Resonant Reactions

The energy range that could be populated in the compound nucleus by capture of the incoming projectile by the target nucleus is for **"direct" reactions:**

- for neutron induced reactions:
  roughly given by the M.B. energy distribution of the incoming projectile
- for charged particle reactions:
  the Gamov window

If in this energy range there is an excited state (or part of it, as states have a width) in the Compound nucleus then the reaction rate will have a resonant contribution. (see Pg. 23/24)

If the center of the state is located in (or near) this energy range, then:

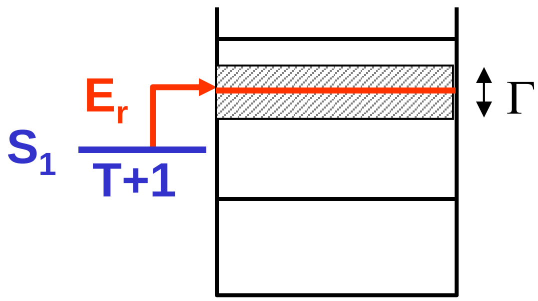
- The resonant contribution to the reaction rate tends to dominate by far
- The reaction rate becomes extremely sensitive to the properties of the resonant state

Reaction: 1 + T ⟶ C ⟶ F+2

With:
- Projectile 1
- Target nucleus T
- Compound nucleus C
- Final nucleus F
- Outgoing particle 2

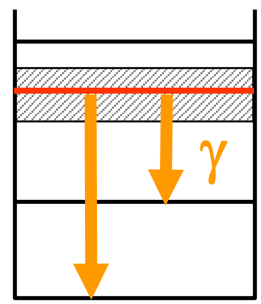
For capture 2 is a γ ray and F=C

Step 1



C

Step 2



C = F

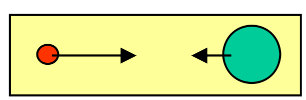
S_1: Particle 1 separation energy in C.

Excited states above S_1 are unbound and can decay by emission of particle 1 (in addition to other decay modes). Such states can serve as resonances

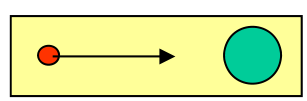
For capture, S_1 = Q-value

E_r: **Resonance energy**. Energy needed to populate the center of a resonance state

Reminder: Center of mass system



Laboratory system



$$E_{CM} = \frac{1}{2}\mu v^2 \qquad \mu = \frac{m_p m_T}{m_p + m_T}$$

$$E_{Lab} = \frac{1}{2} m_p v^2$$

The cross section contribution due to a single resonance is given by the Breit-Wigner formula:

$$\sigma(E) = \pi ƛ^2 \cdot \omega \cdot \frac{\Gamma_1 \Gamma_2}{(E - E_r)^2 + (\Gamma/2)^2}$$

Usual geometric factor

$$= \frac{656.6}{A}\frac{1}{E}\,\text{barn}$$

Spin factor:

