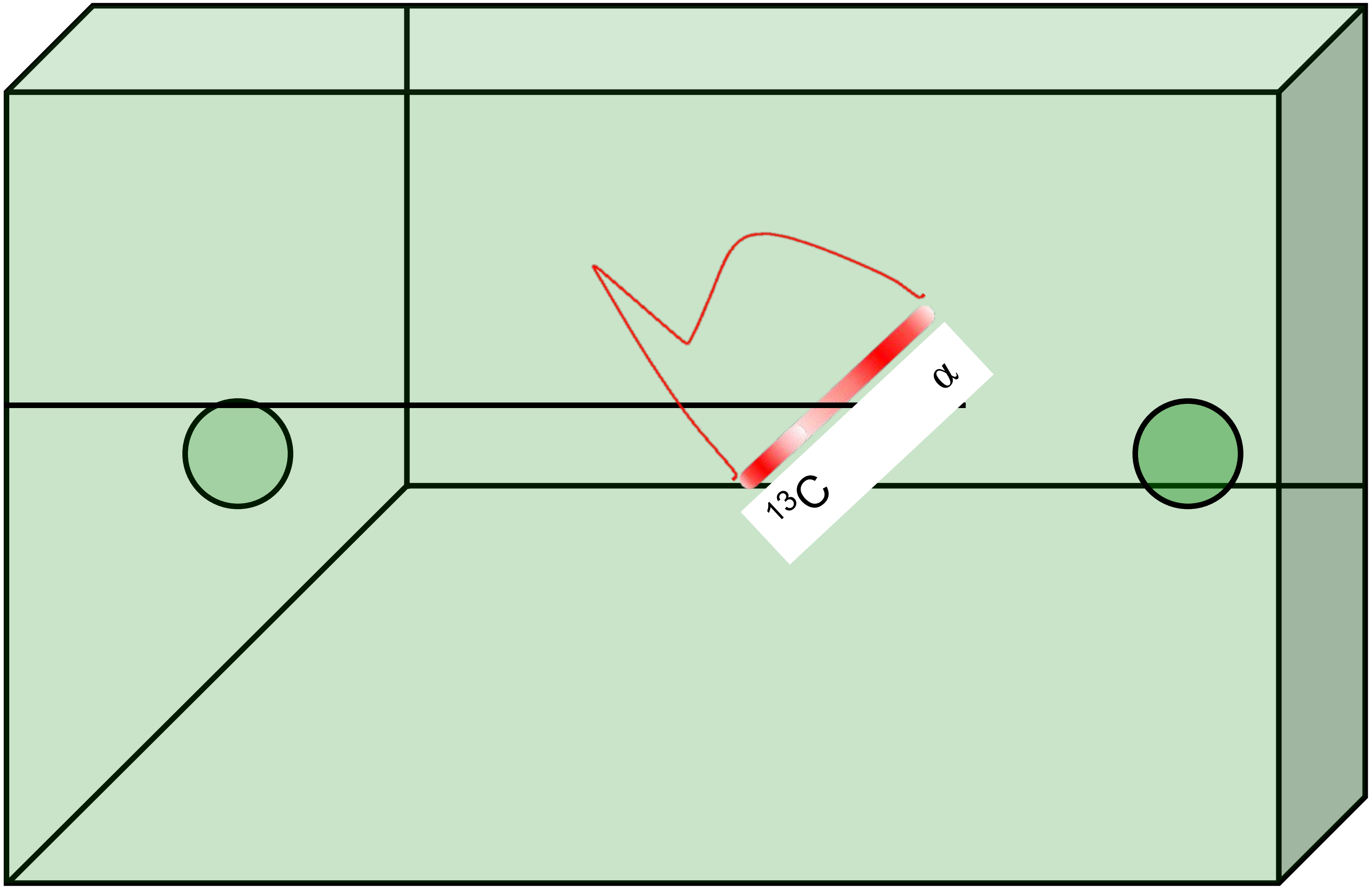


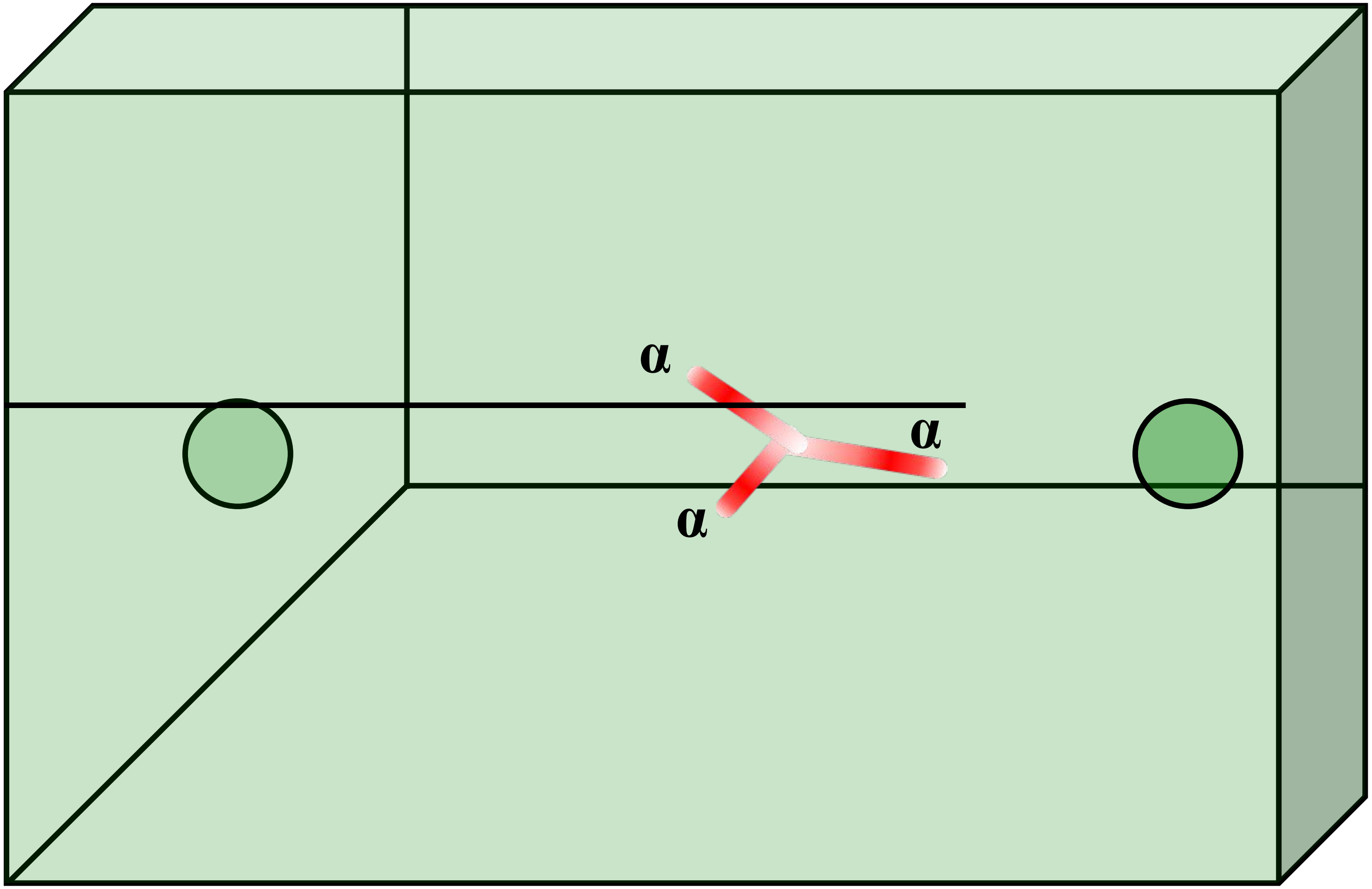


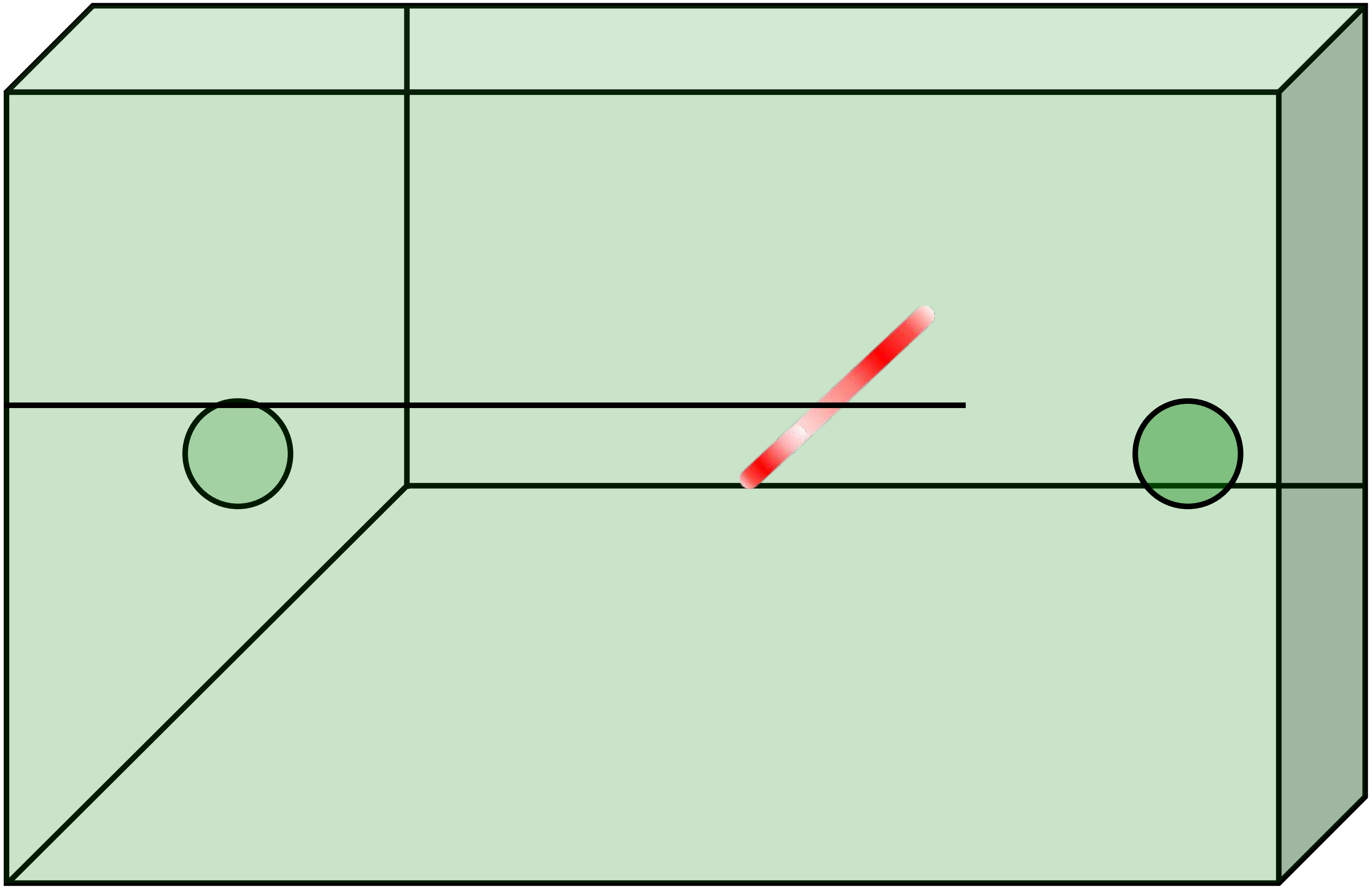
^{12}C or ^{16}O

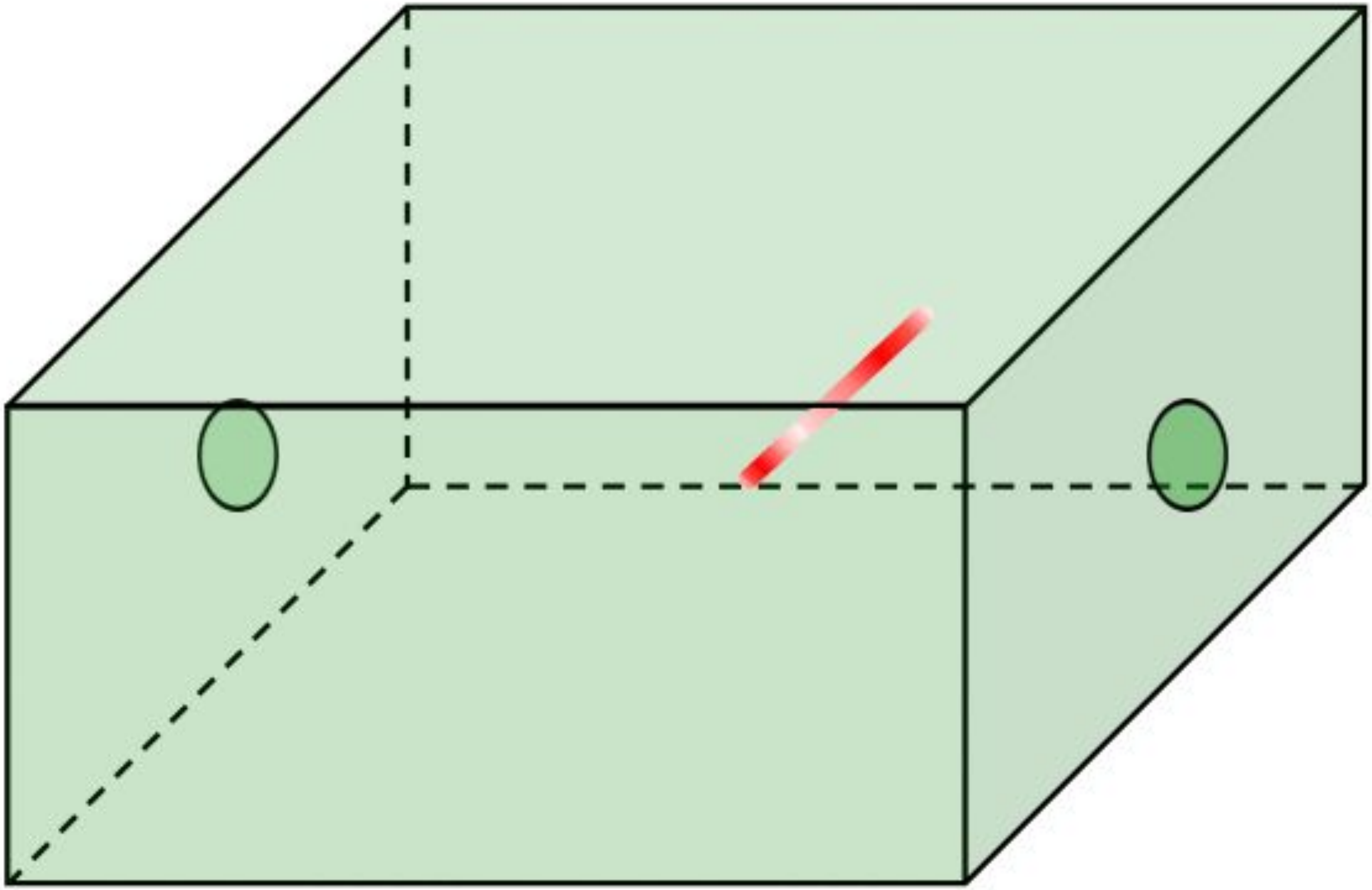

16
O

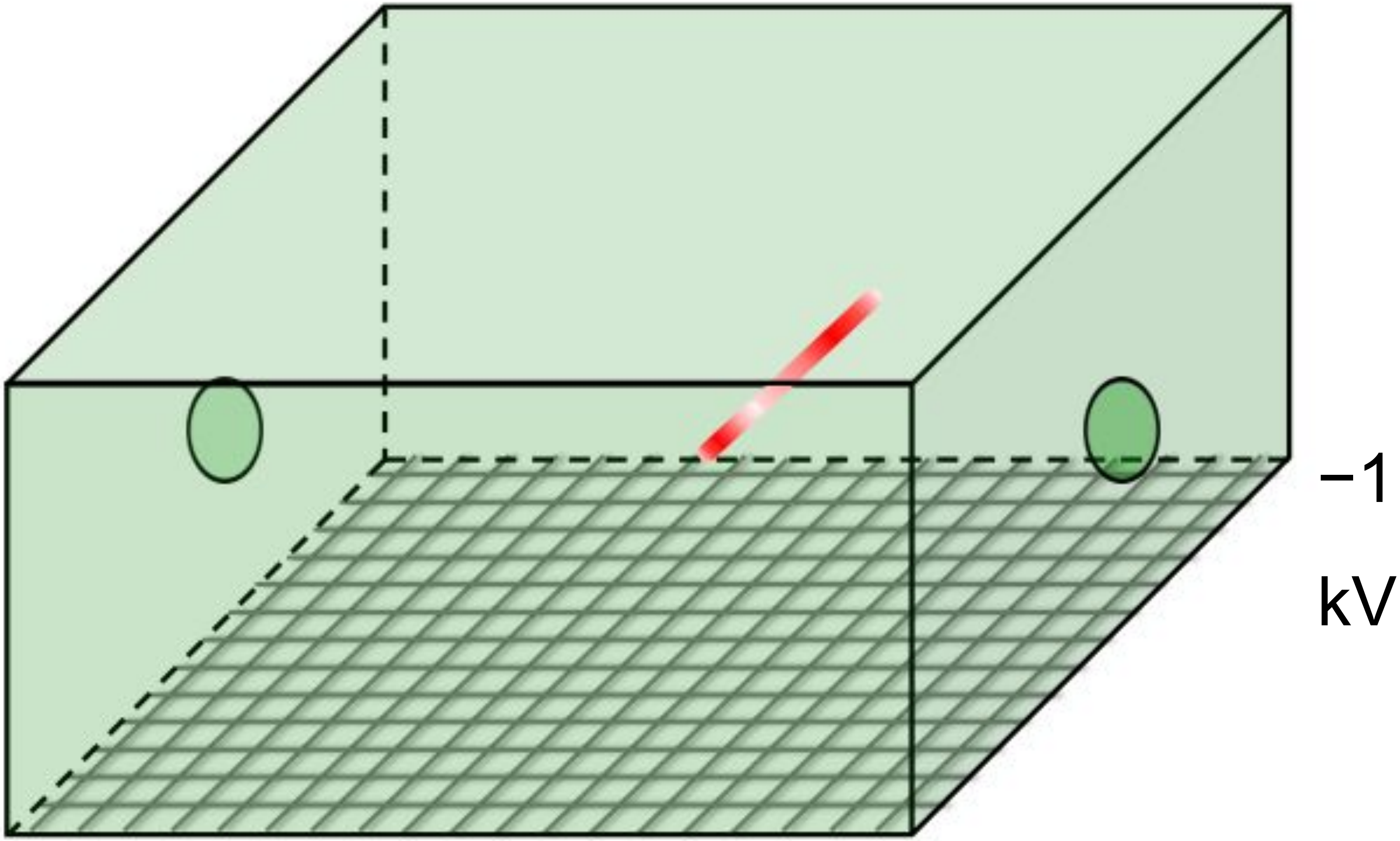

^{4}He
^{13}C


^{13}C
α

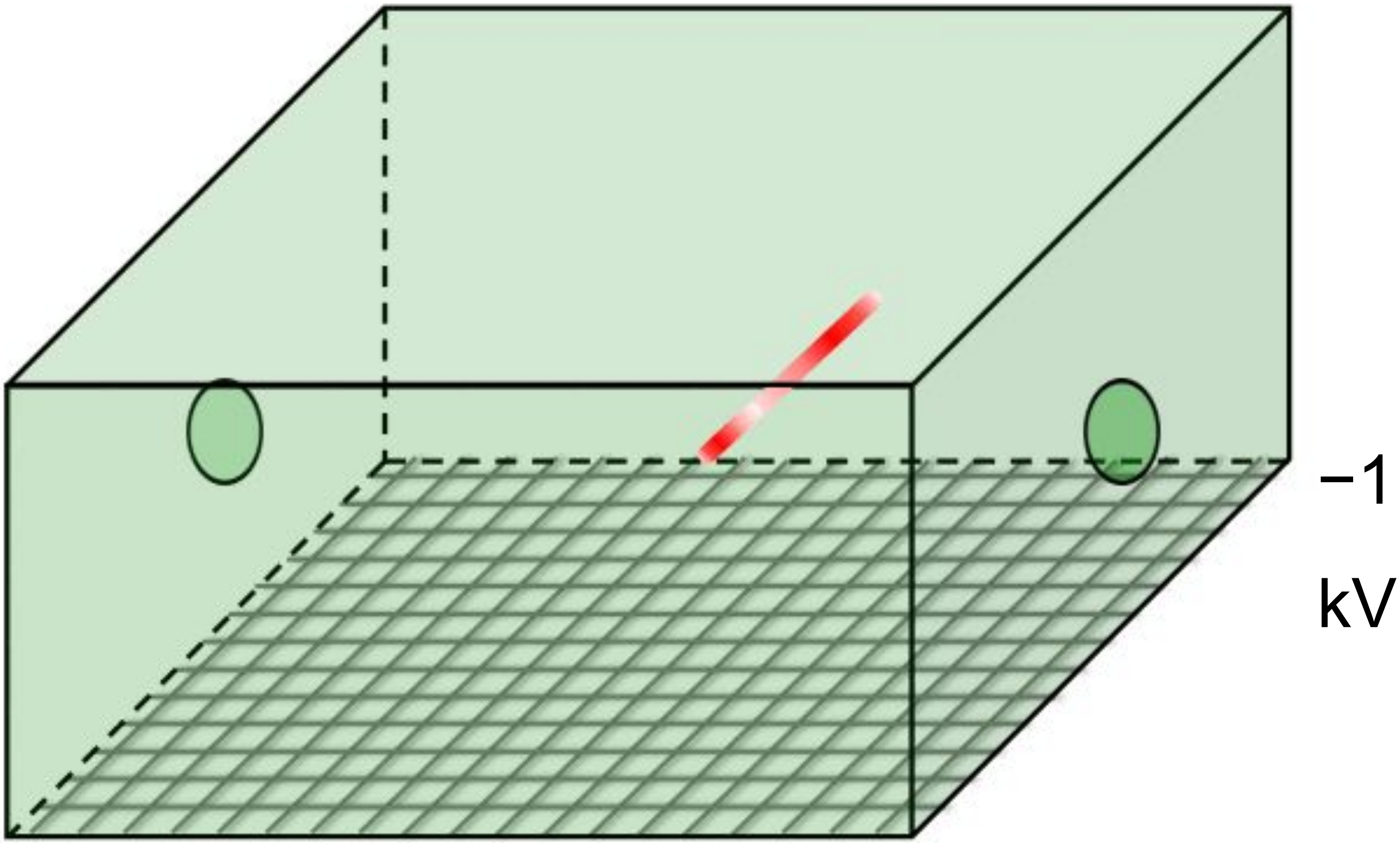

α
α
α

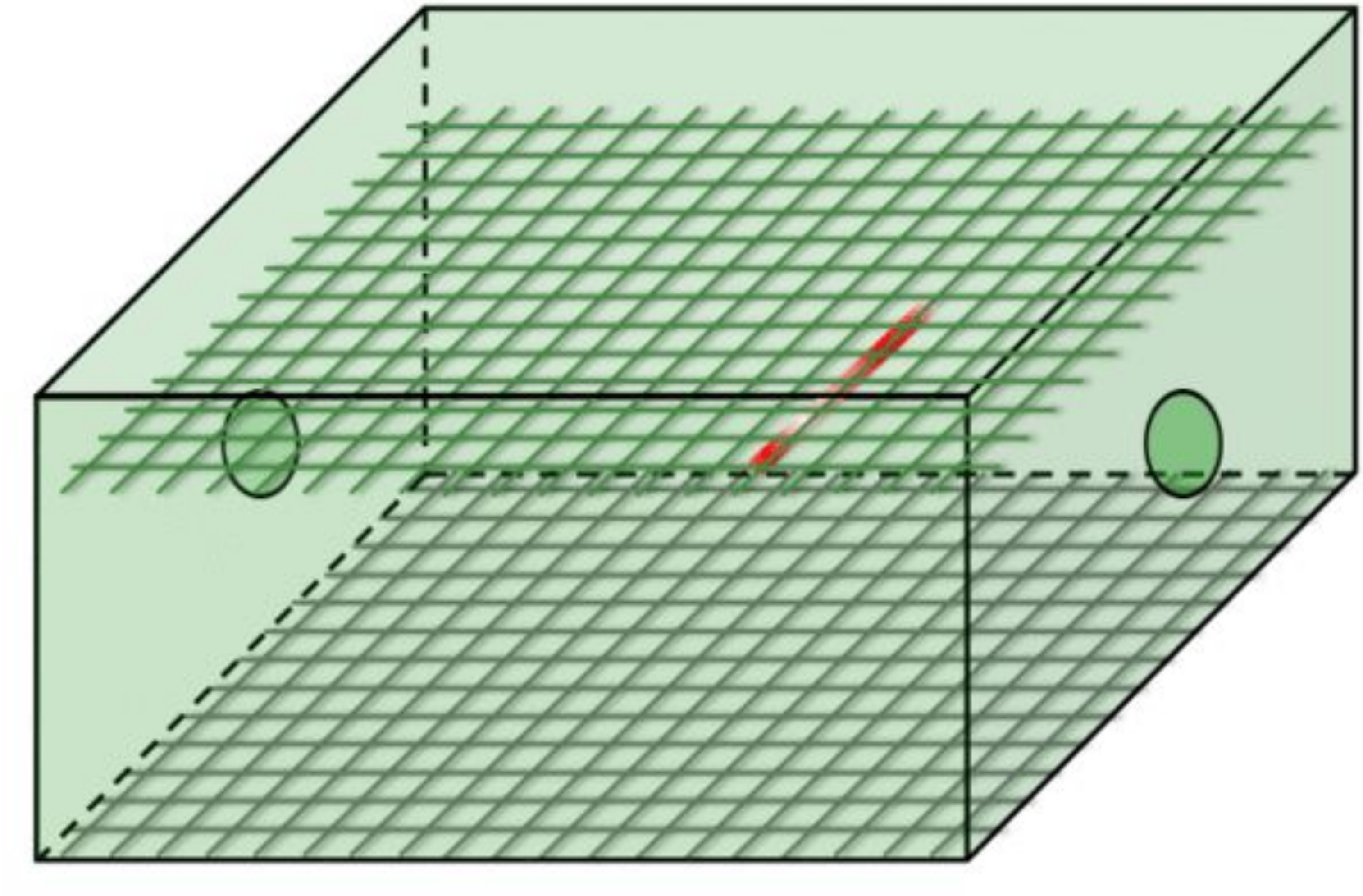






(Repeller) Cathode


−1
kV

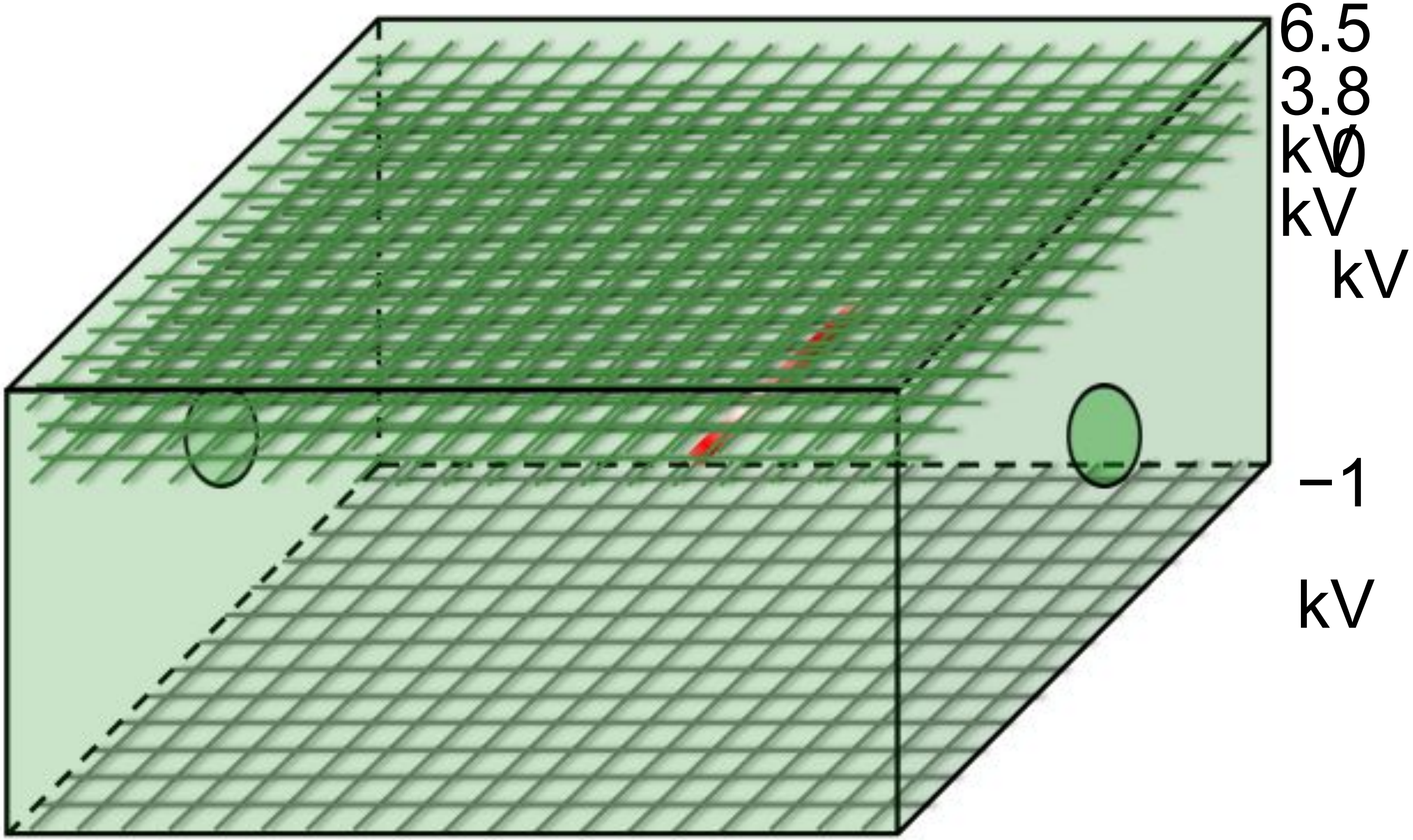


0

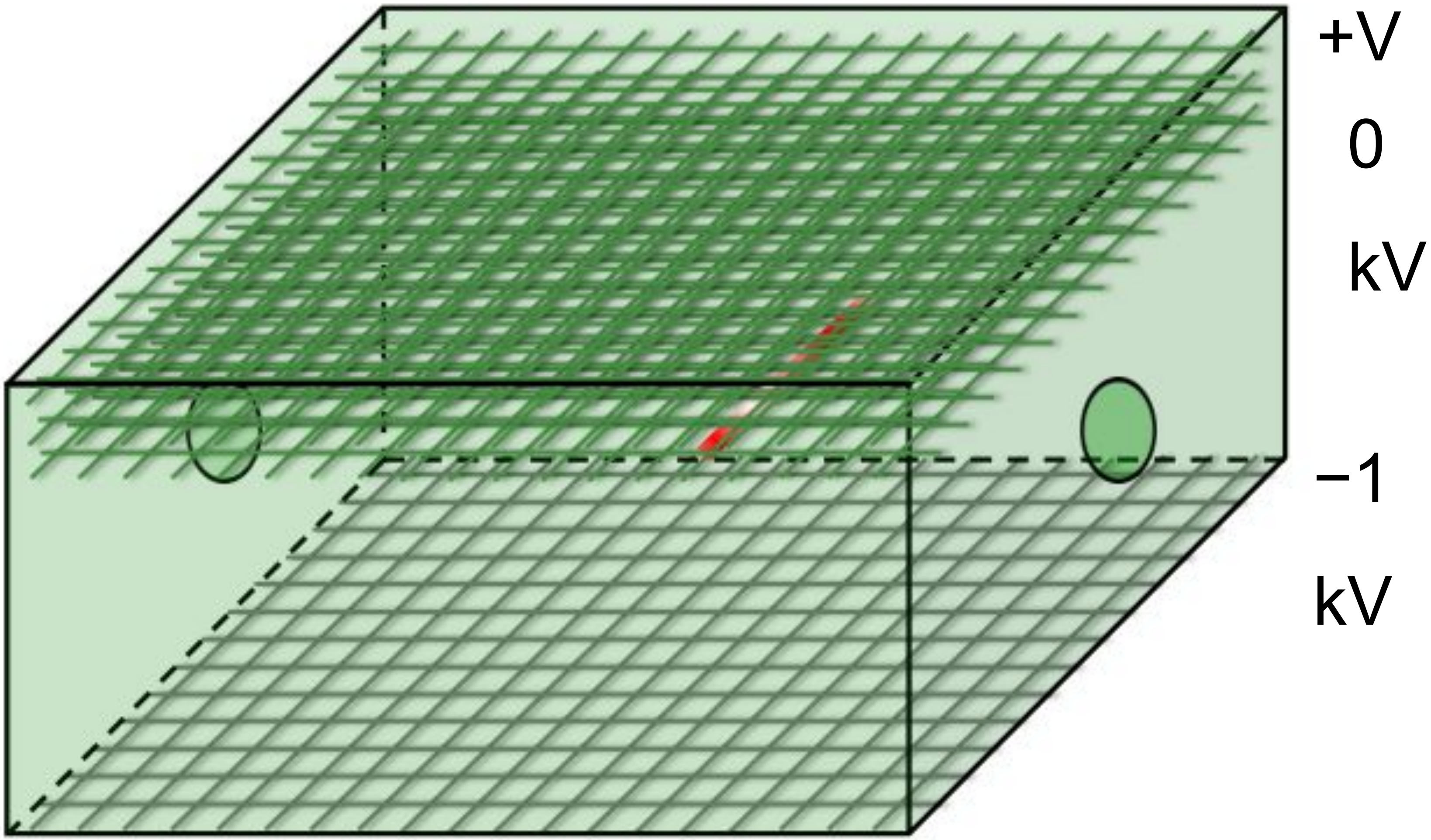
