

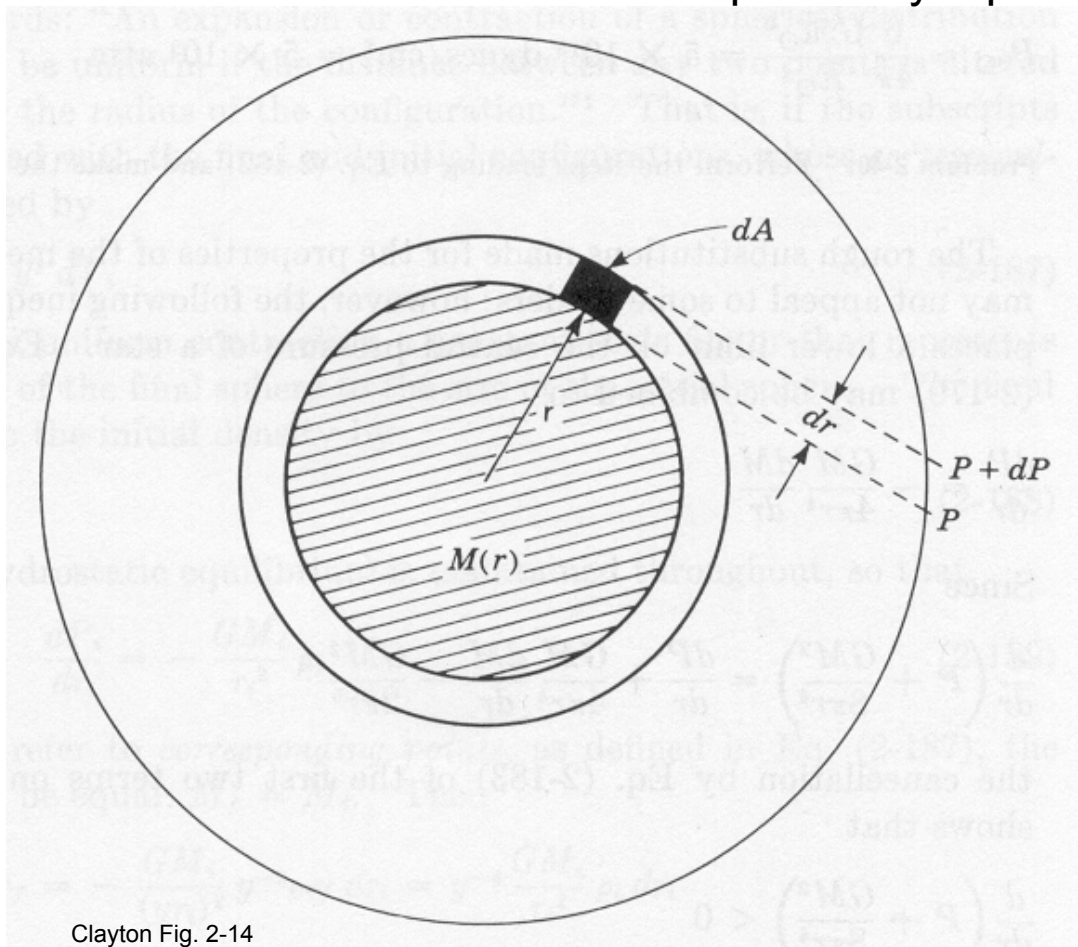
Properties of stars during hydrogen burning

Hydrogen burning is first major **hydrostatic burning phase** of a star:

Star is “stable” - radius and temperature everywhere do not change drastically with time

Hydrostatic equilibrium:

a fluid element is “held in place” by a pressure gradient that balances gravity



Clayton Fig. 2-14

Force from pressure:

$$F_p = PdA - (P + dP)dA \\ = -dPdA$$

Force from gravity:

$$F_G = -GM(r)\rho(r)dAdr / r^2$$

For balance: $F_G = F_P$ need:

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

The origin of pressure: equation of state

Under the simplest assumption of an ideal gas: $P = \rho N_A RT / \mu_I$

→ need high temperature !