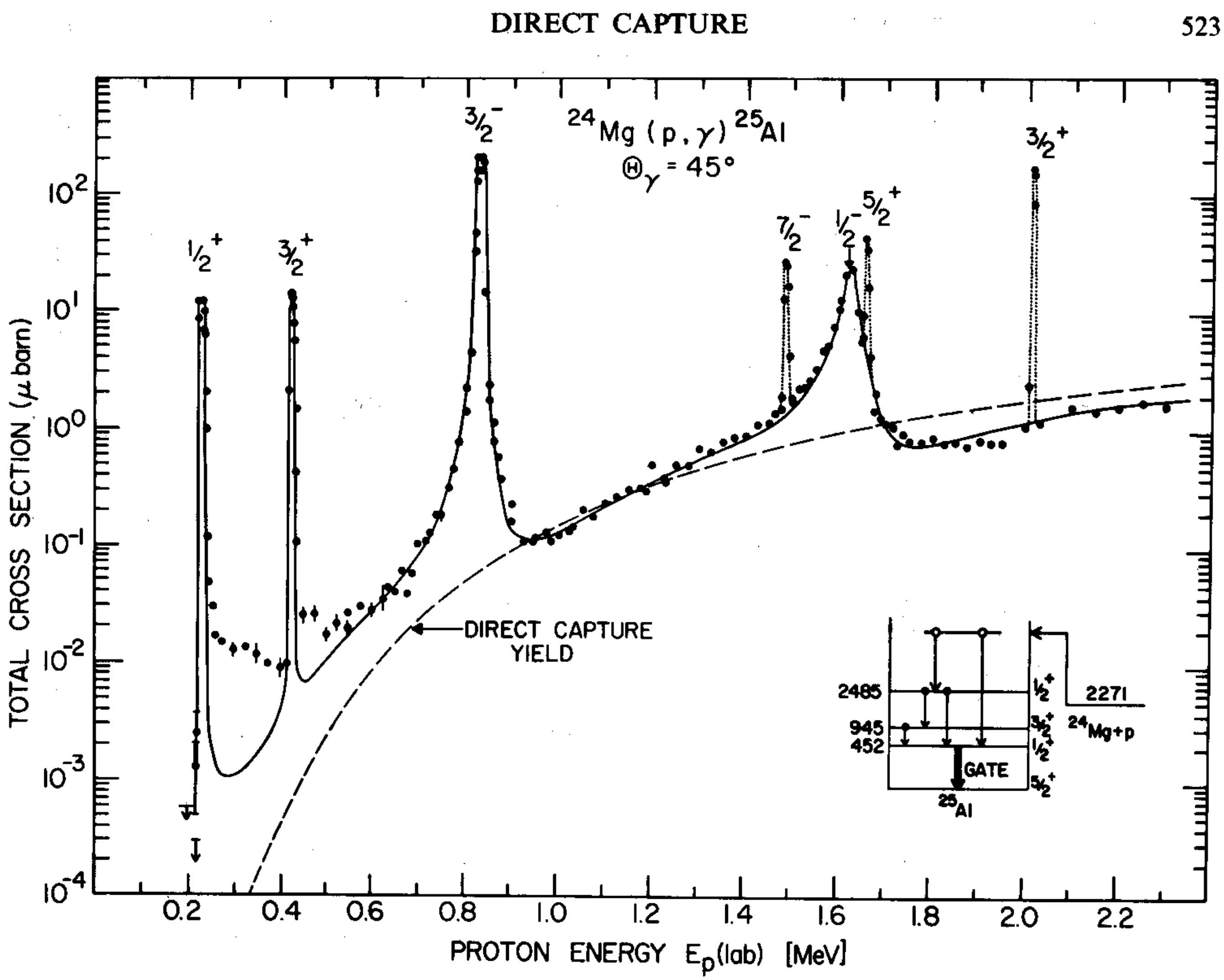
$$\omega = \frac{2J_r + 1}{(2J_1 + 1)(2J_2 + 1)}$$

$\propto \Gamma_1$ Partial width for decay of resonance by emission of particle 1 = Rate for formation of Compund nucleus state

$\propto \Gamma_2$ Partial width for decay of resonance by emission of particle 2 = Rate for decay of Compund nucleus into the right exit channel

Γ Total width is in the denominator as a large total width reduces the relative probabilities for formation and decay into specific channels.

Example:



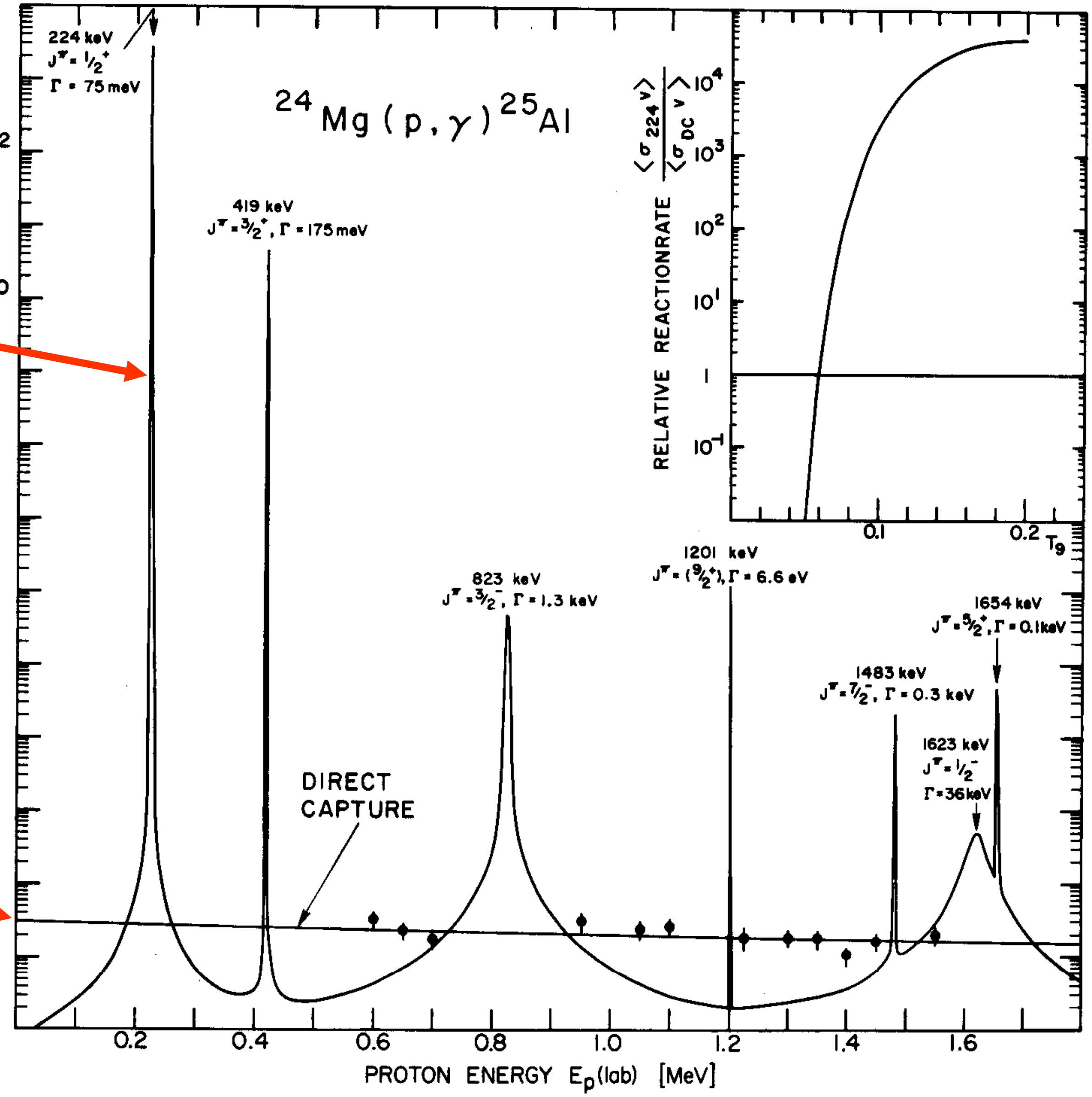
Resonance contributions are on top of direct capture cross sections

… and the corresponding S-factor

Note varying widths !

Not constant S-factor
for resonances
(log scale !!!!)