kV

−1

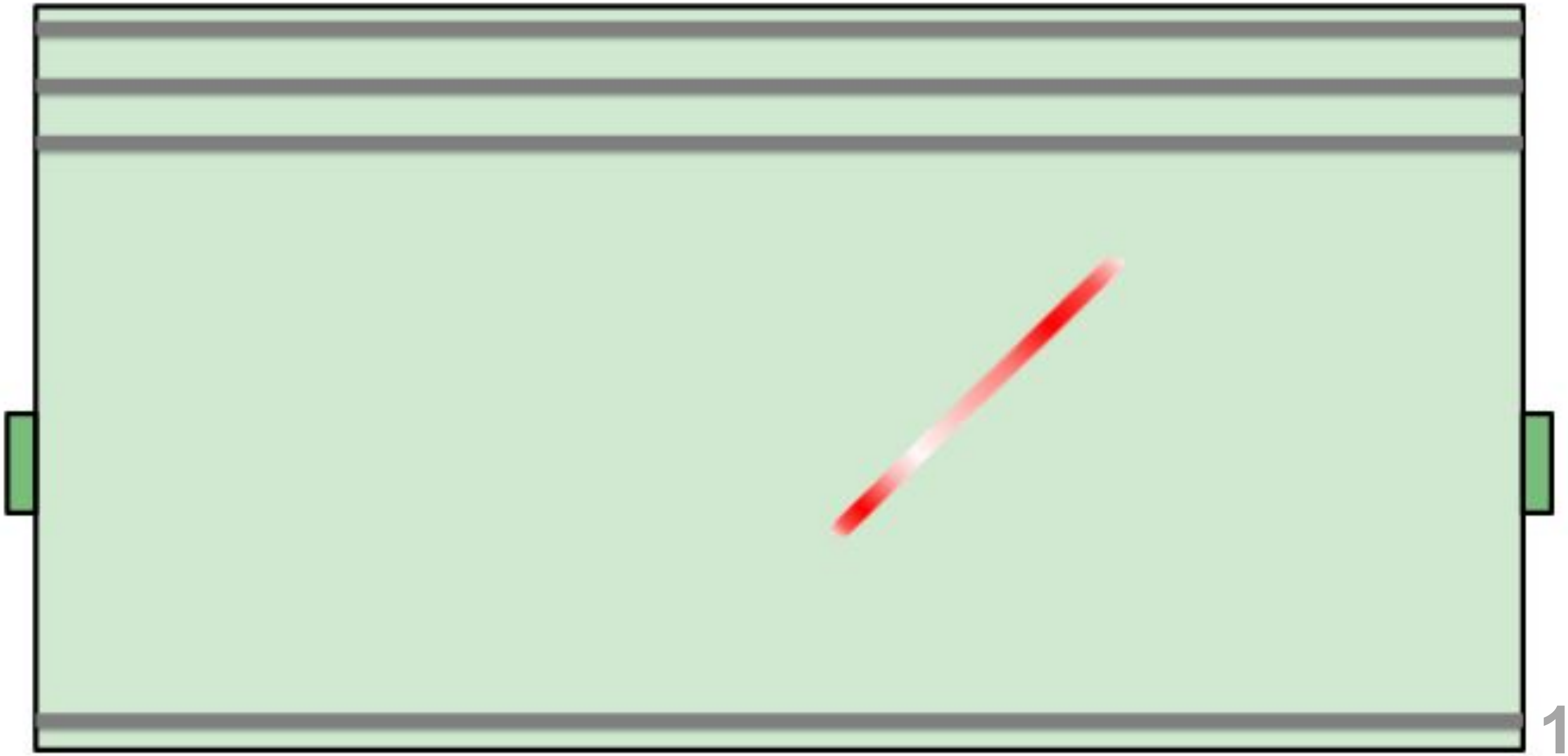
kV

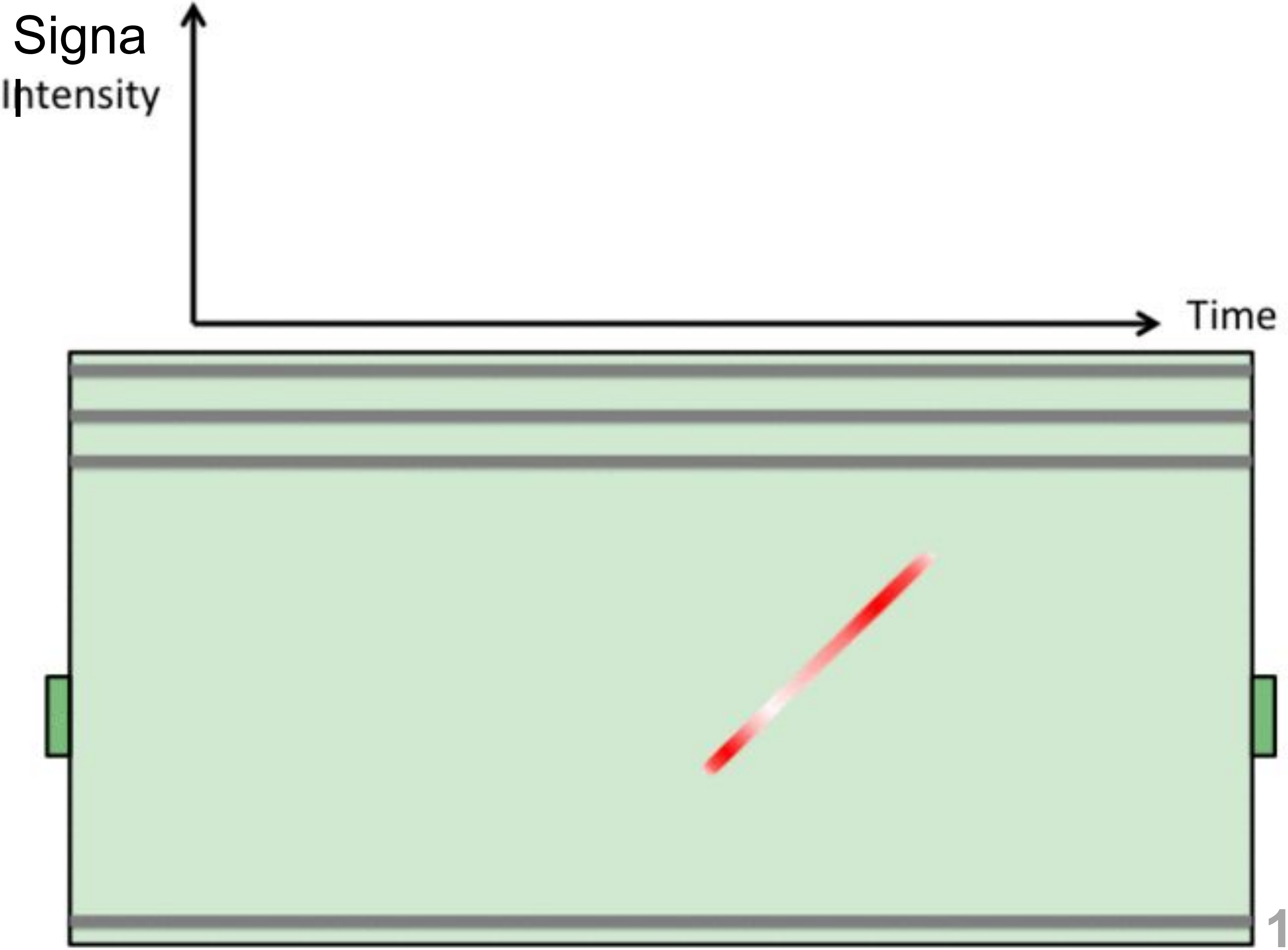

6.5
3.8
kV
0
kV
kV
−1
kV

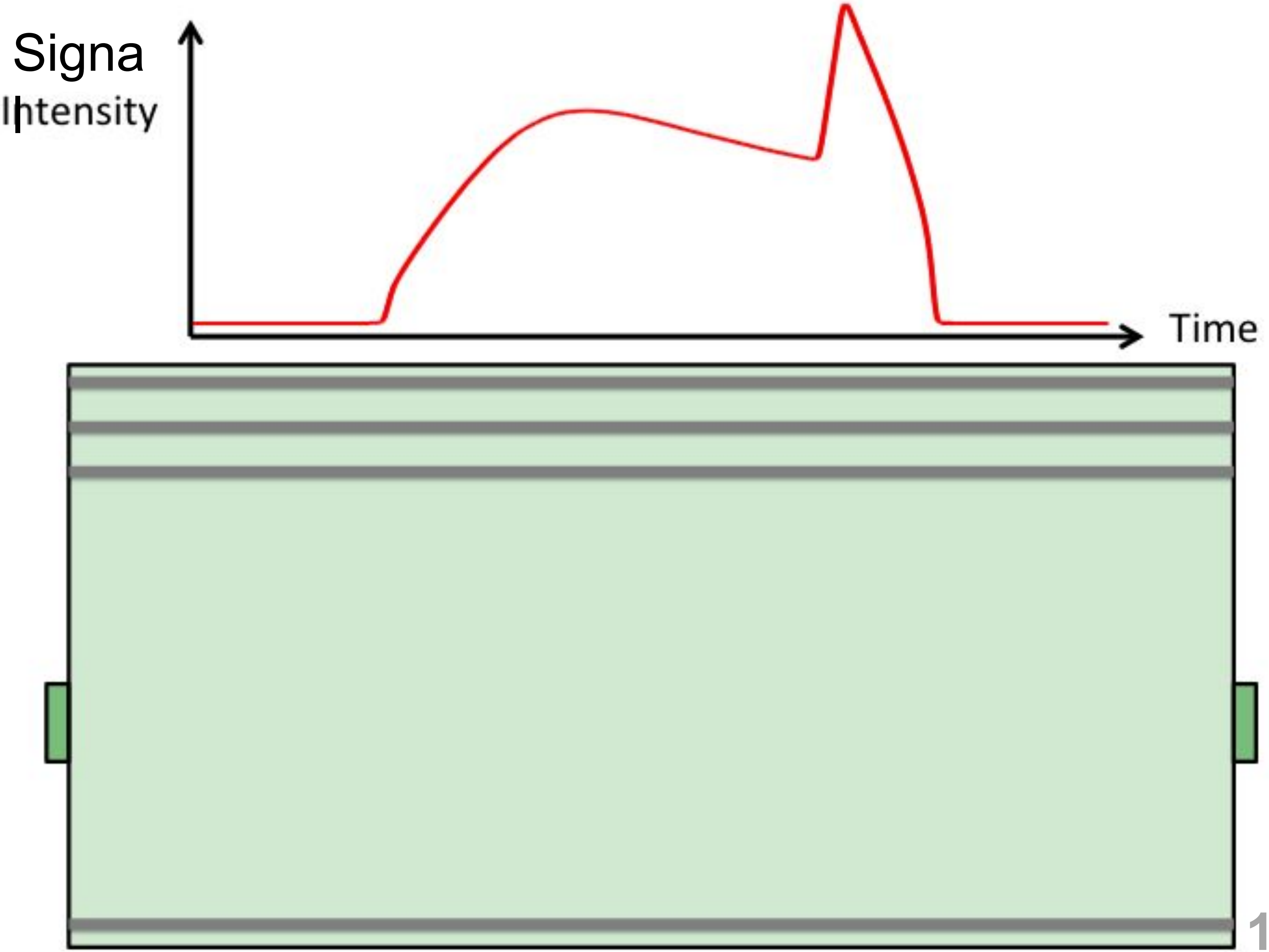
Avalanche grids /micro patterned anode

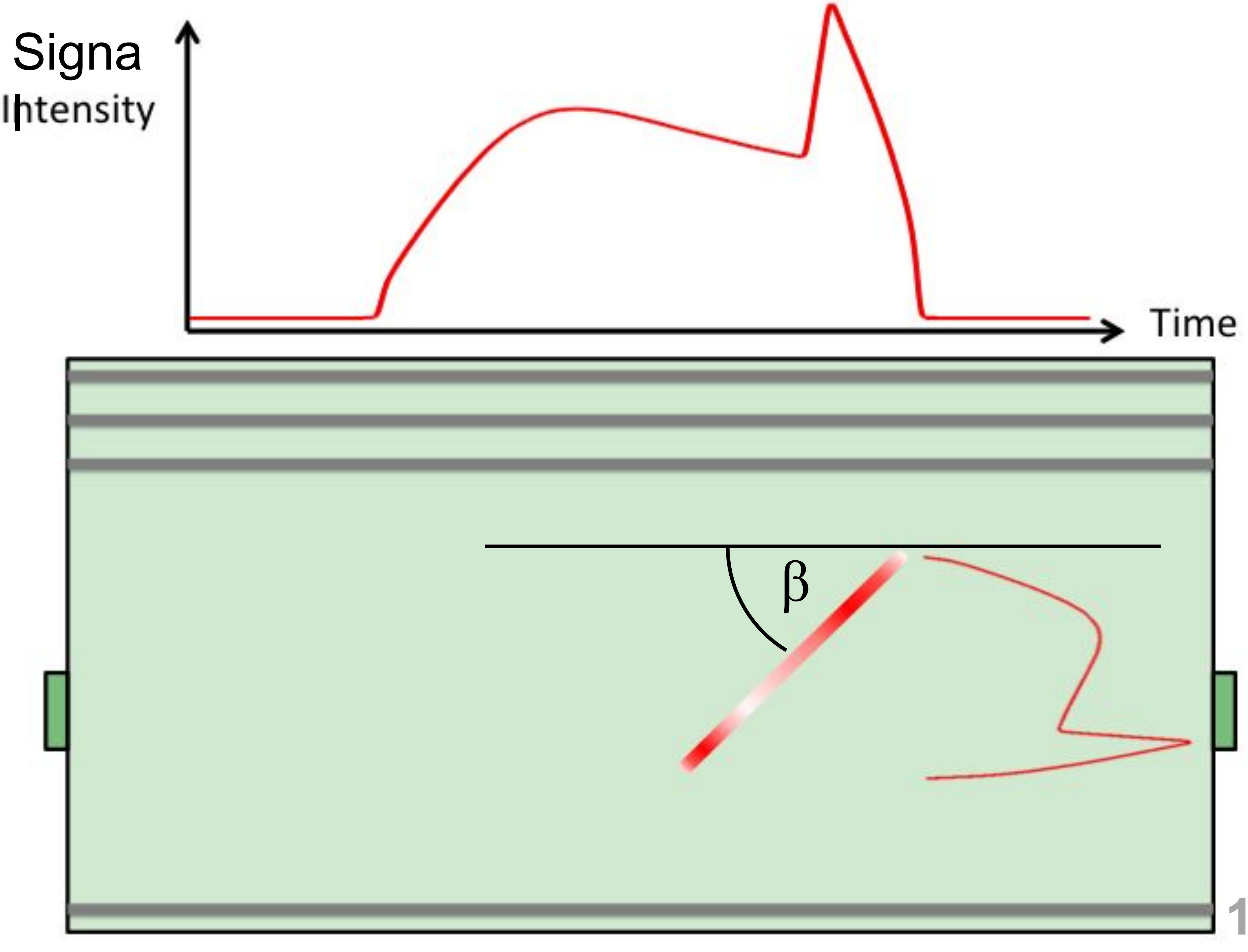


Side view

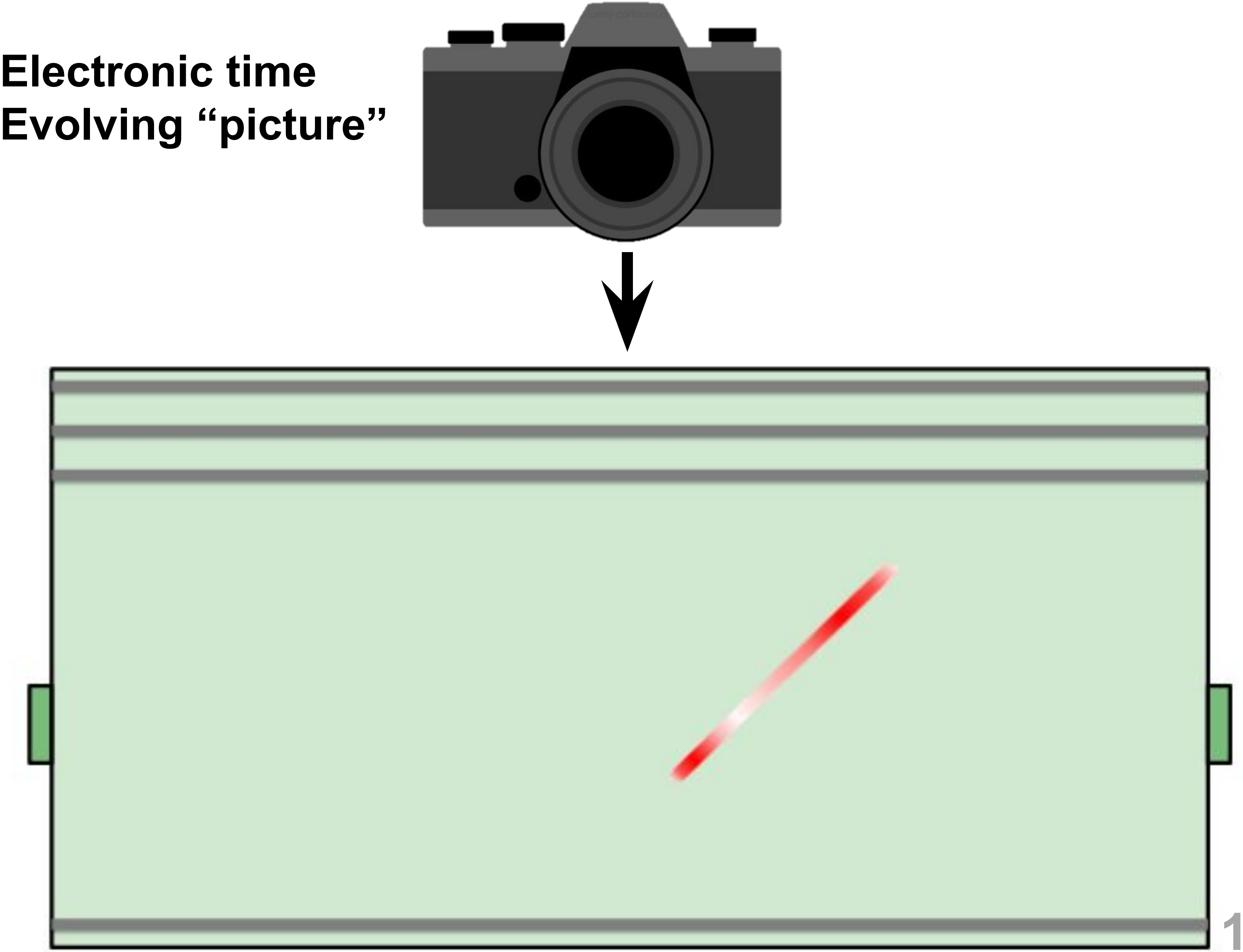


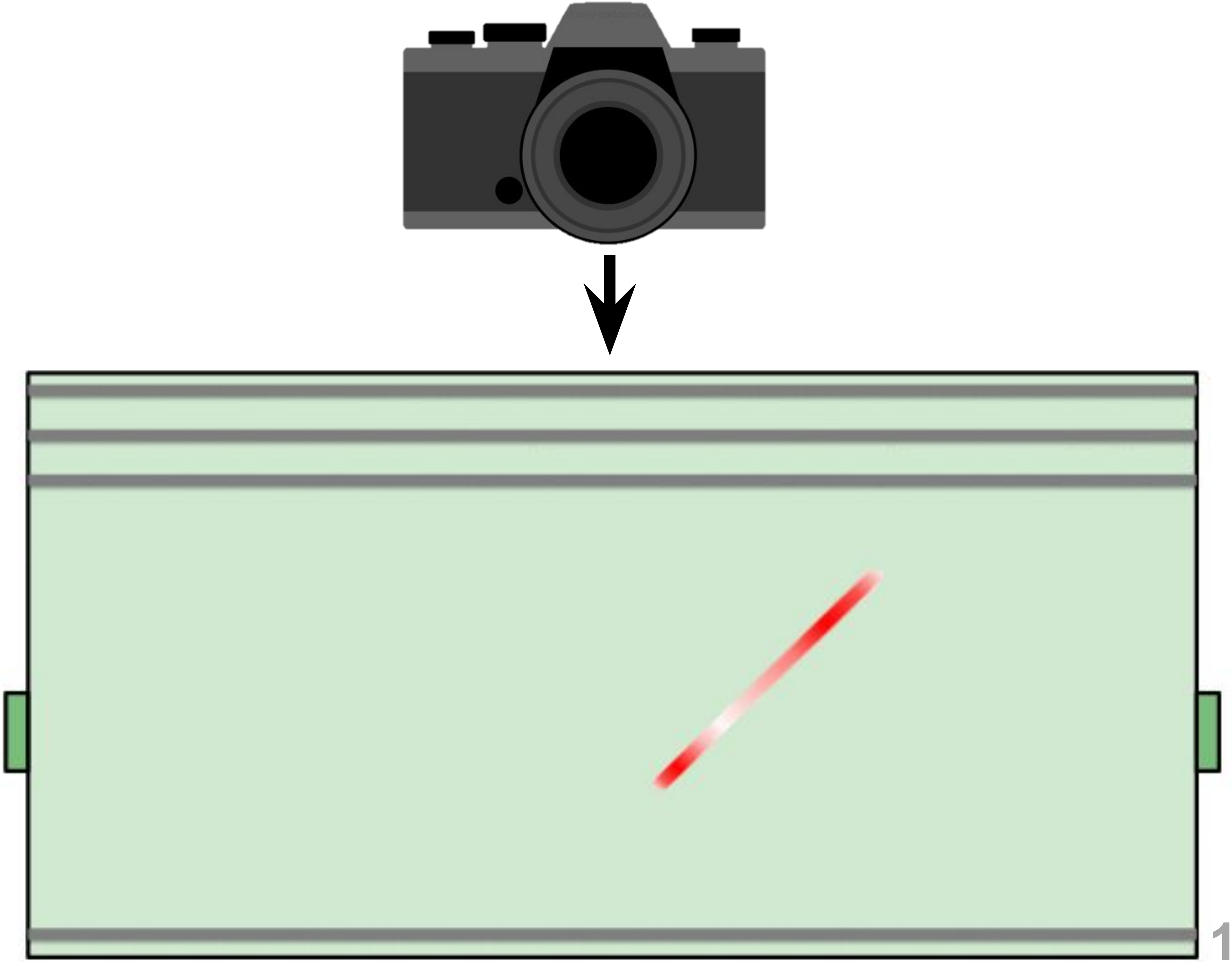

Signa
Intensity
Time

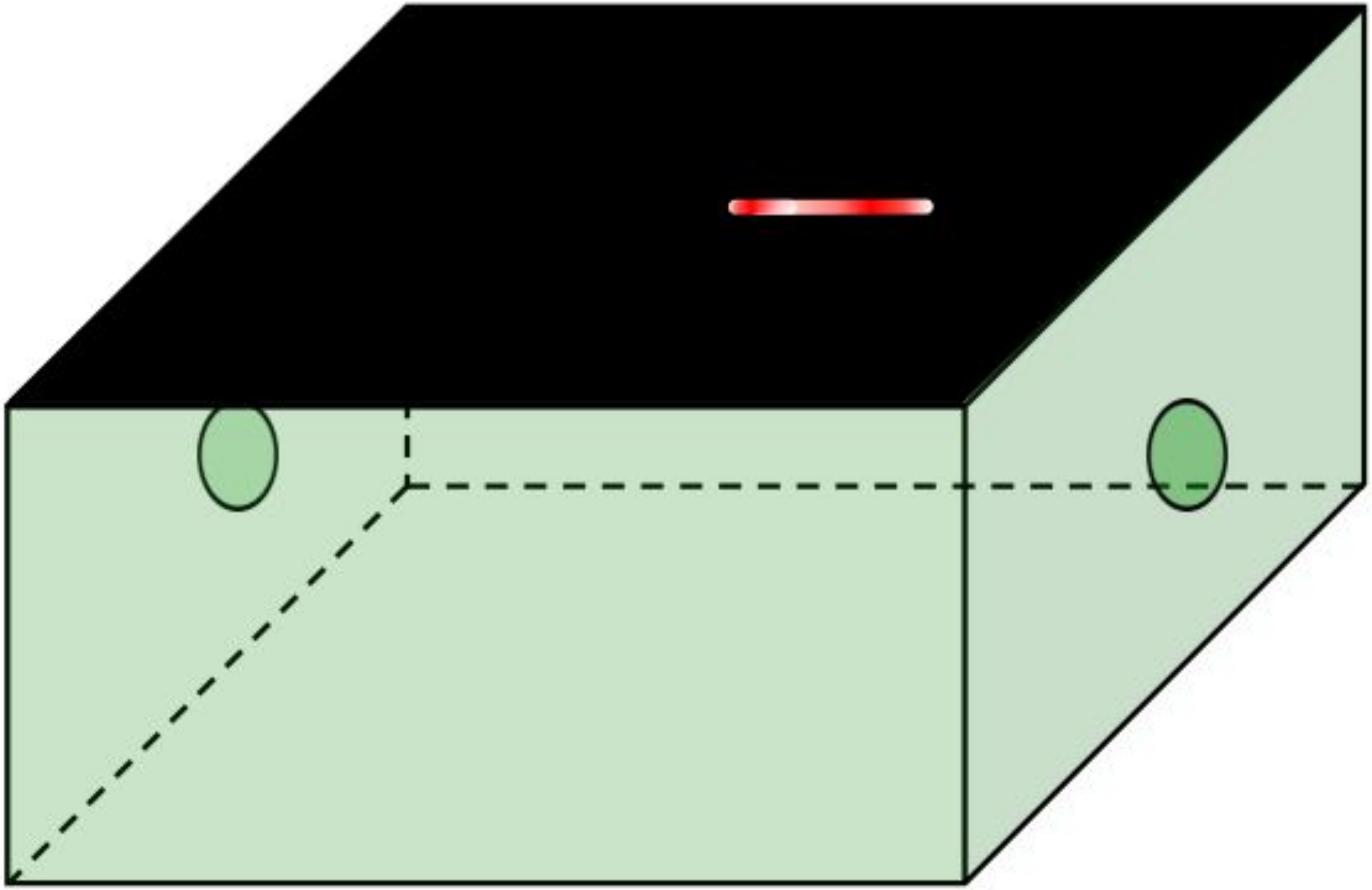

Signal Intensity
Time

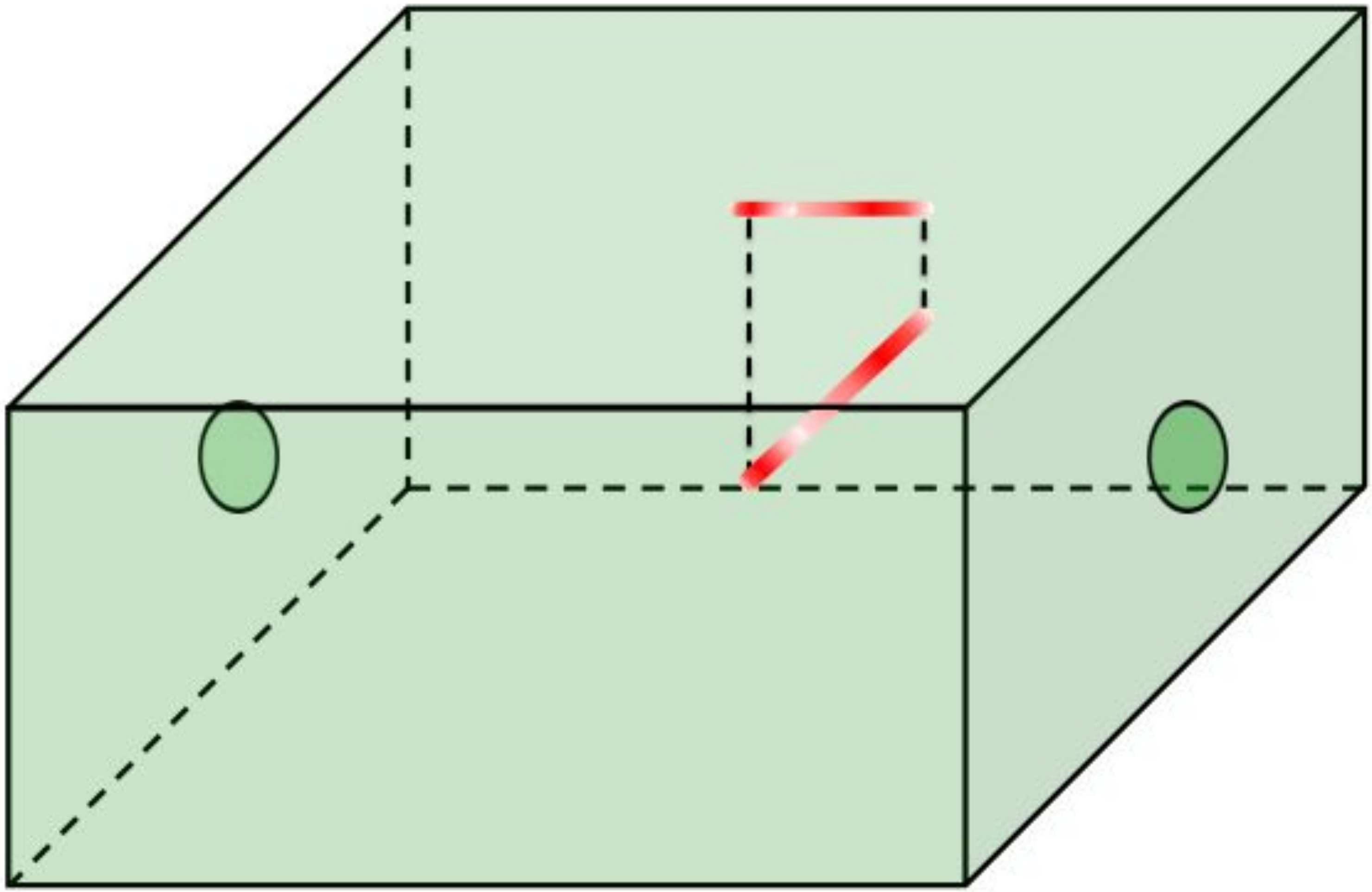