Keeping the star hot:

The star cools at the surface - energy loss is luminosity L

To keep the temperature constant everywhere luminosity must be generated

In general, for the luminosity of a spherical shell at radius r in the star energy :
streaming outward (assuming steady state $dS/dt = 0$)

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho \varepsilon \quad (\text{energy equation})$$

where ε is the energy generation rate (sum of all energy sources and losses)
per g and s

Luminosity is generated in the center region of the star (L(r) rises) by nuclear reactions and then transported to the surface (L(r)=const)

Energy transport to the surface - cooling:

The star will settle into a hydrostatic and thermal equilibrium, where cooling is balanced by nuclear energy generation and there is no time dependence of any state variables.

The generated heat will then exactly match the outgoing energy flow (luminosity) at any point in the star.

Heat flows from hot to cold

→ temperature gradient is required to carry the luminosity outward:

Therefore $T(r)$ and $P(r)$ drop towards the surface → $\rho(r)$ also drops

Possible mechanisms of heat transport:

1. Conduction (not important at low densities in normal stars)
2. Radiative diffusion
3. Convection

Radiative energy transport:

Effectiveness depends on opacity κ :

unit cm^2/g – could call it specific cross section,
for example luminosity L in a layer r gets attenuated by photon absorption with
a cross section σ :

$$L = L_0 e^{-\sigma n r} = L_0 e^{-\kappa \rho r}$$

Photon mean free path l : $l = \frac{1}{\kappa \rho}$ (about 1cm in the sun)

Required temperature gradient:

$$\frac{dT}{dr} = - \frac{3\kappa\rho}{4acT^3} \frac{L(r)}{4\pi r^2}$$

Luminosity per cm^2

a: radiation density constant
= $7.56591 \times 10^{-15} \text{ erg/cm}^3/\text{K}^4$

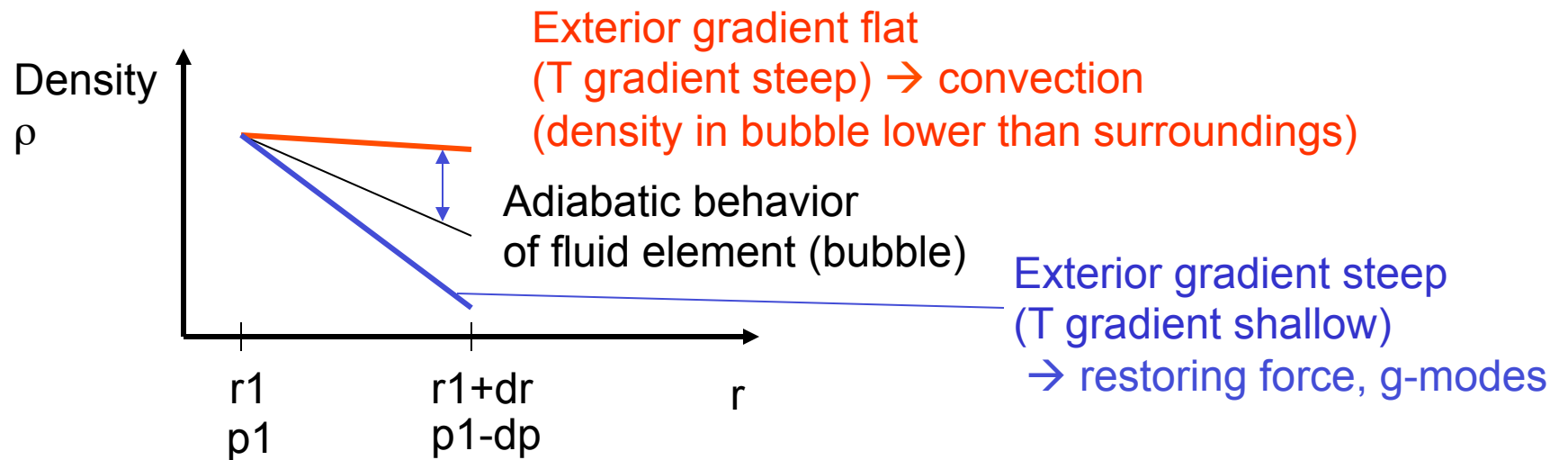
→ Large gradients needed for

- large luminosity at small r (large L/cm^2)
- large opacity

Convective energy transport:

takes over when necessary temperature gradient is too steep
hot gas moves up, cool gas moves down, within convective zone
fluid elements move adiabatically (adiabatic temperature gradient) driven by
temperature dependent bouyancy