Signa
Intensity
Time
β

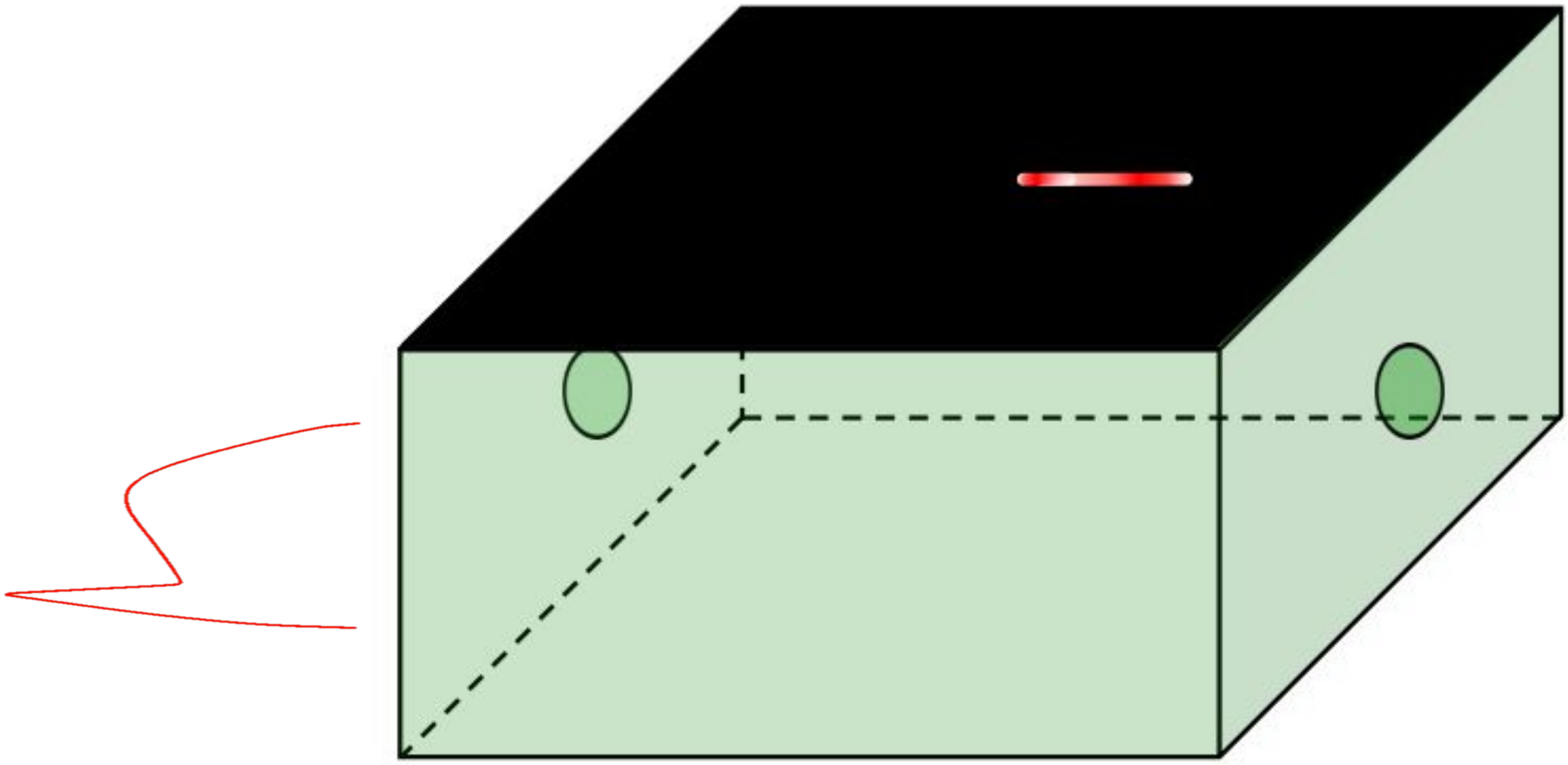
Electronic time
Evolving "picture"







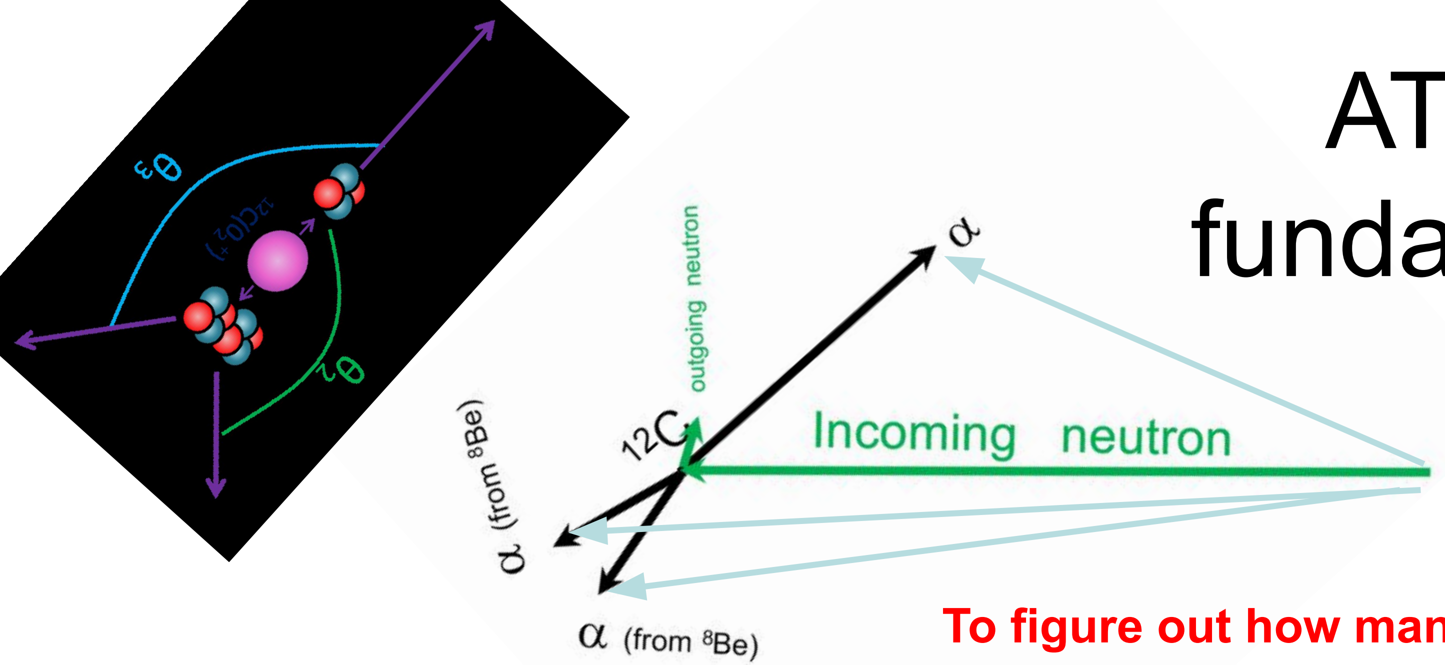




Combining the 2D image and the 1D time projection

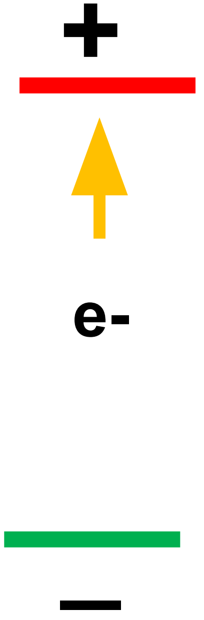
☐ <u>3D path of the track – angular distributions</u>

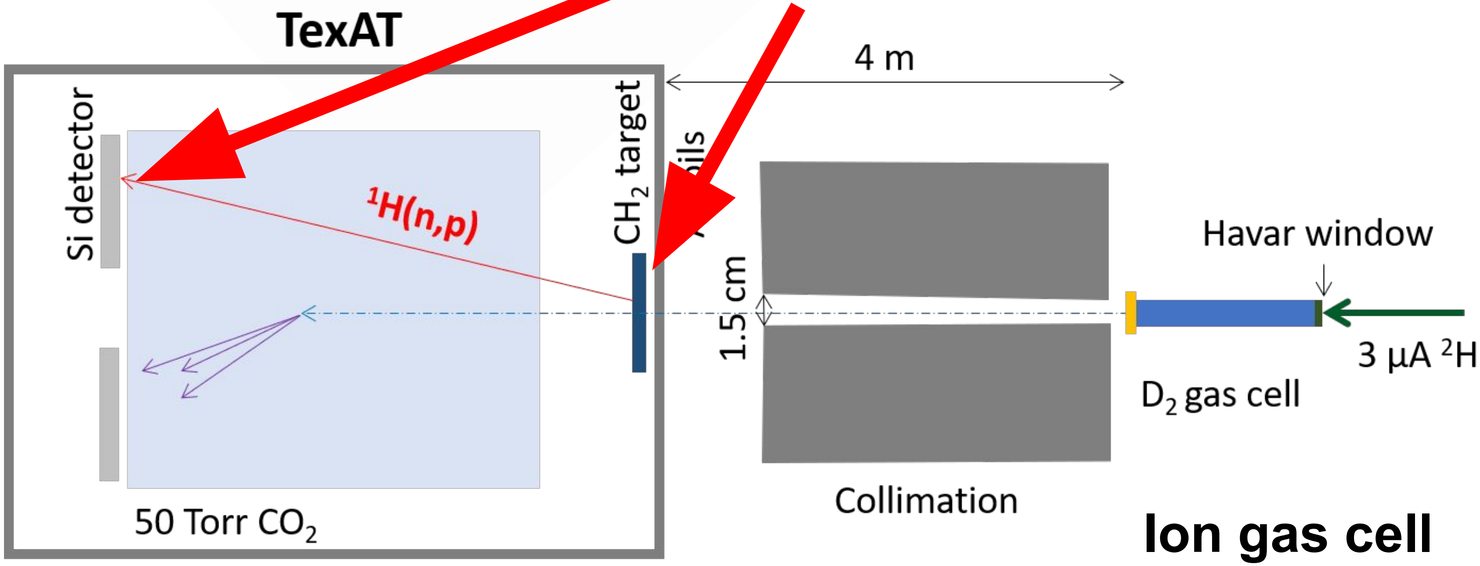
AT-TPC fundamentals



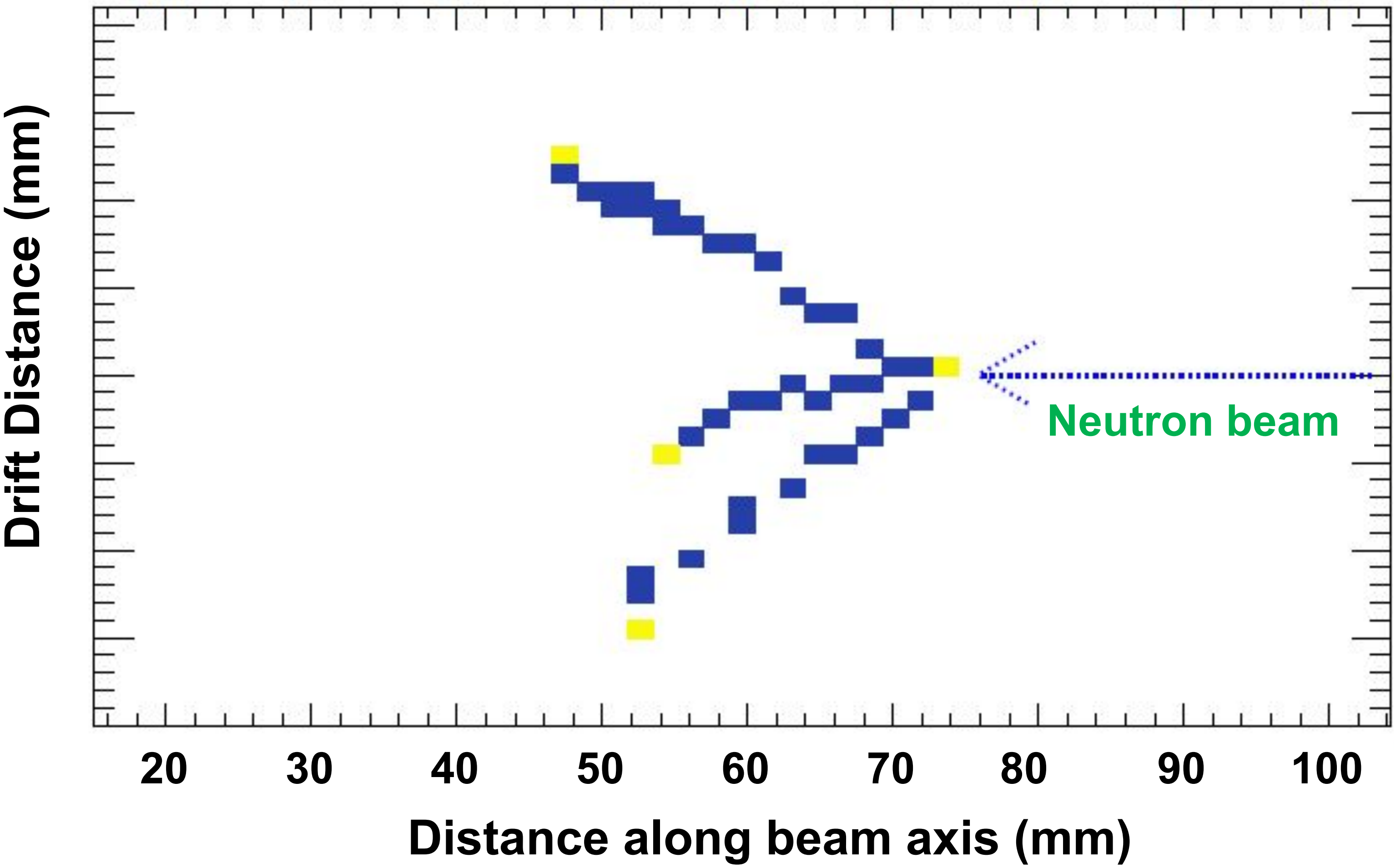
To figure out how many neutrons … primary method (n,p)

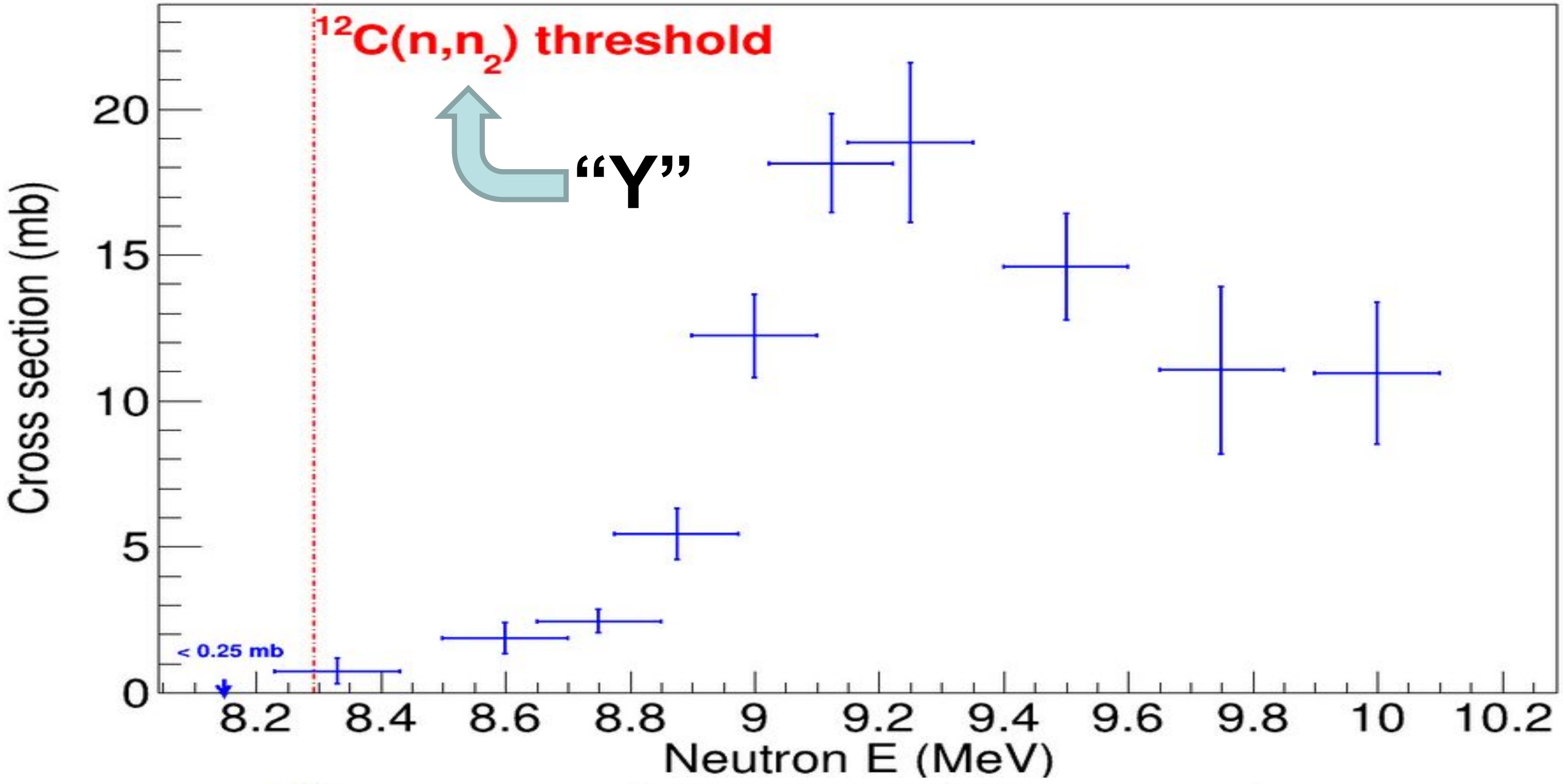
Micro pattered anode



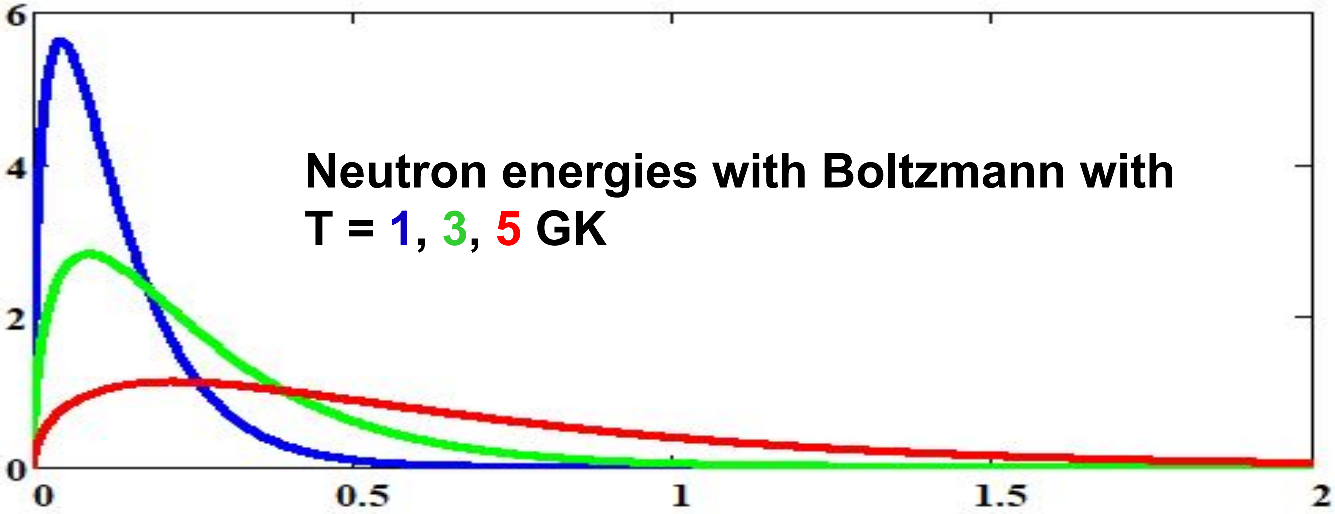


Ion gas cell
$D(d,n)^3He$


Drift Distance (mm)
Neutron beam
20
30
40
50
60
70
80
90
100
Distance along beam axis (mm)



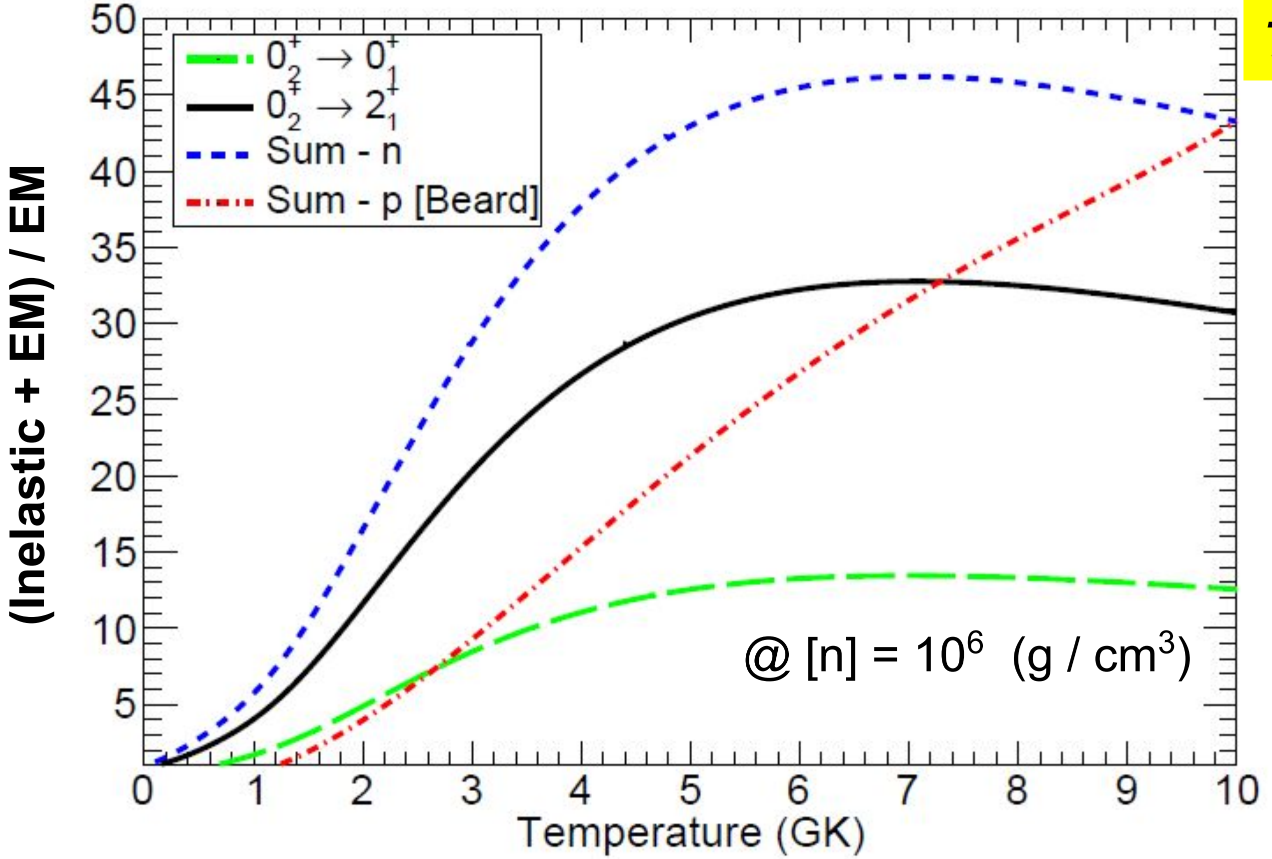
“Y”



Neutron energies with Boltzmann with T = 1, 3, 5 GK

The bottom line …..to get amplification need: high [n] & HOT

Nature Com.,
***13**, 2151 (2022)*
***TAMU** + OU + WU …*



@ [n] = 10^6 (g / cm^3)

Neutron-upscattering enhancement of the triple-alpha process

J. Bishop*,[1] C.E. Parker,[1] G.V. Rogachev,[1,2,3] S. Ahn†,[1] E. Koshchiy,[1] K. Brandenburg,[4] C.R. Brune,[4] R.J. Charity,[5] J. Derkin,[4] N. Dronchi,[6] G. Hamad,[4] Y. Jones-Alberty,[4] Tz. Kokalova,[7] T.N. Massey,[4] Z. Meisel,[4] E.V. Ohstrom,[6] S.N. Paneru,[4] E.C. Pollaco,[8] M. Saxena,[4] N. Singh,[4] R. Smith,[9] L.G. Sobotka,[5,6,10] D. Soltesz,[4] S.K. Subedi,[4] A.V. Voinov,[4] J. Warren,[4] and C. Wheldon[7]

Summary

Some advice

Do not focus on one type of reaction

Do not focus on one facility – it's a big world out there with considerable opportunity at small facilities

Link reactions and structure

Get familiar with a Hauser-Feshbach code

Elastic not boring: prize to the young person who figures out how to treat elastic scattering that is continuous in both energy and angle.

Alignment in intermediate energies could be further exploited (for structure)

Up scattering (e.g. on isomer beams) not fully exploited.

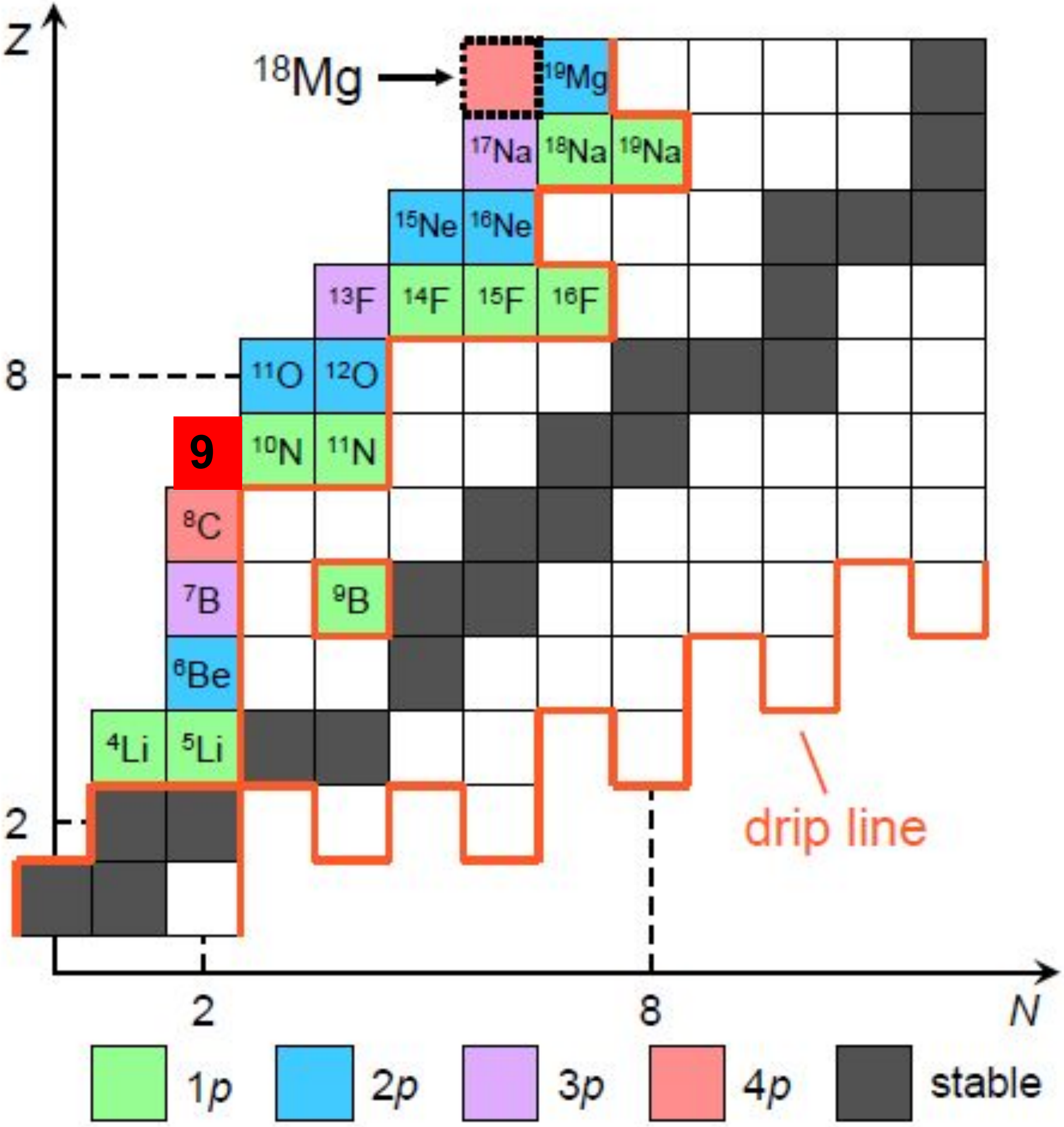
Invariant-mass spectroscopy
on the p-rich side has:

discovered may new isotopes

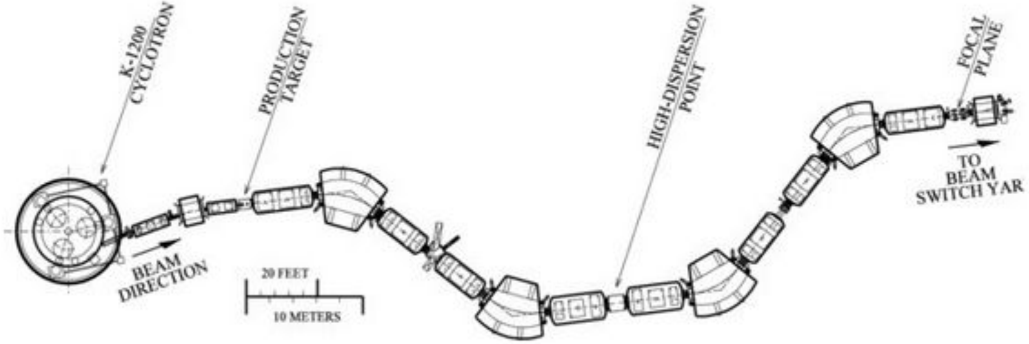
systematized p^n decay

found dozens of new state

refined several dozen energies, widths and spins.

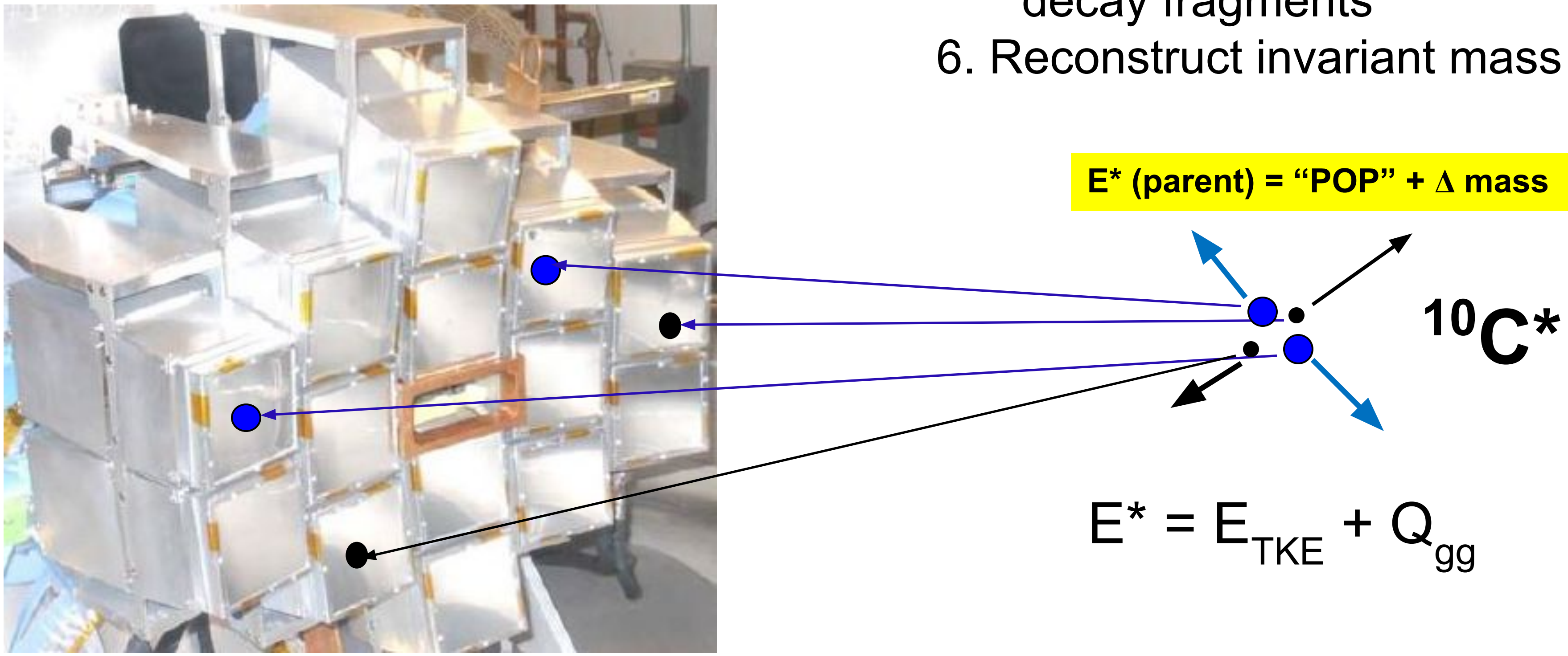


Invariant-mass spectroscopy – how it is done.



1. Accelerate primary
2. Fragment primary
3. Select desired secondary
4. Direct to 2^{ndary} beam to 2^{ndary} target in front of HiRA
5. Detect all charged particle decay fragments
6. Reconstruct invariant mass

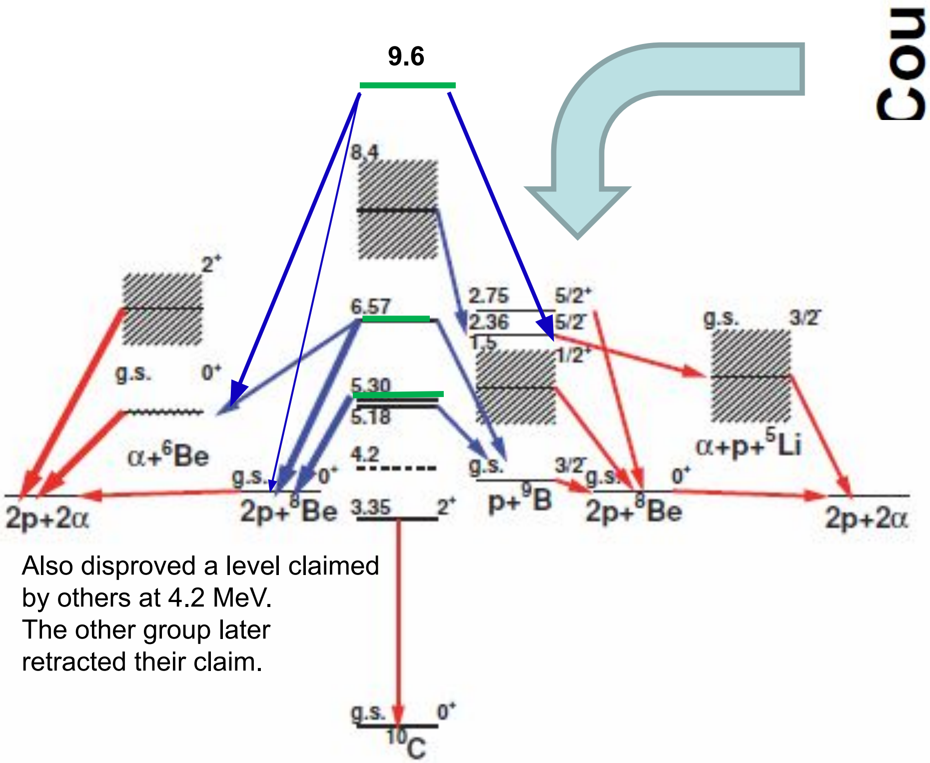
E* (parent) = "POP" + Δ mass



$$E^* = E_{TKE} + Q_{gg}$$

~ 14,000 pixels, all "telescoped" ……. Specialty ASIC (HINP) runs the show.

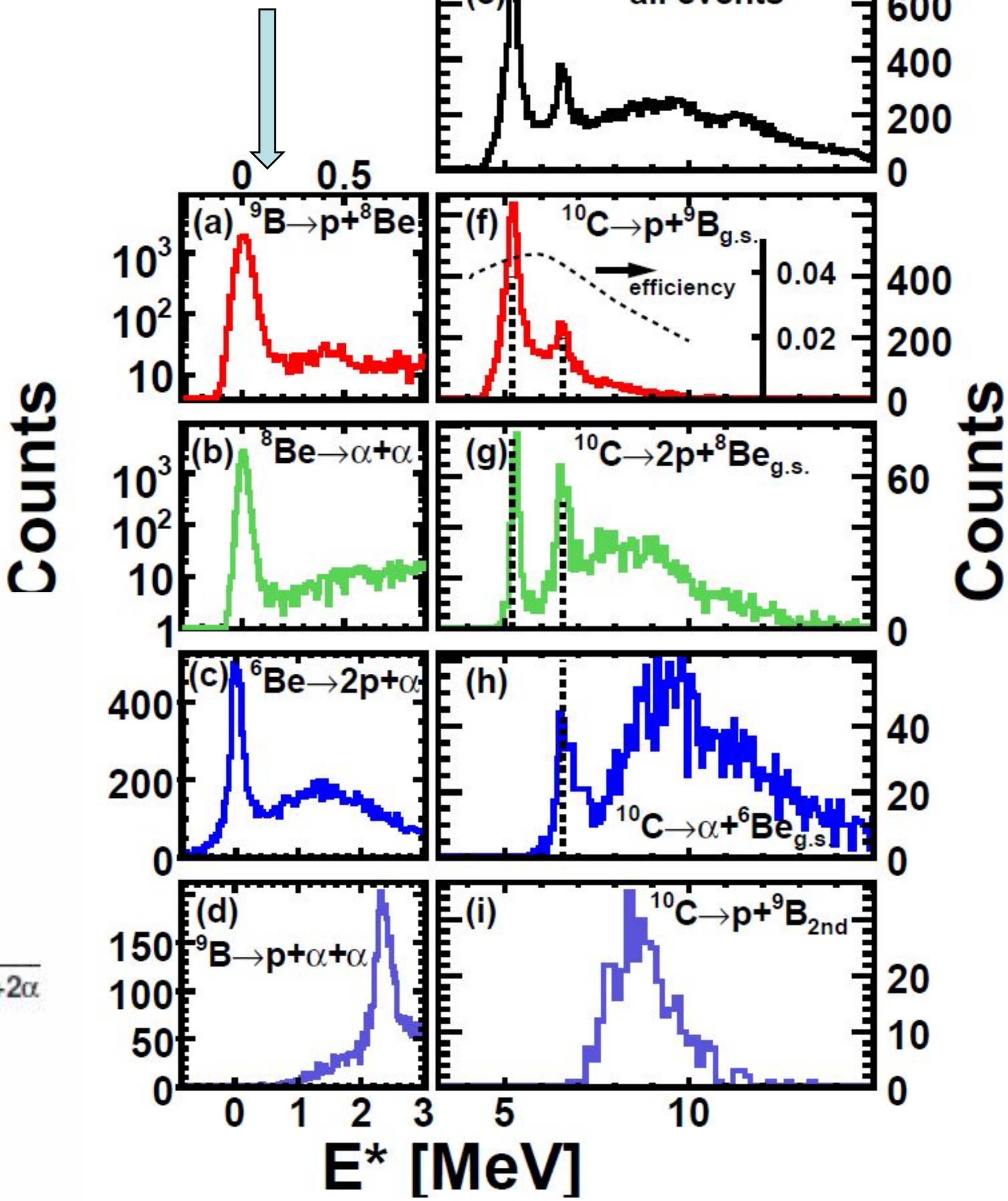
1) Determined the decay paths for known and new levels in ^{10}C using…. 4-particle and sub event (2- and 3-particle) energy correlations.