Motivational consideration:

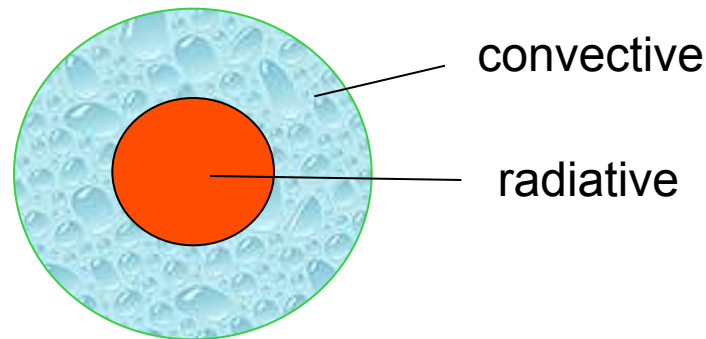


A more rapid drop in T over dr leads to a comparably lower T and higher density at same pressure p_1-dp (for example ideal gas) → flatter density gradient

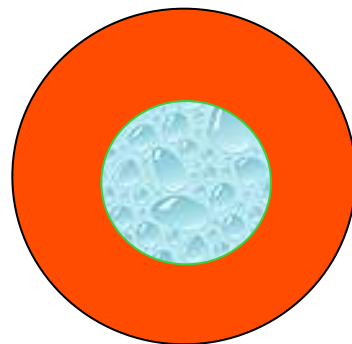
Convection occurs for flat density gradients, steep temperature gradients

Convection also mixes abundances → a convection zone has uniform composition (as long as convection timescale \ll nuclear reaction timescale)

Stars with $M < 1.2 M_{\odot}$ have radiative core and convective outer layer (like the sun):



Stars with $M > 1.2 M_{\odot}$ have convective core and radiative outer layer:



(convective core about 50% of mass for $15M_{\odot}$ star)

Mathematically one can show that the condition for convection is:

$$\left. \frac{P}{T} \frac{dT}{dP} \right|_{Star} > \left. \frac{P}{T} \frac{\partial T}{\partial P} \right|_{Adiabat}$$

(see Ed Brown's notes for stellar evolution course)

This is the Schwarzschild criterion

If there are compositional gradients a modification is the Ledoux criterion

Issues:

- Often implemented in simplified way in 1D models (mixing length theory)
- Uncertainty, for example with convective under and overshoots
- Multi-D simulations possible now locally for certain aspects
(e.g. MSU's Bob Stein for part of solar convective zone)
but not for entire stars
- Details of convective processes turn out to be FUNDAMENTAL to understand stars
 - s-process
 - i-process (boundary effects at end of convective zone)