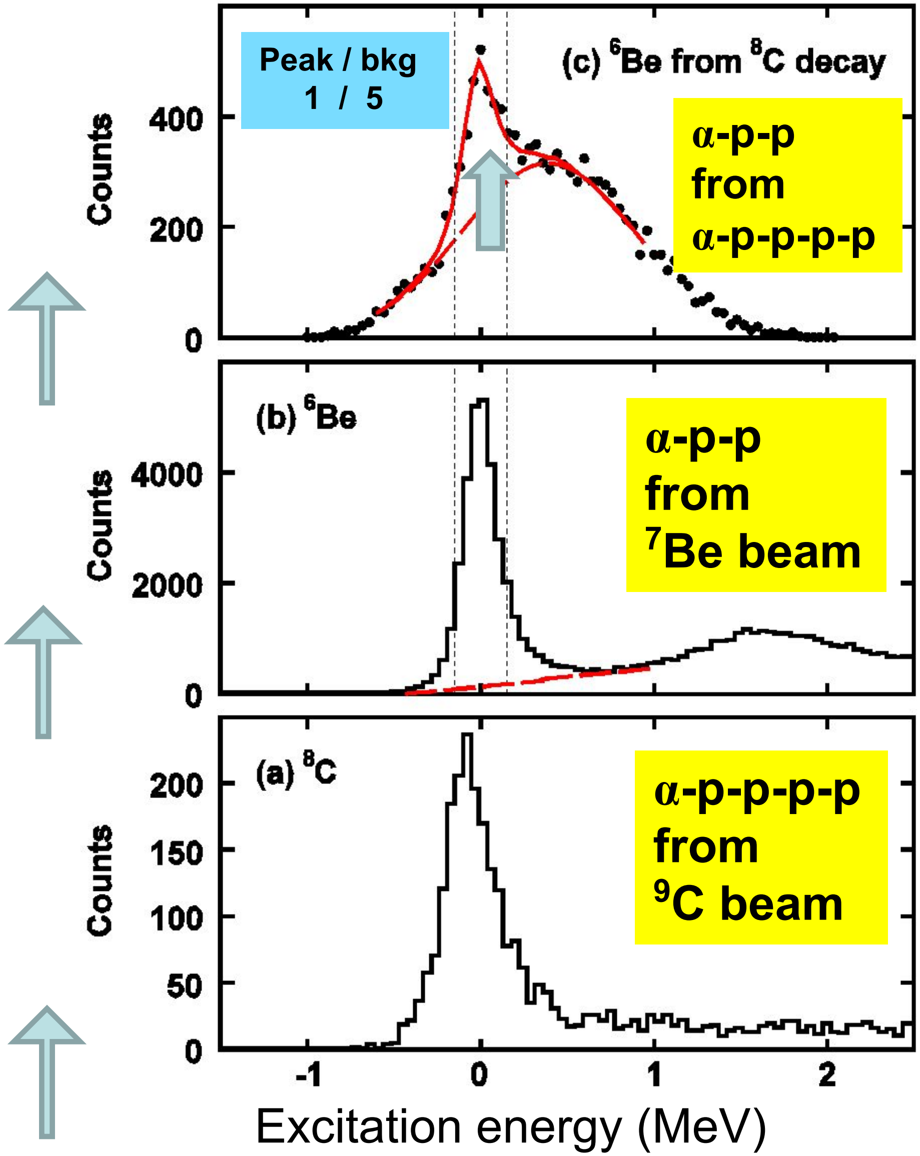
Also disproved a level claimed by others at 4.2 MeV. The other group later retracted their claim.

2,3 - particle intermediates

4-particle (ααpp)



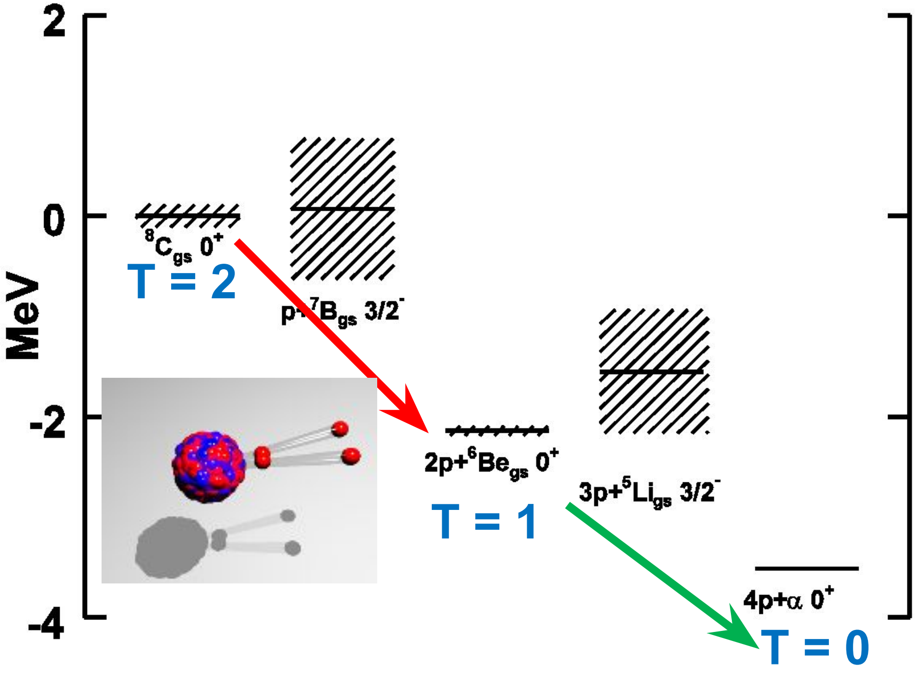
^{8}C decay



α-p-p
from
α-p-p-p-p

α-p-p
from
^{7}Be beam

α-p-p-p-p
from
^{9}C beam



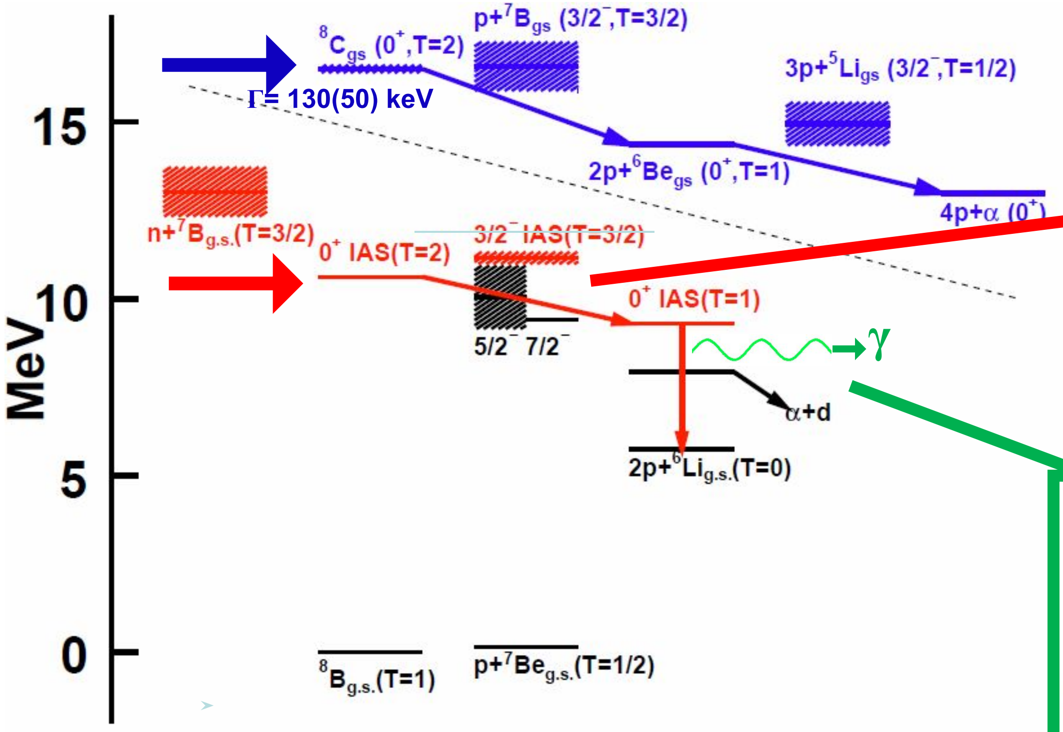
^{6}Be is the (7 zs, ~ 90 keV) intermediate,
i.e. ^{8}B □ [^{6}Be] + 2p + [α +2p] +2p

We studied the 3-body correlation for ^{6}Be decay AND the 3-body correlations for ^{8}C decay.

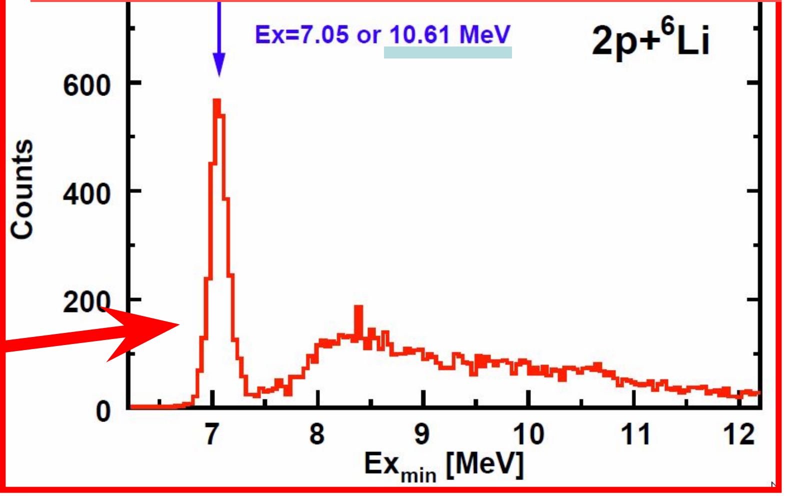
In ~ 1/3 of the events only ONE of the six combinations lies in the ^{6}Be peak. For these events we can assign protons to first and second steps.
□ enhancement at small rel. mom.

2p-2p decay and IAS□IAS 2p decay

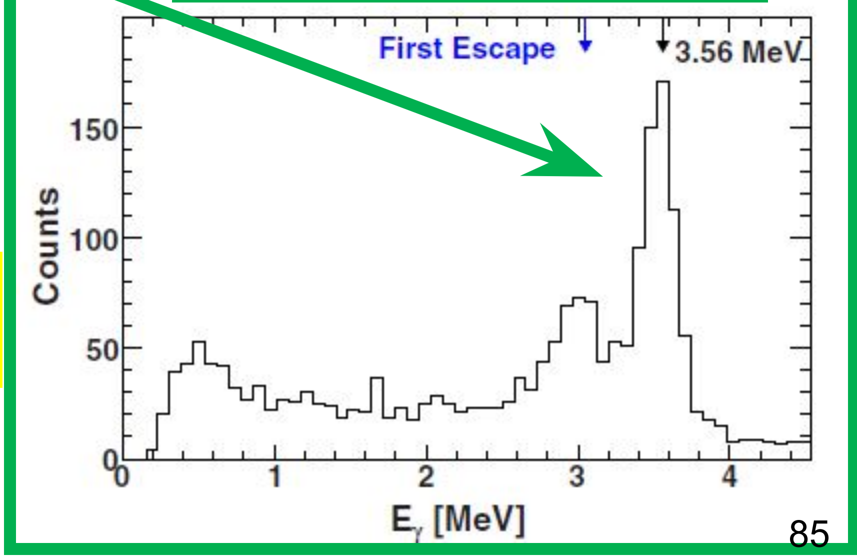
TOP ^{9}C □ ^{8}C$_{gd}$ (0+, T=2) +n
BOT ^{9}C □ ^{8}B$_{IAS}$ (0+, T=2) +p



^{8}B reconstruction from ^{6}Li+p+p

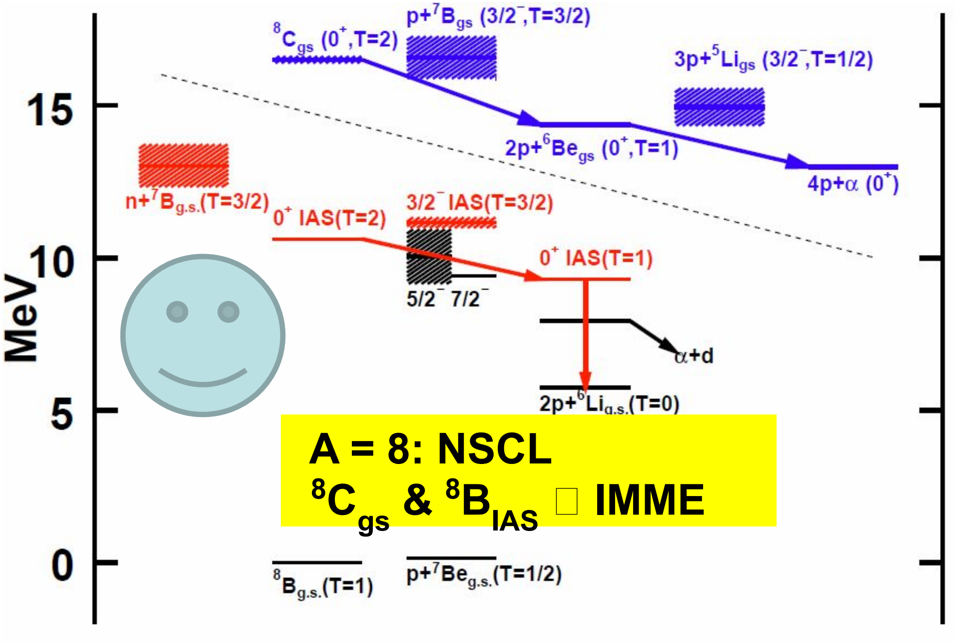


^{6}Li$_{IAS}$ □ Li$_{gs}$ + gamma



1p and 1n decays are forbidden by either energy or isospin

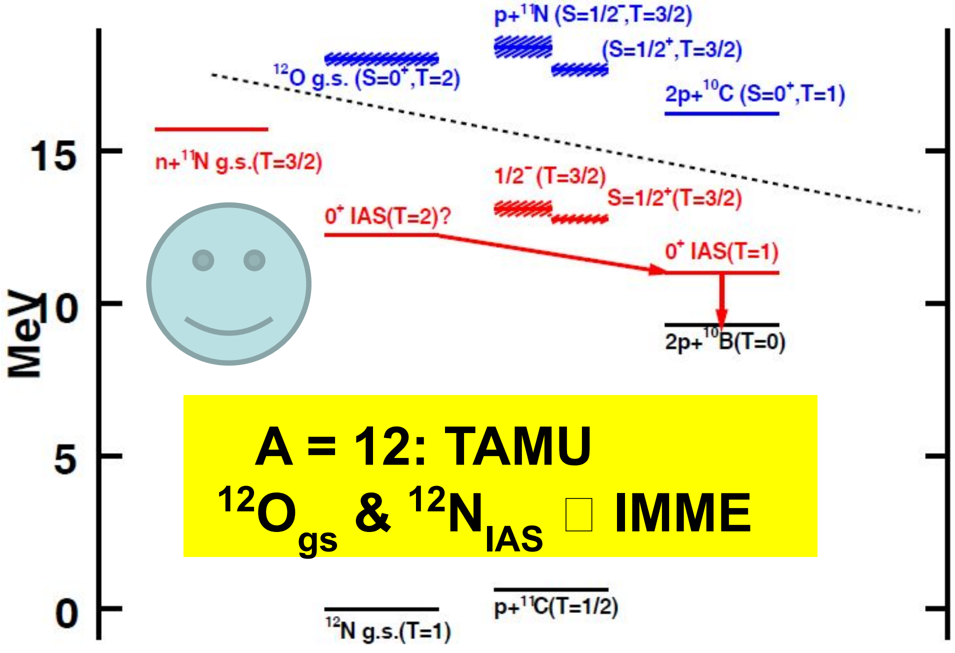
R. J. Charity, et al., Phys. Rev. C **82**, 041304(**R**) (2010).
K. Brown, et al., Phys. Rev. C **90**, 027304 (2014).



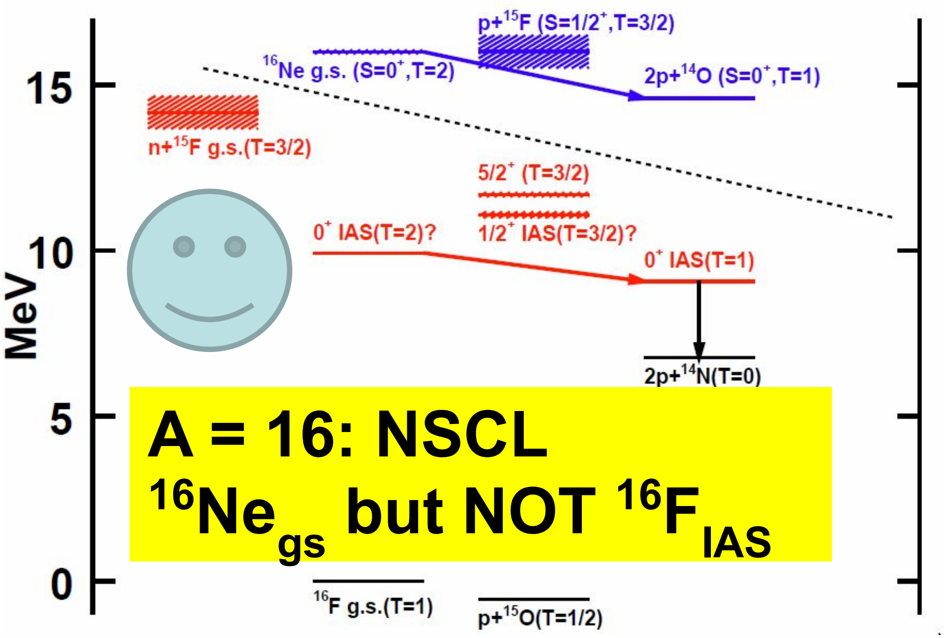
A = 8: NSCL
$^8C_{gs}$ & $^8B_{IAS}$ □ IMME

We have actually found two cases of IAS □ IAS (true?) 2p decays

We are still puzzled over why we did not find third decay for $^{16}F_{IAS}$

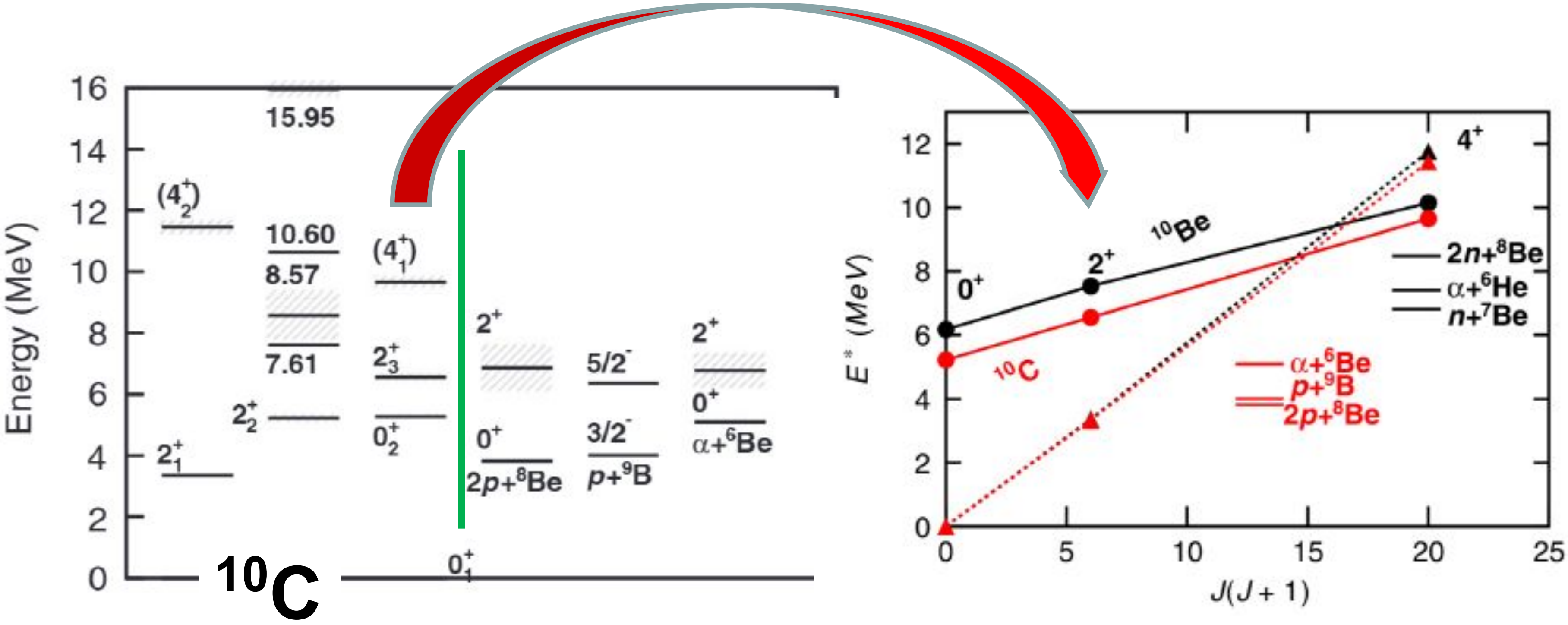


A = 12: TAMU
$^{12}O_{gs}$ & $^{12}N_{IAS}$ □ IMME



A = 16: NSCL
$^{16}Ne_{gs}$ but NOT $^{16}F_{IAS}$

0^+_2 and its (tentative) rotational band
“Dead ringer” for ^{10}Be 0^+_2 mirror band

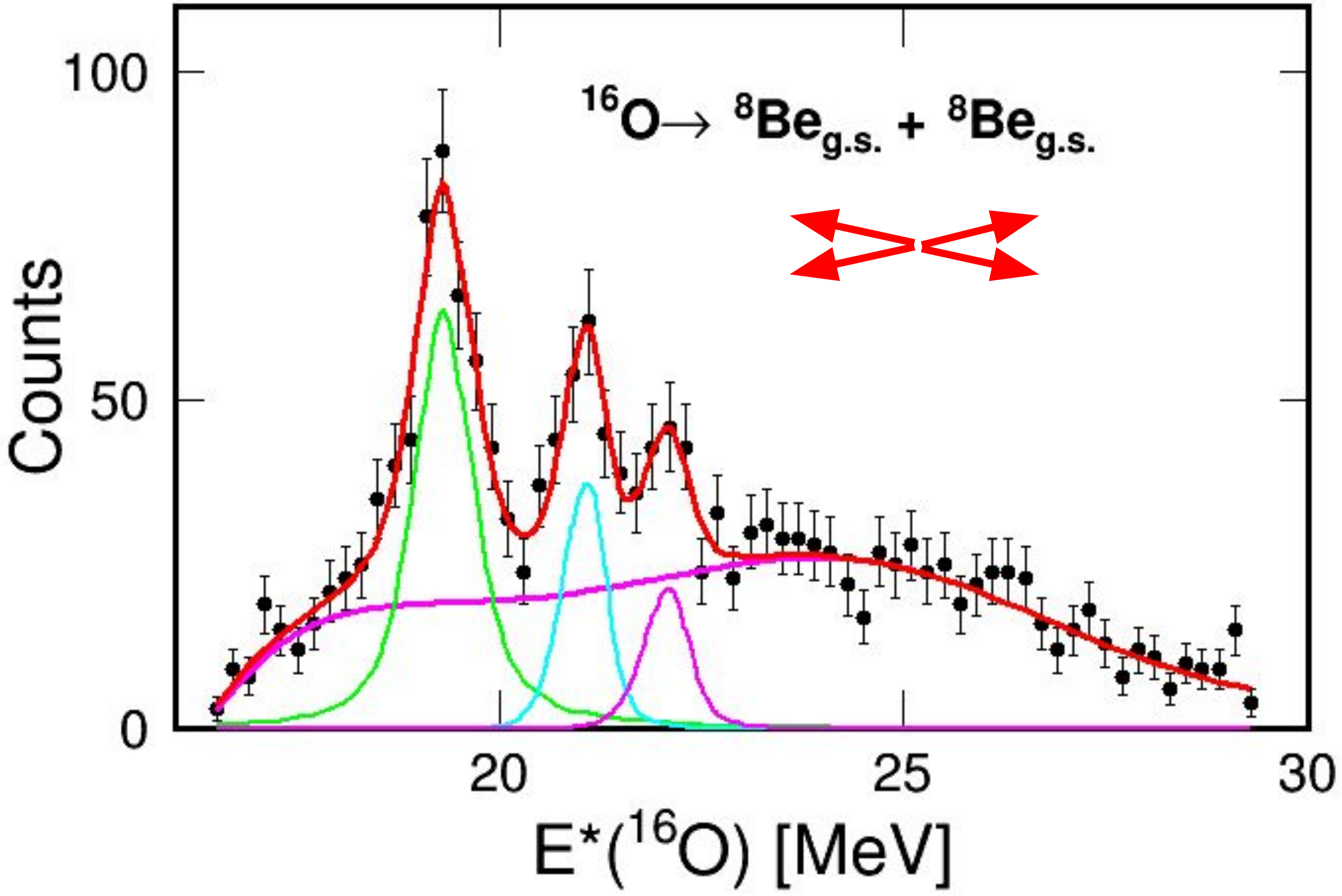


(spin) assignments made by comparing 2p momentum correlations to those from 2p emitters of known spin and significant reduced alpha widths (like mirror).

R.J. Charity et al., PRC **105**, 014314 (2022).

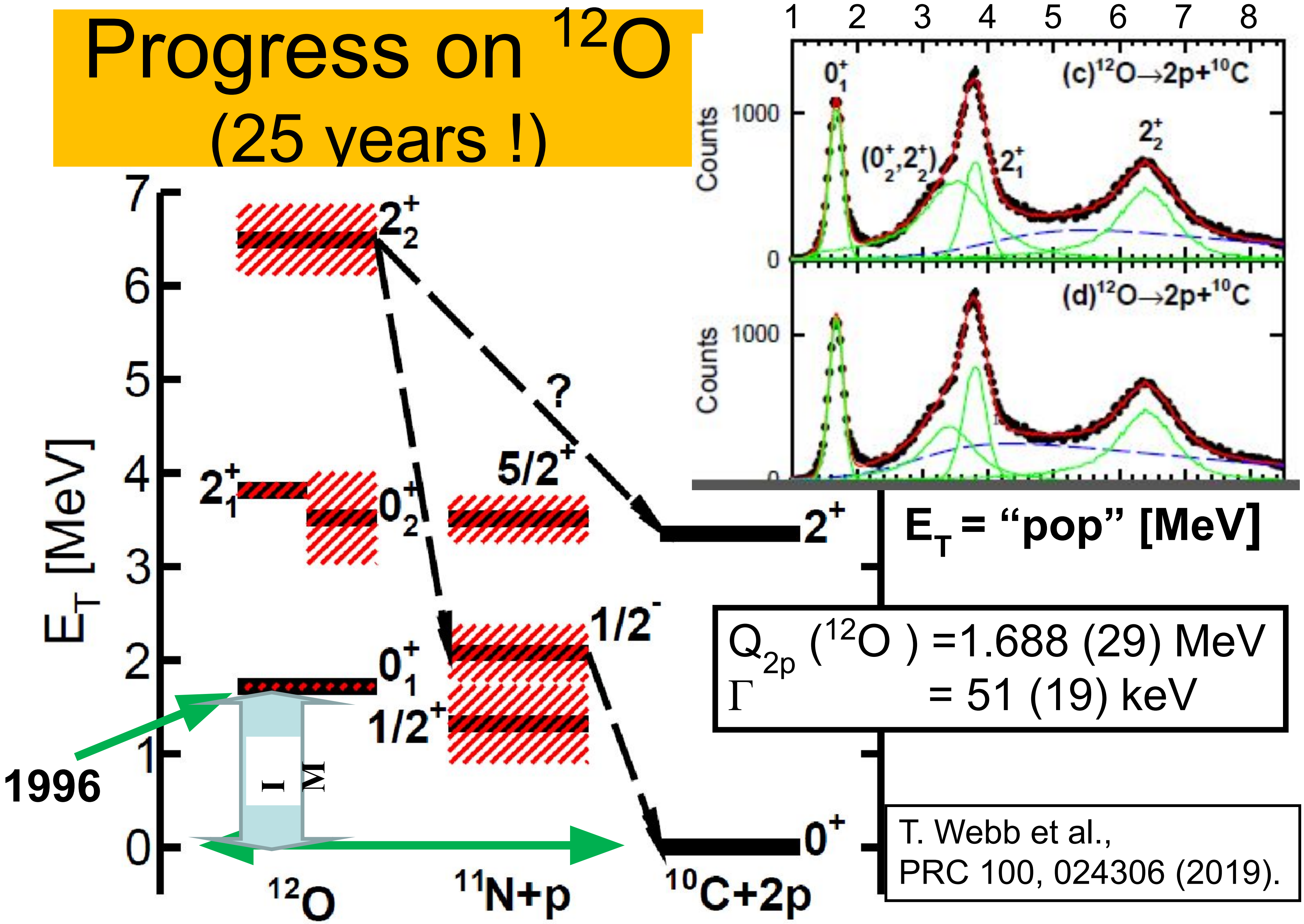
Side ^{16}O story: fission (i.e. two ^{8}Be)

$E^* = 19.2, 20.9, 22.0 \quad \Gamma < 400$ keV



R.J. Charity et al., PRC **99**, 044304 (2019).

Progress on ^{12}O (25 years !)



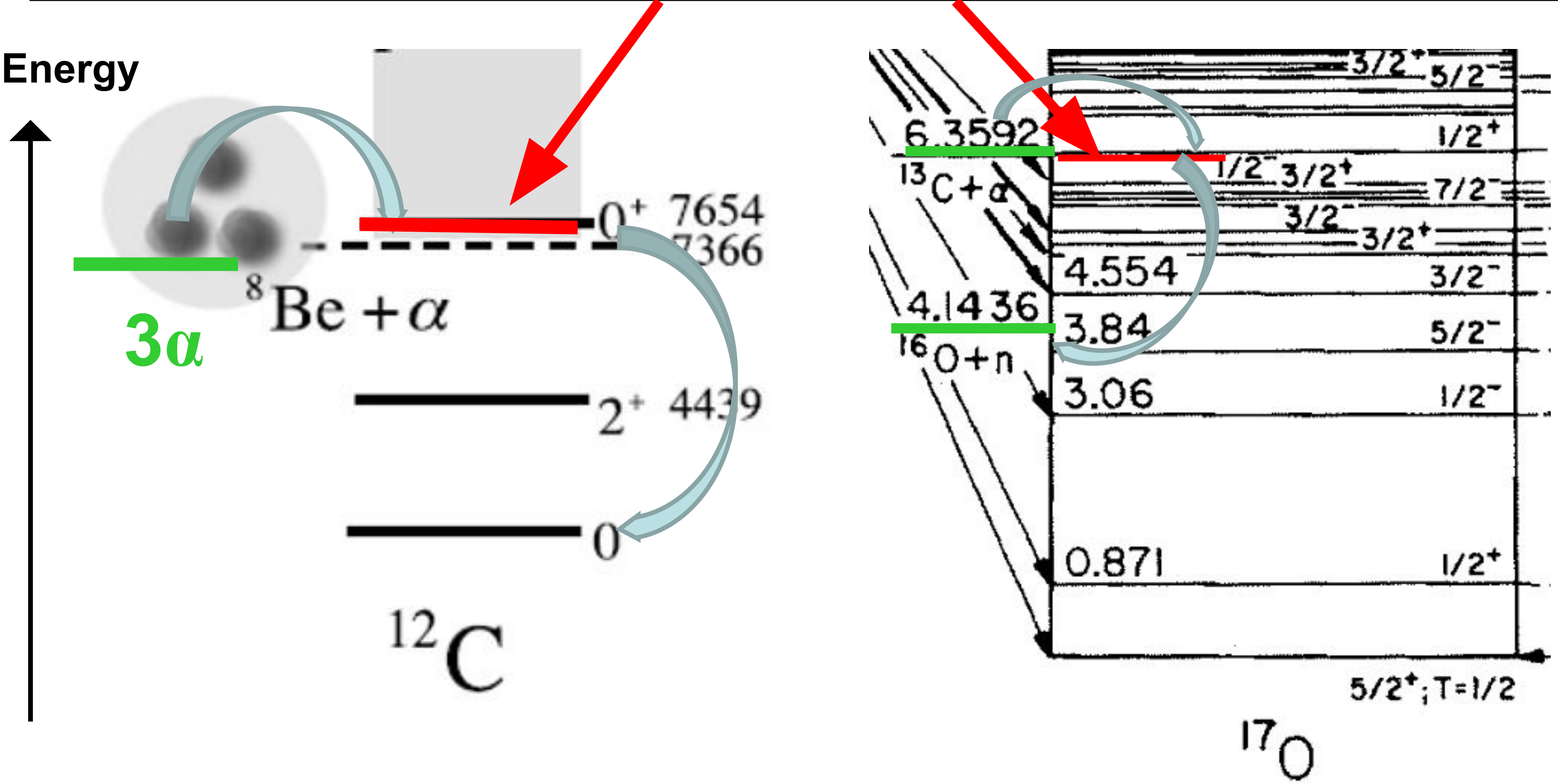
E_T = "pop" [MeV]

Q_{2p} (^{12}O) =1.688 (29) MeV
Γ = 51 (19) keV

T. Webb et al.,
PRC 100, 024306 (2019).

Two Anthropic motivating cases of resonances near thresholds

$^{12}C^*$ ^{17}O



Generates seed for all ALL heavier elements

generates neutrons for the s-process ~ ½ of heavy element synthesis

These **resonances are doorways BUT are we just “lucky”?**

Variants

Momentum Achromat Recoil Separator
(MARS)

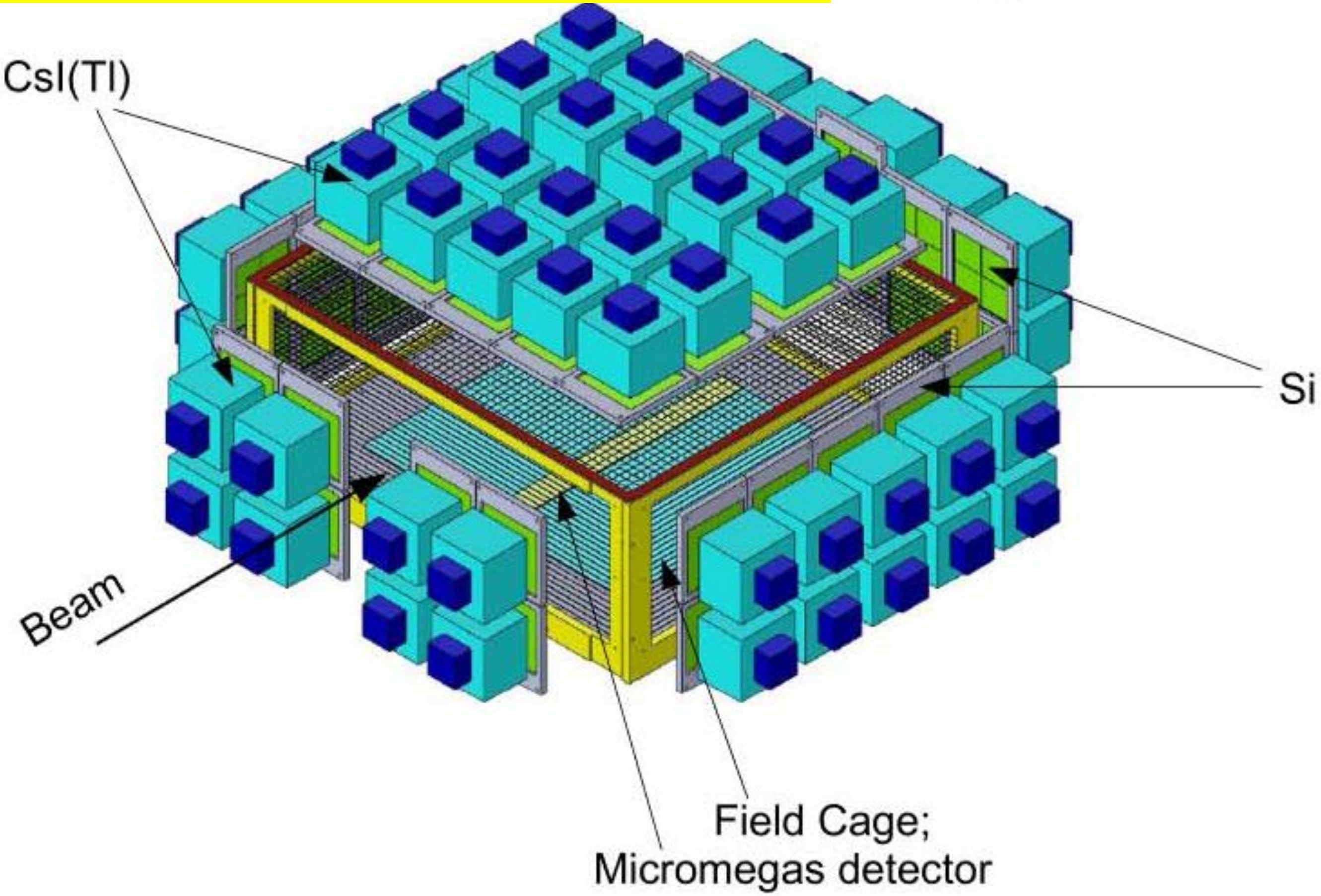
Many
don
Bes
Hav

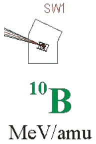
Some
Provi

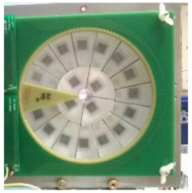
Nuclear Inst. and Methods in Physics Research, A 957 (2020) 163398

Tex-AT
Texas Active Target TPC

Detectors: Si (58 total);
CsI(Tl) (58 total);
Micromegas: 1024 channels



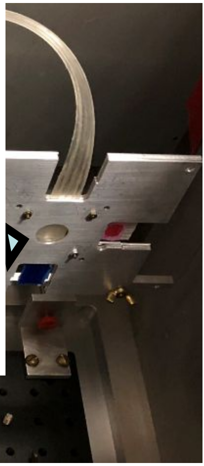




on in Si,

Somet
CAES...
(Gade and Weisshaar)

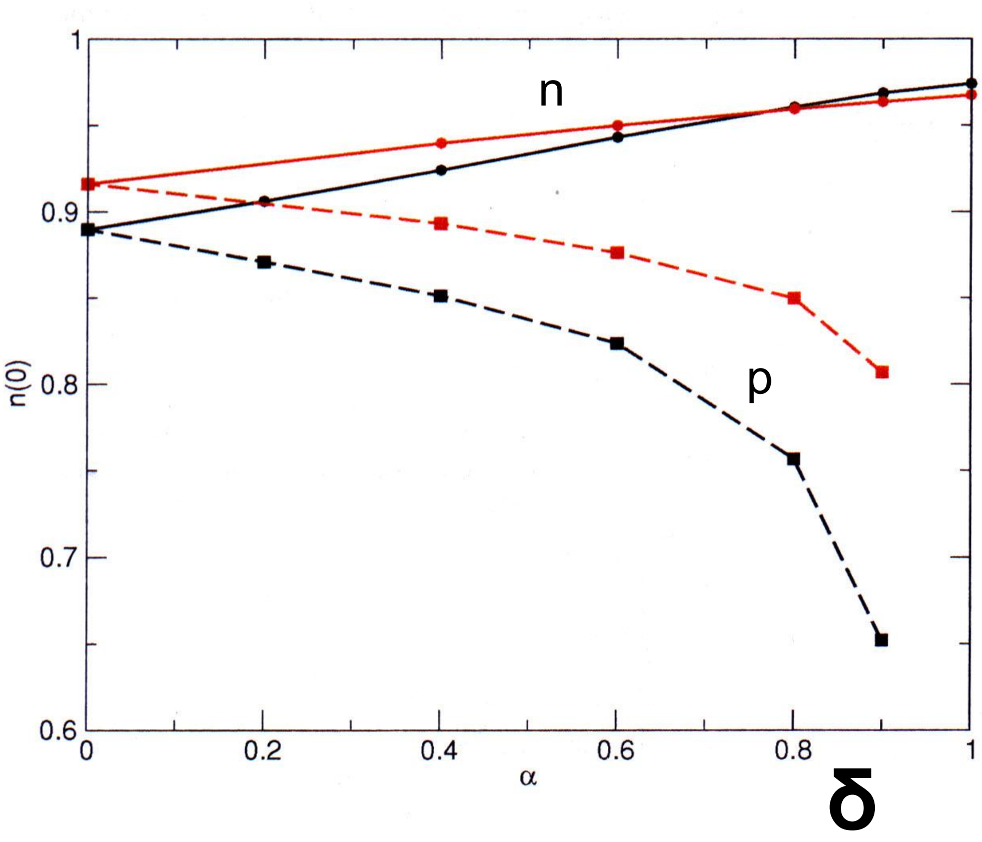
(SFA - WU)



SP occupation (asymmetry)

SCGF calc for NM

Fick, Muther, Rios, A. Polls, and A. Ramos and PRC **71**, 014313 (2005).



With increasing asymmetry one expects:

(the occupation at k = 0)

n_n ⇑ and n_p ⇓

Neutrons become less correlated

Protons become more correlated

$\rho = 0.16\ \mathrm{fm}^{-3}$, $\rho = 0.32\ \mathrm{fm}^{-3}$

How are we doing this? Example from TAMU

□ using K500 cyclotron and the **MARS** separator

From enriched Carborane $C_2[^{10}B_{10}]H_{12}$

ECR source

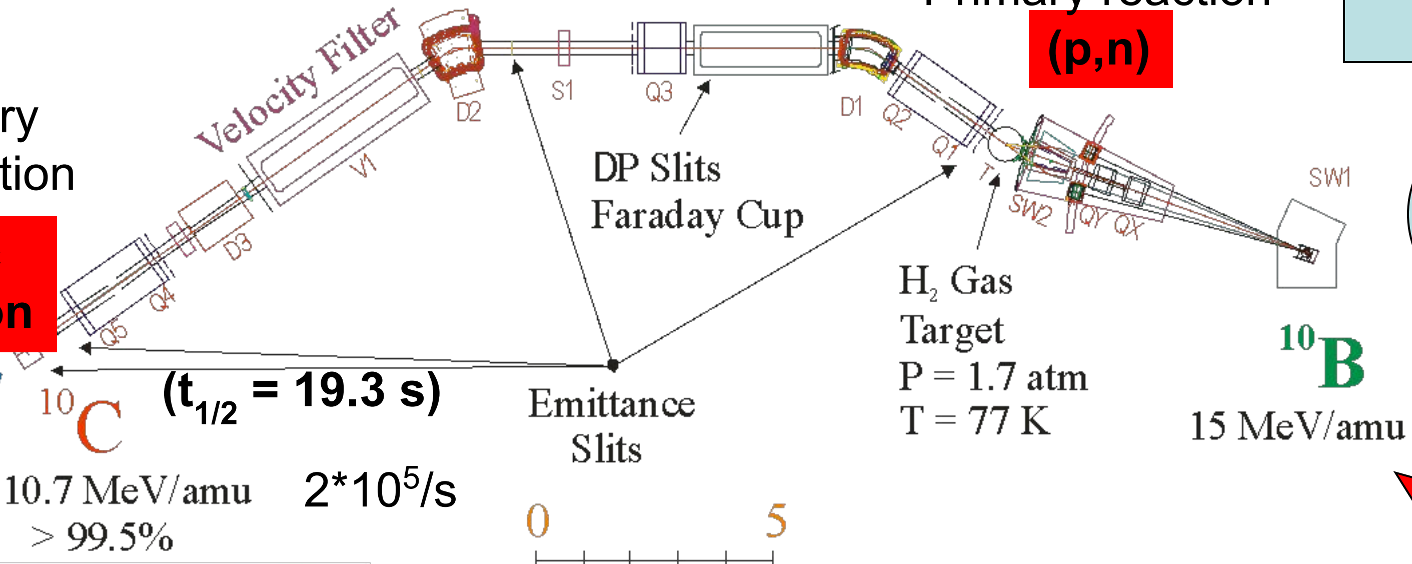
K-500 cyc

Primary reaction

(p,n)

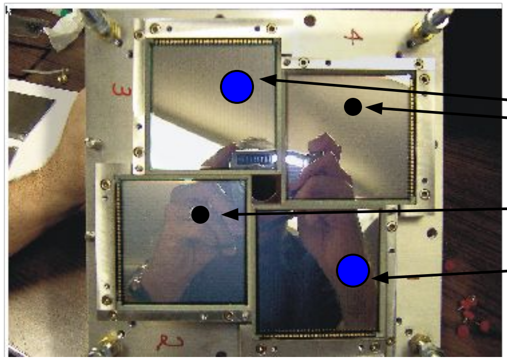
Secondary reaction

Inelastic excitation



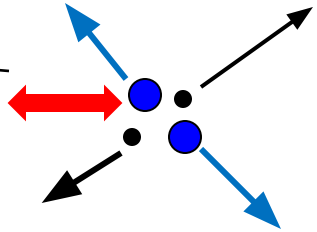
($t_{1/2}$ = 19.3 s)

$2*10^5$/s



4 ΔE-E Si "telescopes" from the HiRA array

E* (parent) = "POP" – Δ mass



A 4-particle correlation experiment !

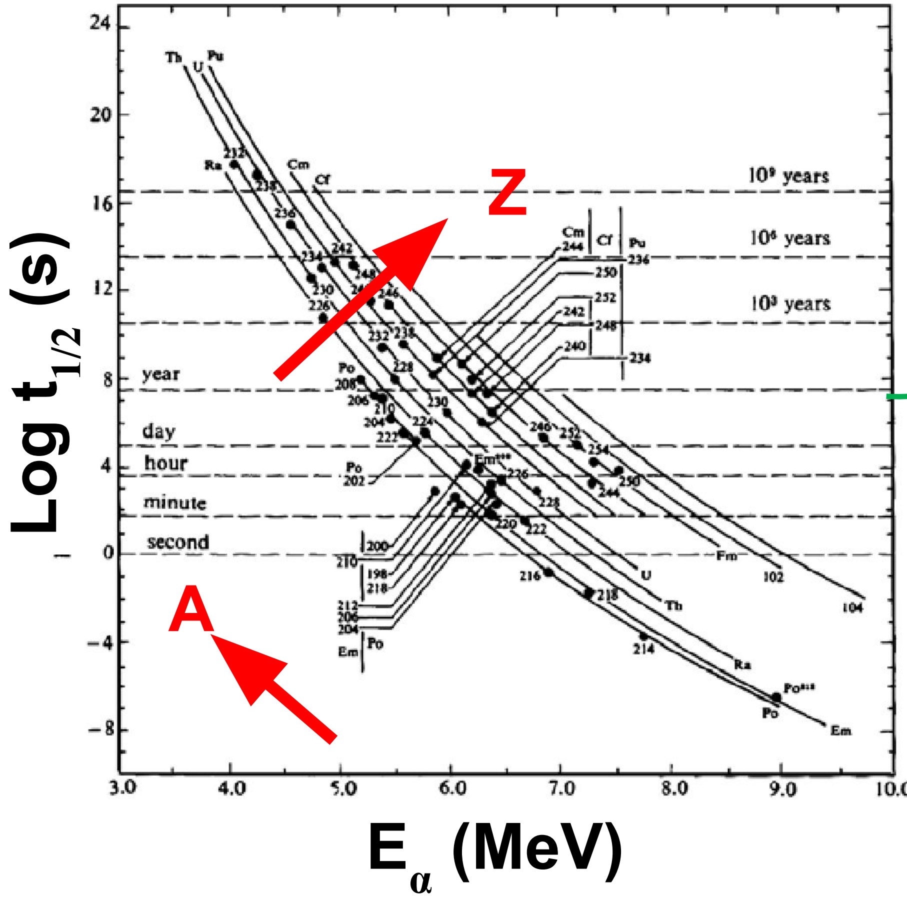
$$E^* = E_{T(KE)} - \Delta M$$

□ Time, Energy, and Particle resolving "CAMERA" with 4k pixels □

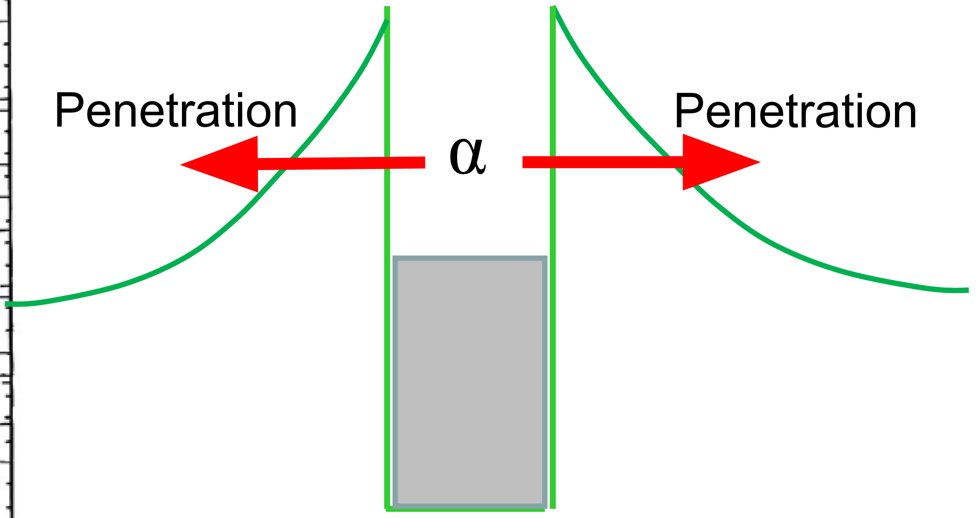
Next Topic: Action decomposition

Barrier penetration □ Dual penetration

Lesson 1: G. Gamow, Z. Phys **51**, 204 (**1928**) explained Geiger-Nutall relation



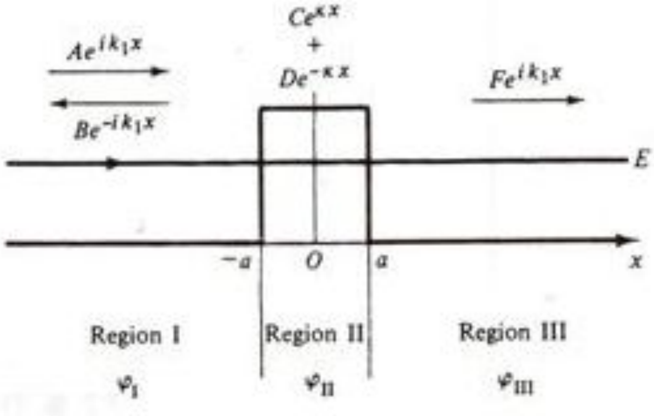
He used a “cusp” Coulomb potential



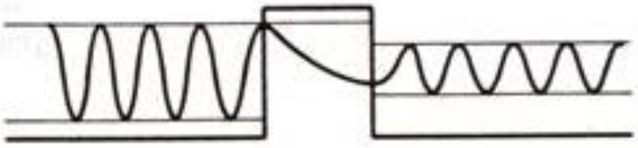
Z goes up – Coul. Barrier goes up so lifetime goes up

A goes up - E_α (Q) goes down so lifetime goes up

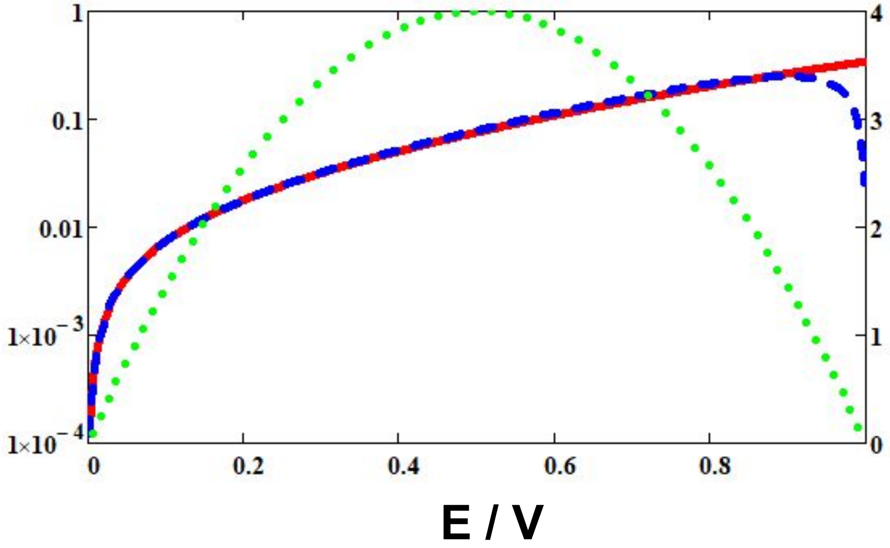
Lesson 2: “transmission” through a square well