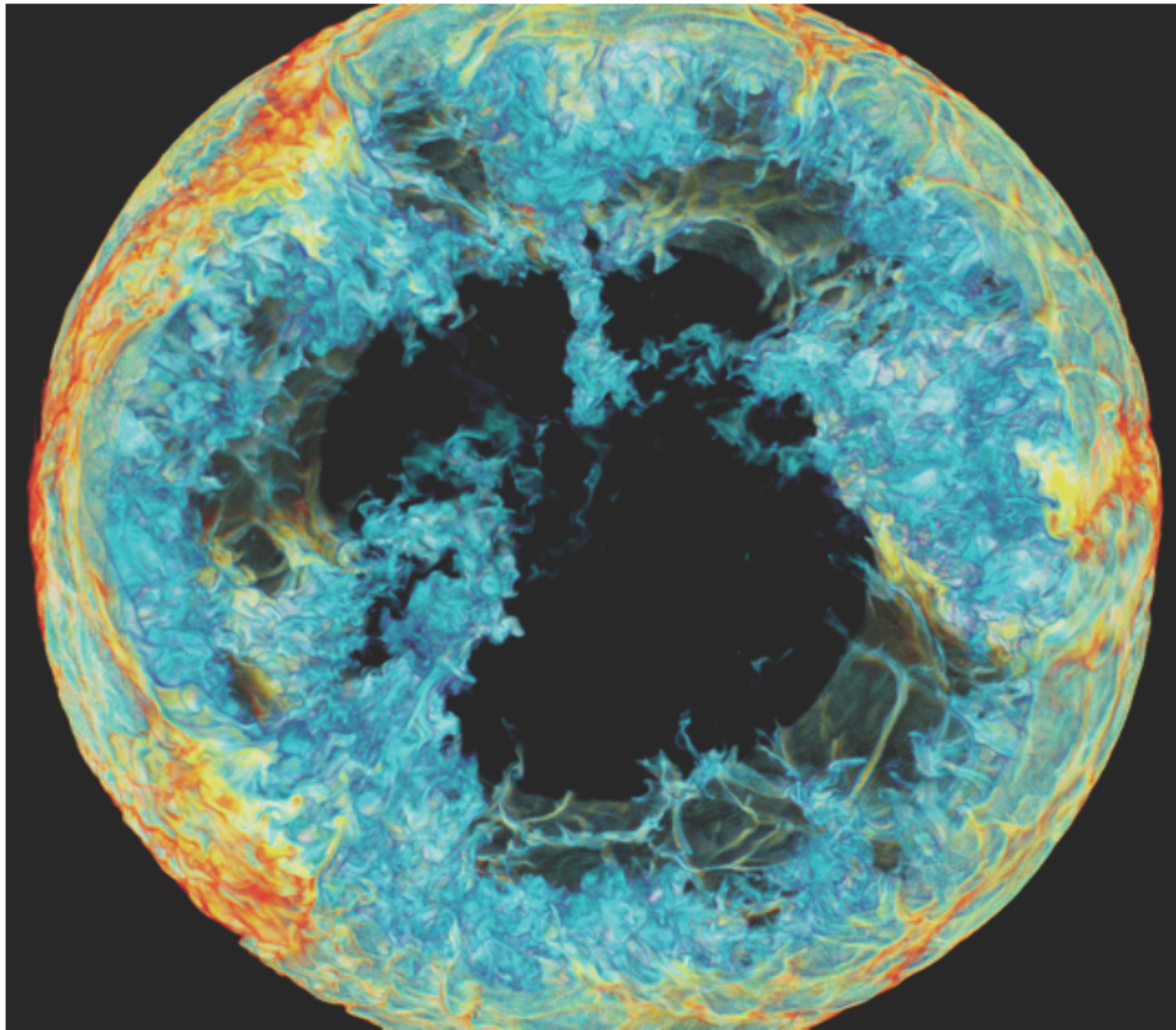


Figure 1: Visualization of entrainment processes at a convection boundary in a stellar hydrodynamics simulations. Shown is the concentration of H-rich material that is mixed from a stable layer (outside the spherical circumference, not seen) into an underlying Carbon-12 rich convection layer inside a star. The heterogeneous concentration distribution is the result of mixing processes that are critical for the production of the elements in stars. This run was performed on a 768^3 grid over 1.47 million time steps on the WestGrid cluster orcinus (April 2012), running on 2056 cores for a total of 500,000 CPU hours. Credit: Paul Woodward, LCSE – U Minnesota / Falk Herwig, U Victoria.

Stars

Multi-messenger Observations

GAIA

→ Luminosity

KEPLER

→ Seismology

SNO+

→ Neutrinos

Accelerator Facilities

DIANA

LENA, HIGS, StAna
LANSCE, FRANZ, nTOF
FRIB, CARIBU

Samples of stars

Stardust

Pre-solar grains

- How do stars mix, rotate, and generate magnetic fields?
- Which stars go supernova?
Structure before it explodes?
- What are the elements stars make?
As a function of metallicity?
- A new process? i-process
- What is the sun's metallicity?