(a)



(b)



$$\frac{1}{T} = 1 + \frac{1}{4}\left(\frac{k_1^2 - \kappa^2}{k_1\kappa}\right)^2 \sinh^2(2\kappa a)$$

$$\frac{1}{T} = 1 + \frac{1}{4}\,\frac{V^2}{E(V - E)}\,\sinh^2(2\kappa a)$$

$$T^{step} \sim e^{-\frac{2}{\hbar}(2\mu(V-E))^{1/2}\Delta x}\left[\frac{4k\kappa}{k^2 + \kappa^2}\right]^2.$$

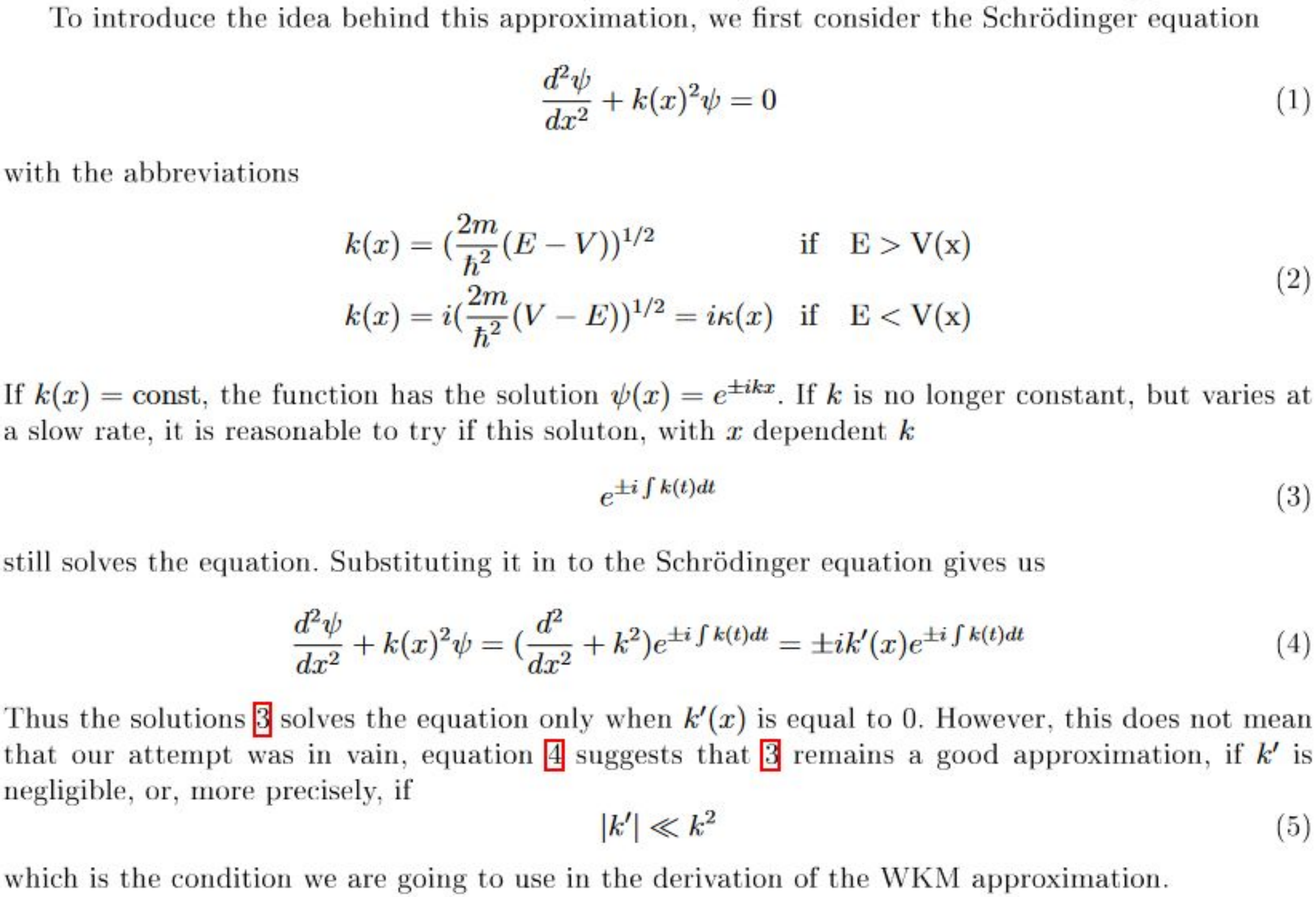
-Forbidden action^2

Lesson 3: (J)WKB barrier penetration (Jefferies) Wentzel, Kramers, Brillouin

As SE, generally, has no analytic solution ……

 simplified pot + small terms (pert theory) OR

 Assume slowly varying potential WKB (appropriate for robust barriers)



To introduce the idea behind this approximation, we first consider the Schrödinger equation

$$\frac{d^2\psi}{dx^2} + k(x)^2\psi = 0 \tag{1}$$

with the abbreviations

$$\begin{aligned} k(x) &= (\frac{2m}{\hbar^2}(E-V))^{1/2} && \text{if} \quad \mathrm{E > V(x)} \\ k(x) &= i(\frac{2m}{\hbar^2}(V-E))^{1/2} = i\kappa(x) && \text{if} \quad \mathrm{E < V(x)} \end{aligned} \tag{2}$$

If $k(x) = \text{const}$, the function has the solution $\psi(x) = e^{\pm ikx}$. If k is no longer constant, but varies at a slow rate, it is reasonable to try if this soluton, with x dependent k

$$e^{\pm i\int k(t)dt} \tag{3}$$

still solves the equation. Substituting it in to the Schrödinger equation gives us

$$\frac{d^2\psi}{dx^2} + k(x)^2\psi = (\frac{d^2}{dx^2} + k^2)e^{\pm i\int k(t)dt} = \pm ik'(x)e^{\pm i\int k(t)dt} \tag{4}$$

Thus the solutions 3 solves the equation only when $k'(x)$ is equal to 0. However, this does not mean that our attempt was in vain, equation 4 suggests that 3 remains a good approximation, if k' is negligible, or, more precisely, if

$$|k'| \ll k^2 \tag{5}$$

which is the condition we are going to use in the derivation of the WKM approximation.

H. Jeffreys, Proc. Lond. Math. Soc. S2-23, 1, 428 (1925). G. Wentzel, Z. Phys. 38, 518 (1926), H. A. Kramers, Z. Phys. 39, 828 (1926).
Any damn QM book {Merzbacher ch 7} or our own CB's book

Lesson 3: WKB summary …..

$$P = e^{-2\{G\}}$$

$$G = \{(\sqrt{2\mu(V-E)} * x)/\hbar\}$$

$$\ldots = \{forbidden.."p-x"area\}$$

$$\mathcal{P} \sim e^{-2\int_a^b \kappa(r)dr} = e^{-2G}.$$

Based on this logic, in 1960 Goldansky not only predicted 2p decay (wo intermediate) but also that this decay should optimize the **product** of penetrabilities.

$$\begin{aligned}\mathcal{P}\mathcal{P}(X_E) &\sim \mathcal{P}(\epsilon_1) \times \mathcal{P}(\epsilon_2) \\ &\sim e^{-\frac{2}{\hbar}[\int_a^b (2\mu(V(x)-X_E E))^{1/2} dr]} \\ &\times e^{-\frac{2}{\hbar}[\int_{a'}^{b'} (2\mu(V(x)-(1-X_E)E))^{1/2} dr]}\end{aligned}$$

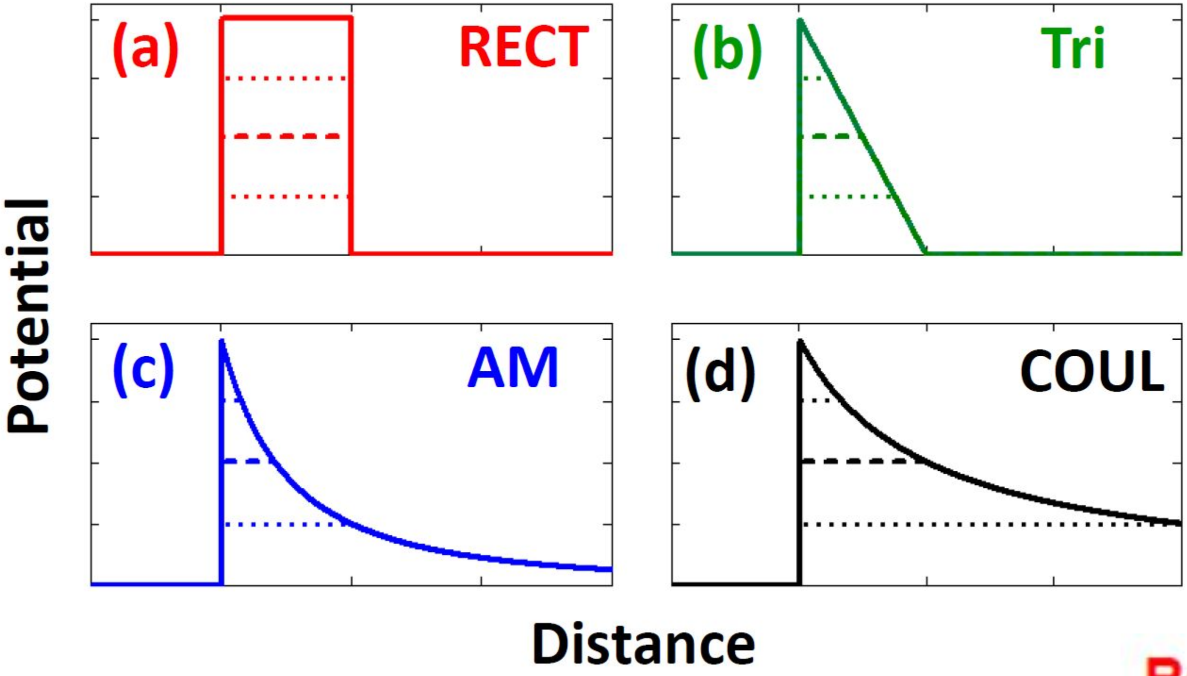
The tail of the Coulomb potential this drives the (dual) barrier penetration to have ~ equal (p) energies. This bias towards equitable E sharing increases as Z increases.

Data had to wait ~ 60 years….

Lesson 4: creep up on reality with a wee bit of geometry

Imagine two particles attacking a potential with the only requirement that they share the total energy.

Consider two cases: 1) energy is shared equally and 2) one gets more than the other.



For NOW compare the **net energy-distance** areas for symmetric vs asymmetric energy division.

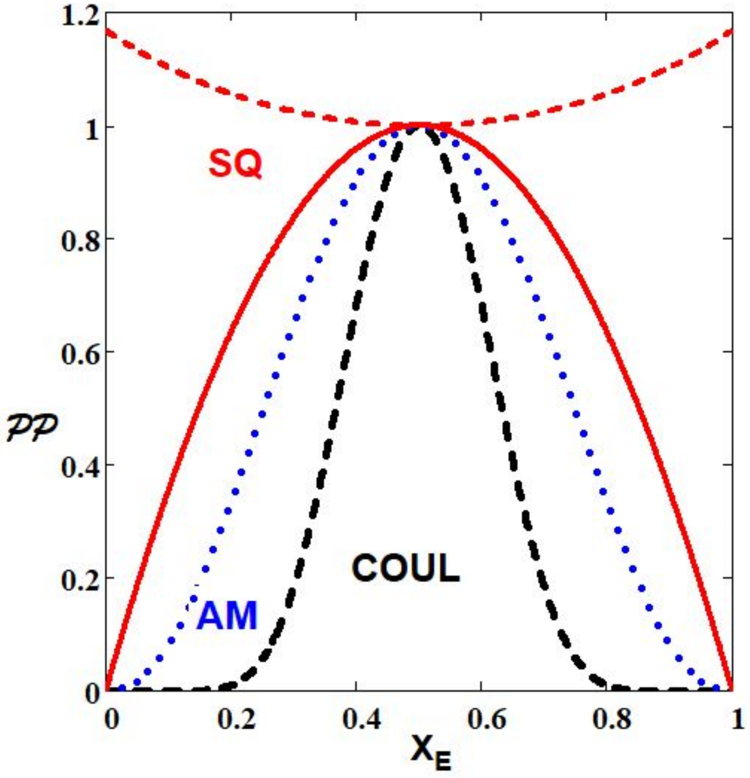
Ratio asy/sym = 1

Ratio asy/sym = 1.25

Ratio asy/sym = 1.69

Ratio asy/sym = 1.74

Lesson 4: now do WKB, i.e. use forbidden actions.



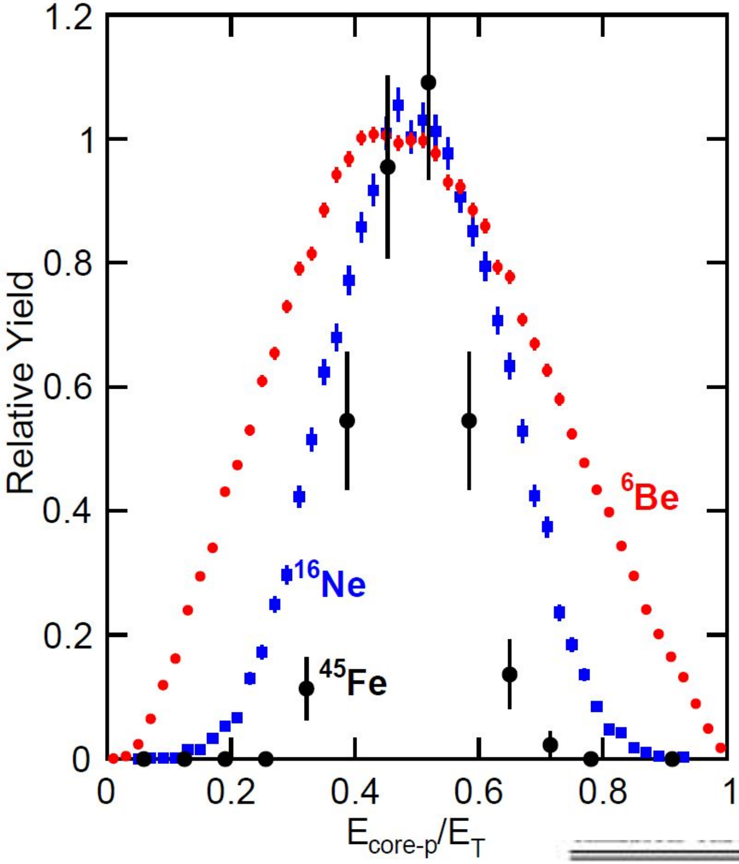
Dual penetration
(□ from action alone □)
when
Potentials have equal action
for X_E = ½
Action = 4 (quanta)

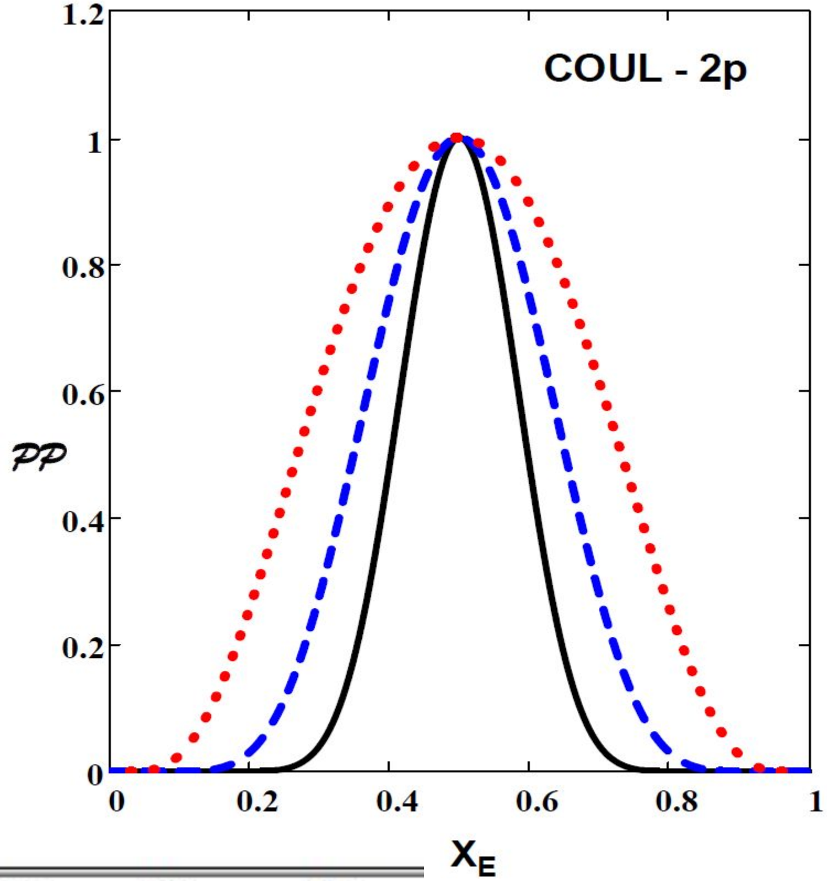
SO…. – PP informs on HOW the Action is accumulated.

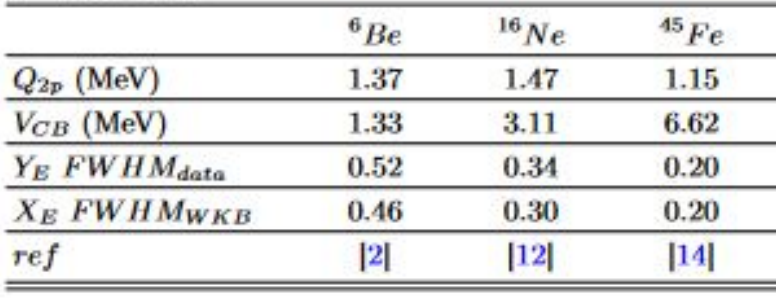
Back to the real world

Experiment **WKB**

with correlations without

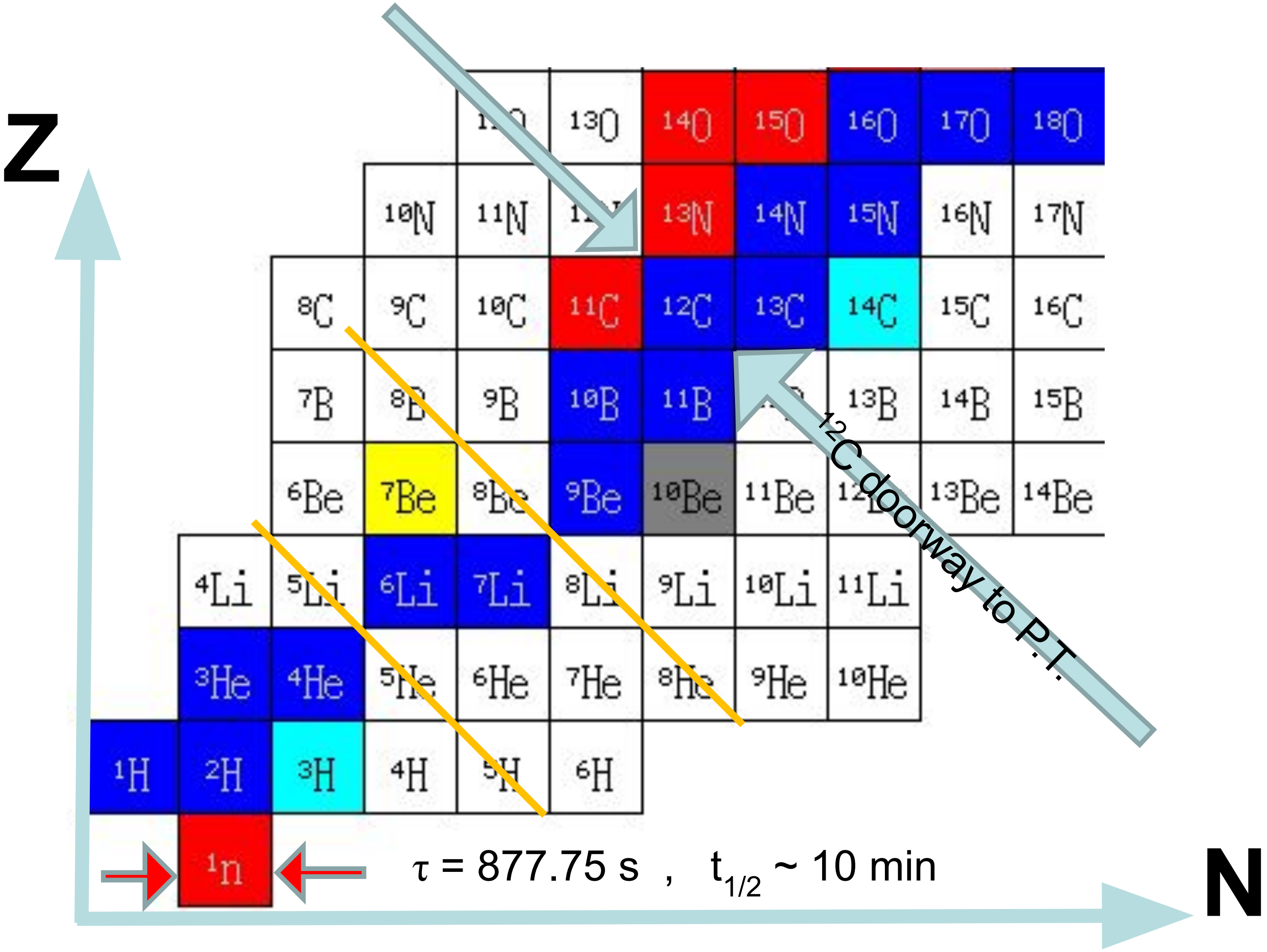






	^6Be	^{16}Ne	^{45}Fe
Q_{2p} (MeV)	1.37	1.47	1.15
V_{CB} (MeV)	1.33	3.11	6.62
Y_E $FWHM_{data}$	0.52	0.34	0.20
X_E $FWHM_{WKB}$	0.46	0.30	0.20
ref	[2]	[12]	[14]

Even more fun with God's quantum dots
a few SHORT STORIES after a primer



τ = 877.75 s , $t_{1/2}$ ~ 10 min

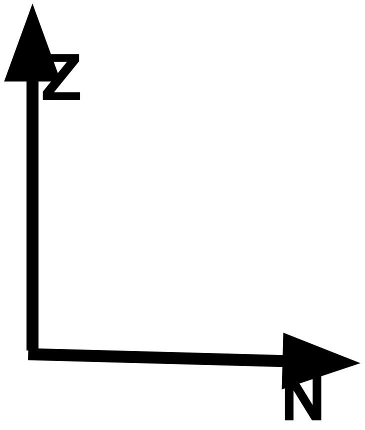
Big-Bang Nucleosynthesis
Big Dud (as far as PT)

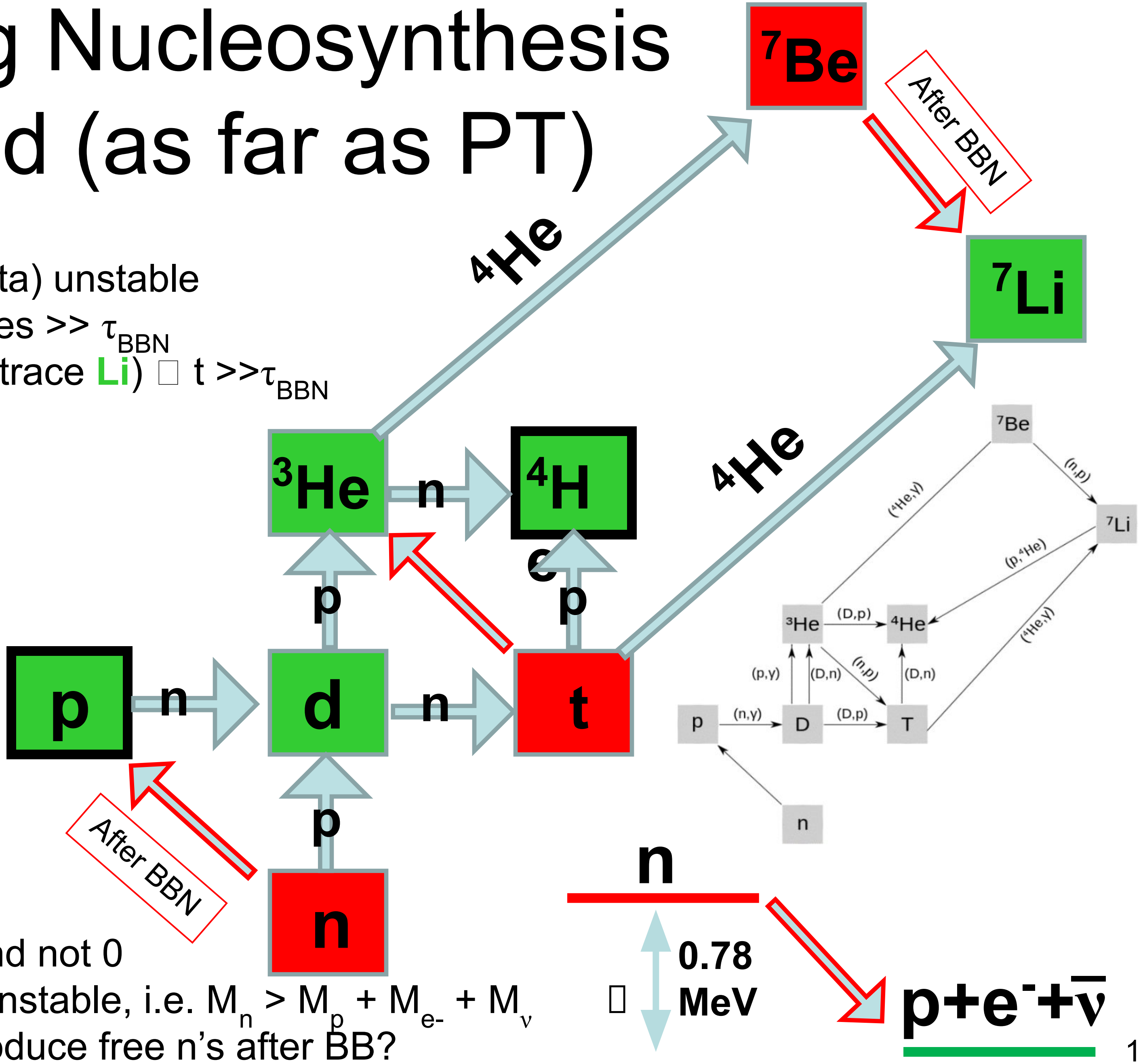
NOTE:

n, **t** and ^{7}**Be** are (beta) unstable

but have lifetimes >> τ_{BBN}

Only **p, d,** ^{3}**He,**^{4}**He** (trace **Li**) □ t >>τ_{BBN}





ALSO NOTE:

The PT starts at 1 and not 0

BECAUSE the n is unstable, i.e. $M_n > M_p + M_{e-} + M_\nu$

How does nature produce free n's after BB?