Theory:

- 3D Modeling
- Nuclear cross section extrapolation

Big Theme:

- Validation

Woodward

The structure of a star in hydrostatic equilibrium:

Equations so far: $\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}$ (hydrostatic equilibrium)

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho \varepsilon \quad (\text{energy})$$

$$L(r) = -4\pi r^2 \frac{4acT^3}{3\kappa\rho} \frac{dT}{dr} \quad (\text{radiative energy transfer})$$

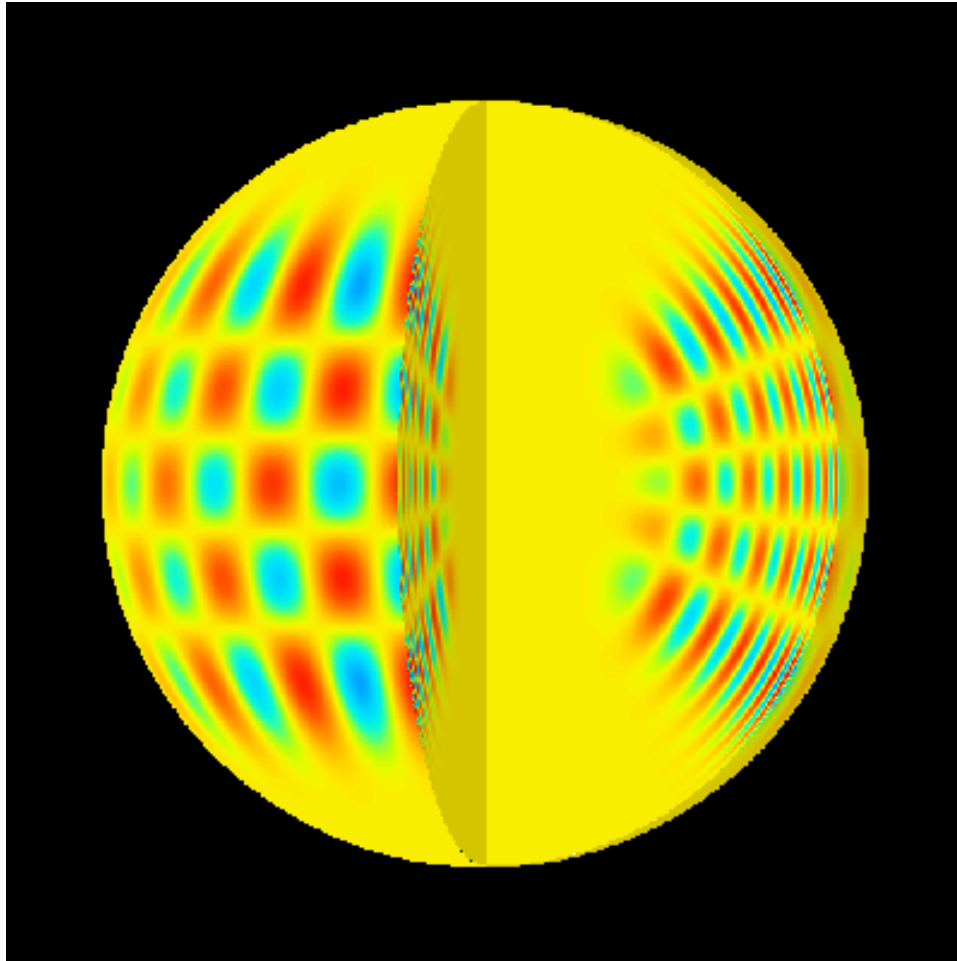
In addition of course: $\frac{dM(r)}{dr} = 4\pi\rho r^2$ (mass)

and an equation of state

BUT: solution not trivial, especially as ε, κ in general depend strongly on composition, temperature, and density

Example: The sun

But - thanks to **helioseismology** one does not have to rely on theoretical calculations, but can directly measure the internal structure of the sun



oscillations with periods
of 1-20 minutes

max 0.1 m/s

Conditions in the sun
(J. Bahcall, BS05
standard solar model)