With only a few exceptions:
stars, either in life or death, produce the rest of PT

With CNO process □
allows for ^{13}C and ^{15}N NMR
With ^{13}C ….. □ neutrons via
$^{13}C + \alpha$ □ ^{17}O □ $^{16}O + n$
these are the n's for s-process

What our sun does (~ 85% truth)

$2\{\, p + p$ □ $d + e^{+} + \nu + Q_1 \}$

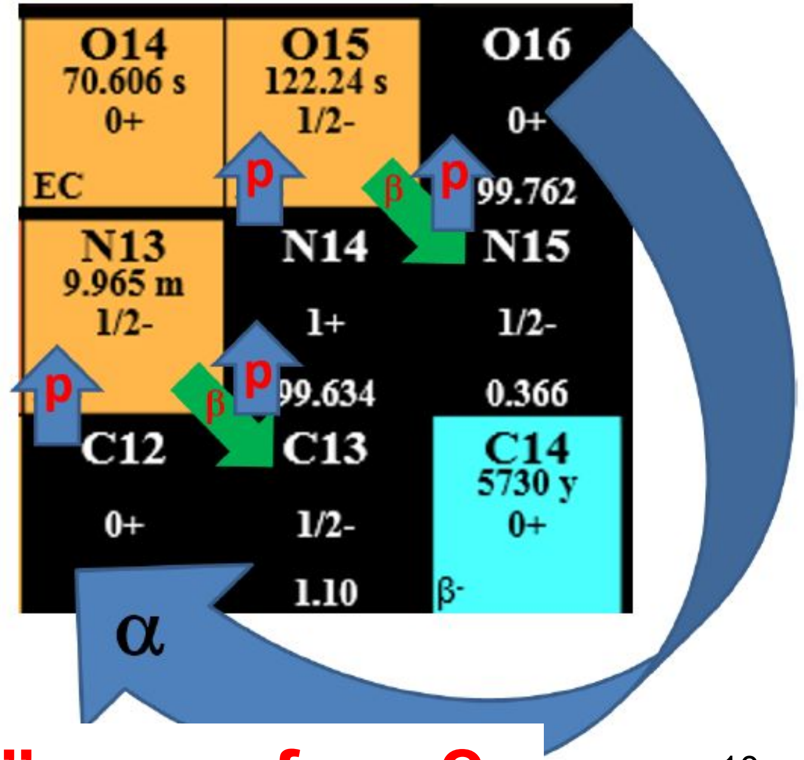
$2\{\, p + d$ □ $^{3}He + Q_2 \}$

$\underline{\{^{3}He + {}^{3}He}$ □ $\underline{^{4}He + 2p + Q_3 \}}$

$4p$ □ $^{4}He + 2e^{+} + 2\nu + Q_{tot}$

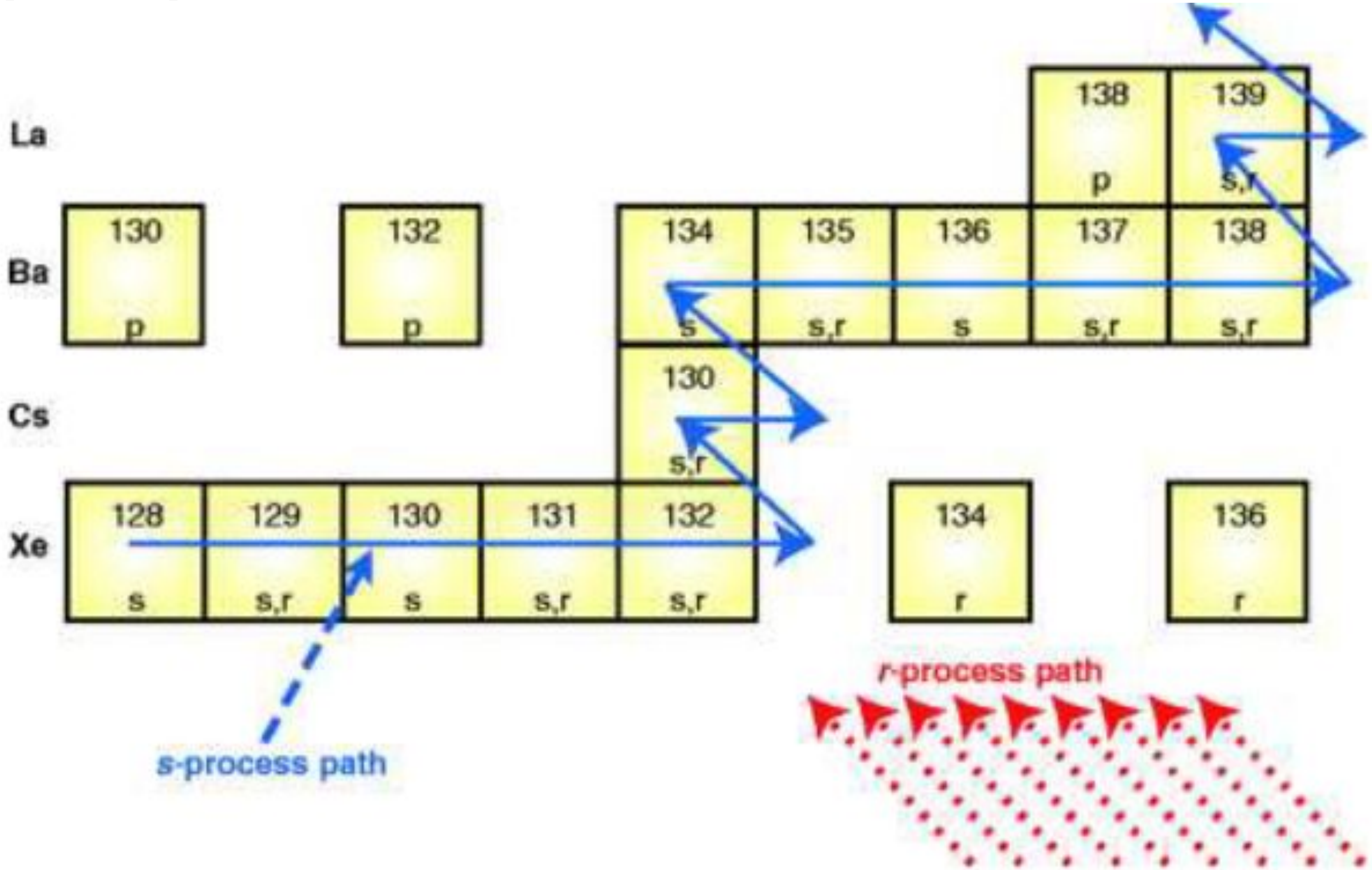
$Q_{tot} = 2Q_1 + 2Q_2 + Q_3 = 27.6$ MeV

To repair part of the lie ……
The sun also uses ^{12}C to catalyze
The "CNO process" that does
EXACTLY the same thing, i.e.
$4p$□$^{4}He + 2e^{+} + 2\nu + Q_{tot}$ **but ALSO**
Gives us ^{13}C & $^{14,15}N$



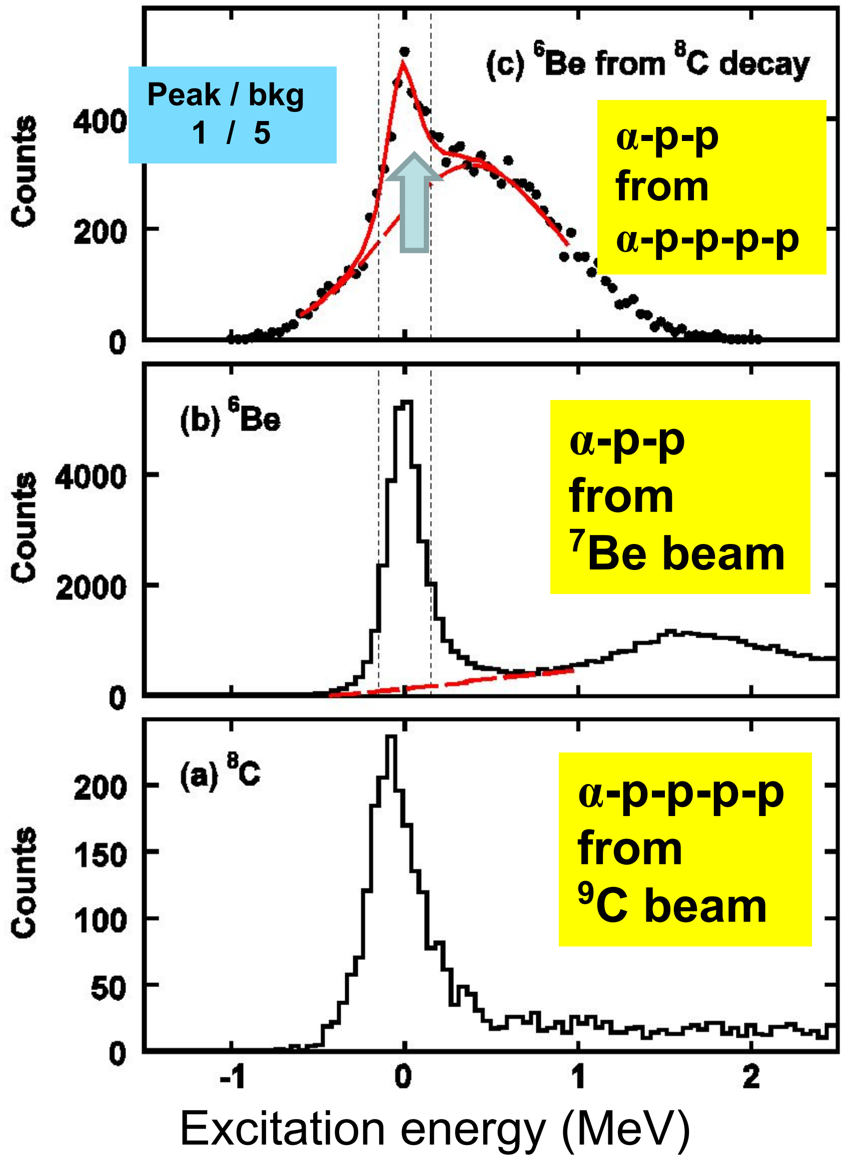
BUT where does the ^{12}C "seed" come from?

Nuclei heavier than Fe come (mostly) from slow and fast n-capture processes



BUT where do the n's (post BBN) for **s**(low) and **r**(apid) n-capture come from?

^{8}C decay

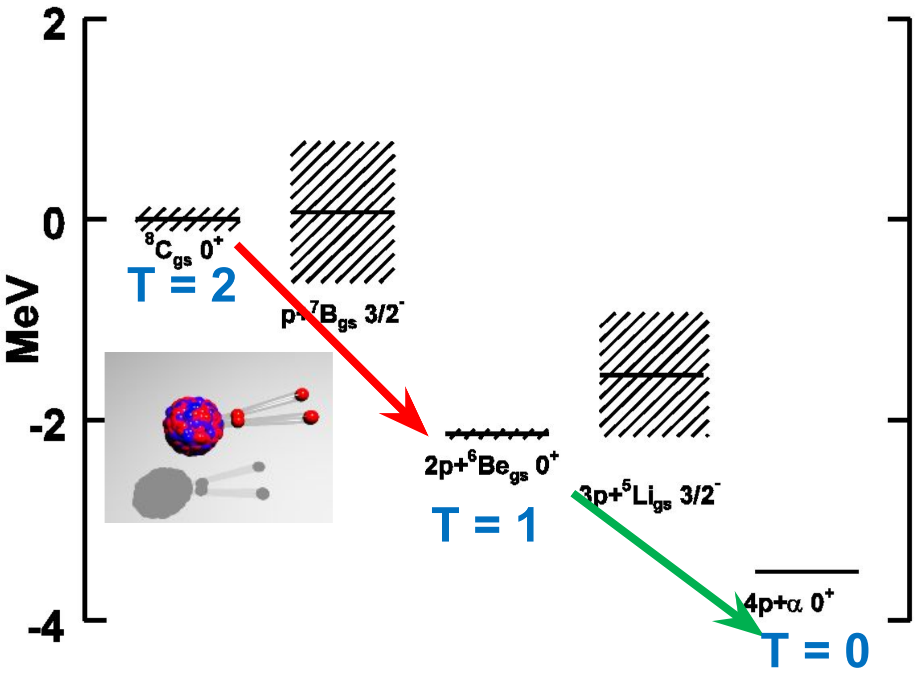


Peak / bkg
1 / 5

α-p-p
from
α-p-p-p-p

α-p-p
from
^{7}Be beam

α-p-p-p-p
from
^{9}C beam



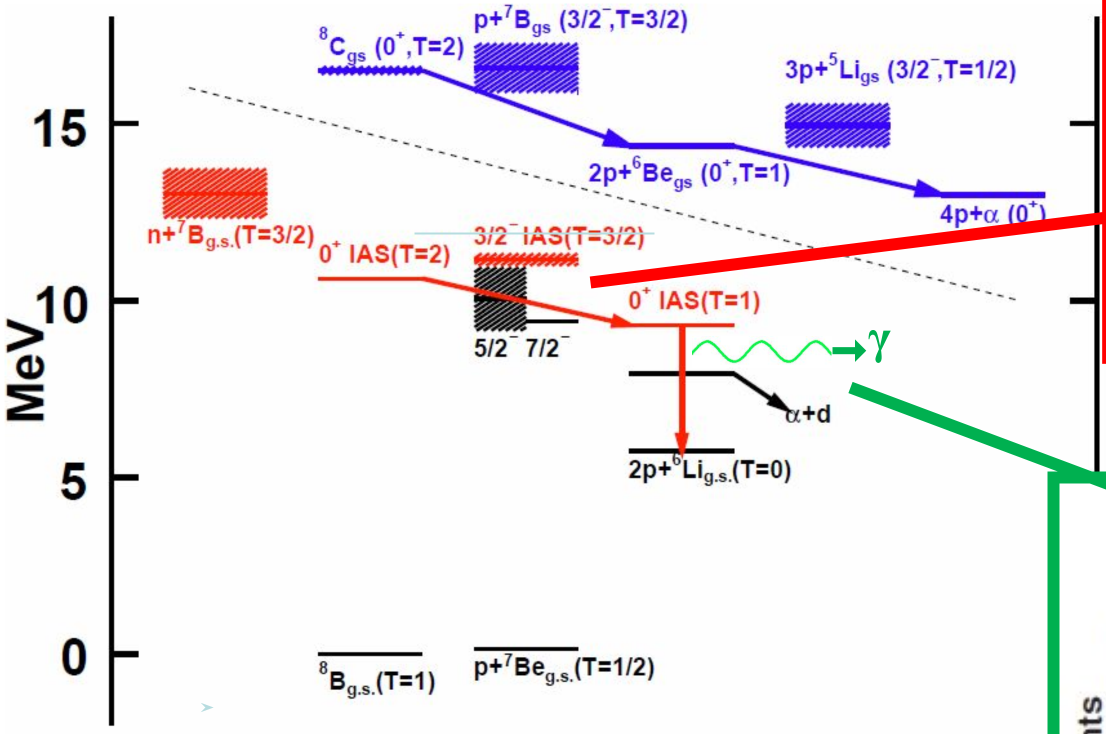
^{6}Be is the (7 zs) intermediate, i.e.
^{8}B □ [^{6}Be] + 2p + [α +2p] +2p

We studied the 3-body correlation for ^{6}Be decay AND the 3-body correlations for ^{8}C decay.

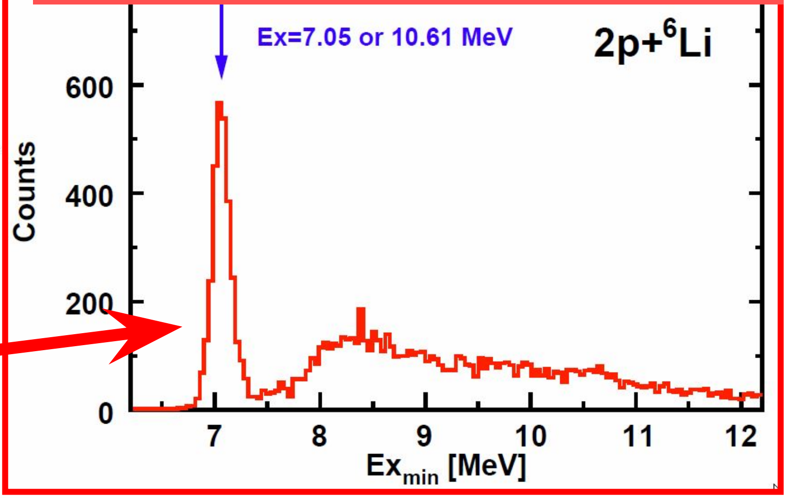
In ~ 1/3 of the events only ONE of the six combinations lies in the ^{6}Be peak. For these events we can assign protons to first and second steps.
□ enhancement at small rel. mom.

2p-2p decay and IAS□IAS 2p decay

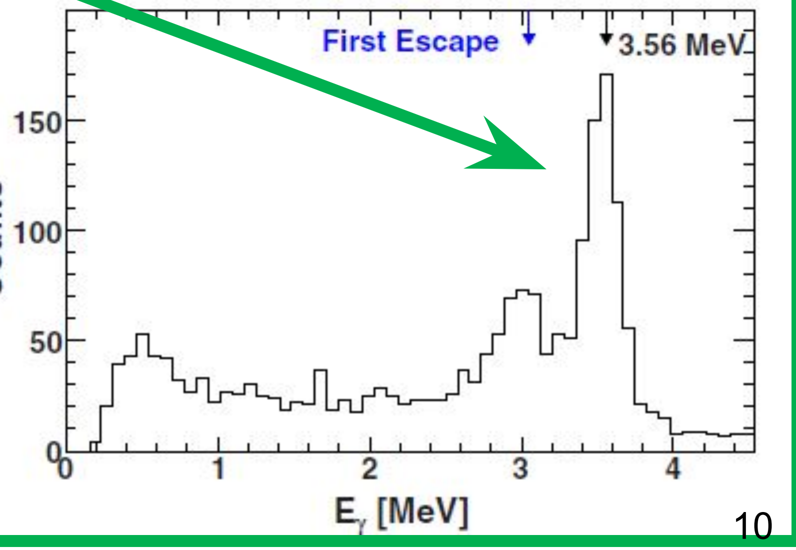
TOP ^{9}C □ ^{8}C$_{gd}$ (0+, T=2) +n
BOT ^{9}C □ ^{8}B$_{IAS}$ (0+, T=2) +p



^{8}B reconstruction from ^{6}Li+p+p

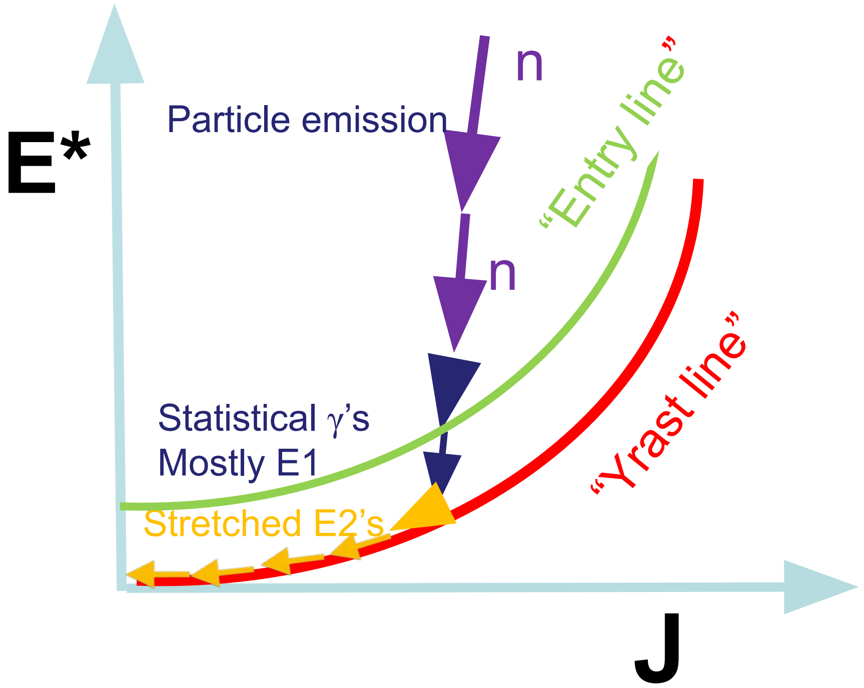


^{6}Li$_{IAS}$ □ Li$_{gs}$ + gamma



1p and 1n decays are forbidden by either energy or isospin

R. J. Charity, et al., Phys. Rev. C **82**, 041304(**R**) (2010).
K. Brown, et al., Phys. Rev. C **90**, 027304 (2014).



E*

J

Particle emission

n

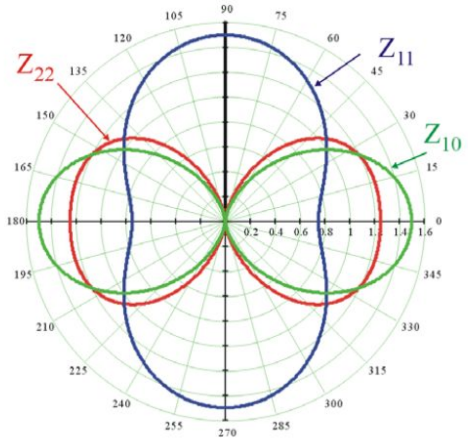
n

“Entry line”

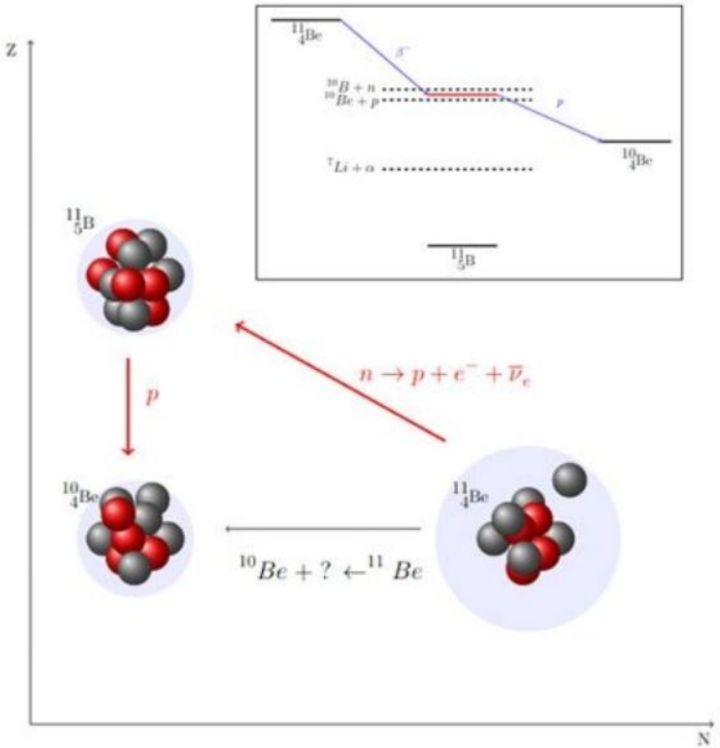
“Yrast line”

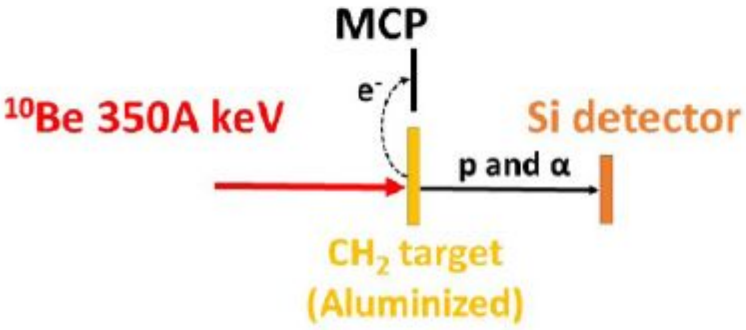
Statistical γ’s
Mostly E1

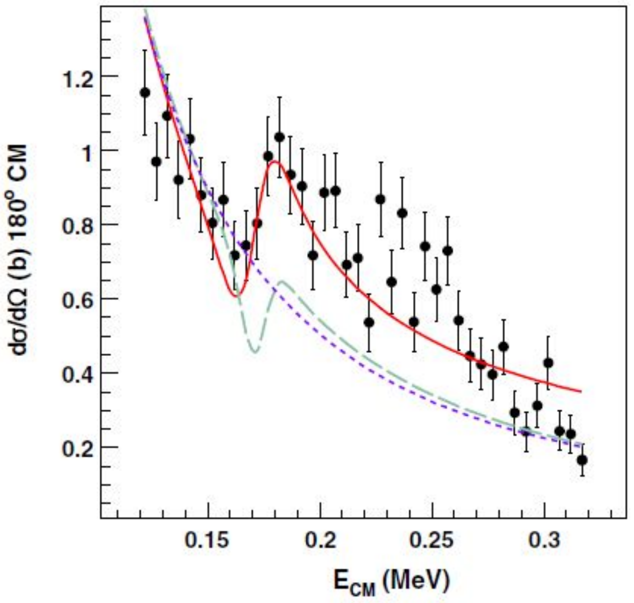
Stretched E2’s



$$Z_{22}(\theta) = \frac{5}{4}(1-\cos^4\theta),$$







PRL**123, 082501 (2019) & 124**, 129902E (2020)

^{10}Be(p,p’) & (d,n)^{11}B Paired PRLs
PRL**129, 012501 & 129**, 012502 (2022)

When Rabi heard of the muon, he asked… “Who ordered that?”
Well, I guess the same entity that ordered the curious beta delayed p emission of ^{11}Be
A resonance embedded in two continua (i.e. ^{10}Be + p and ^{7}Li + α)