~ constant S-factor
for direct capture



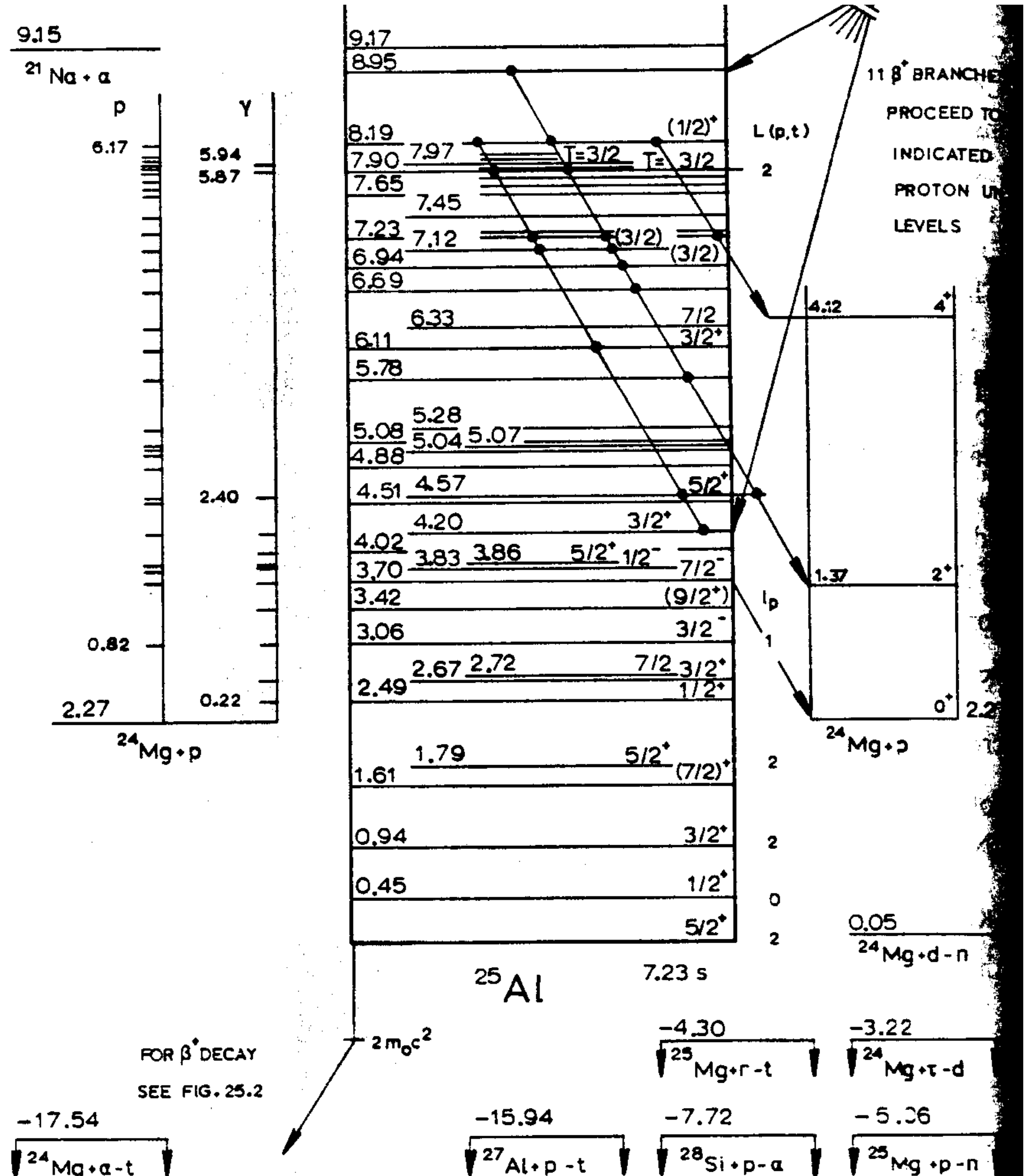


Fig. 25.4. Energy levels of ^{25}Al.

^{25}Al energy levels:

Each resonance corresponds to a level. **For example:**

E_r=3.06 MeV – 2.27 MeV
=790 keV in CM System !

In Lab system:

$E_{r\ LAB}$=25/24 * 0.790 MeV
= 0.823 MeV

So with 823 keV protons on a 24Mg target at rest one would hit the resonance (See Pg. 58)

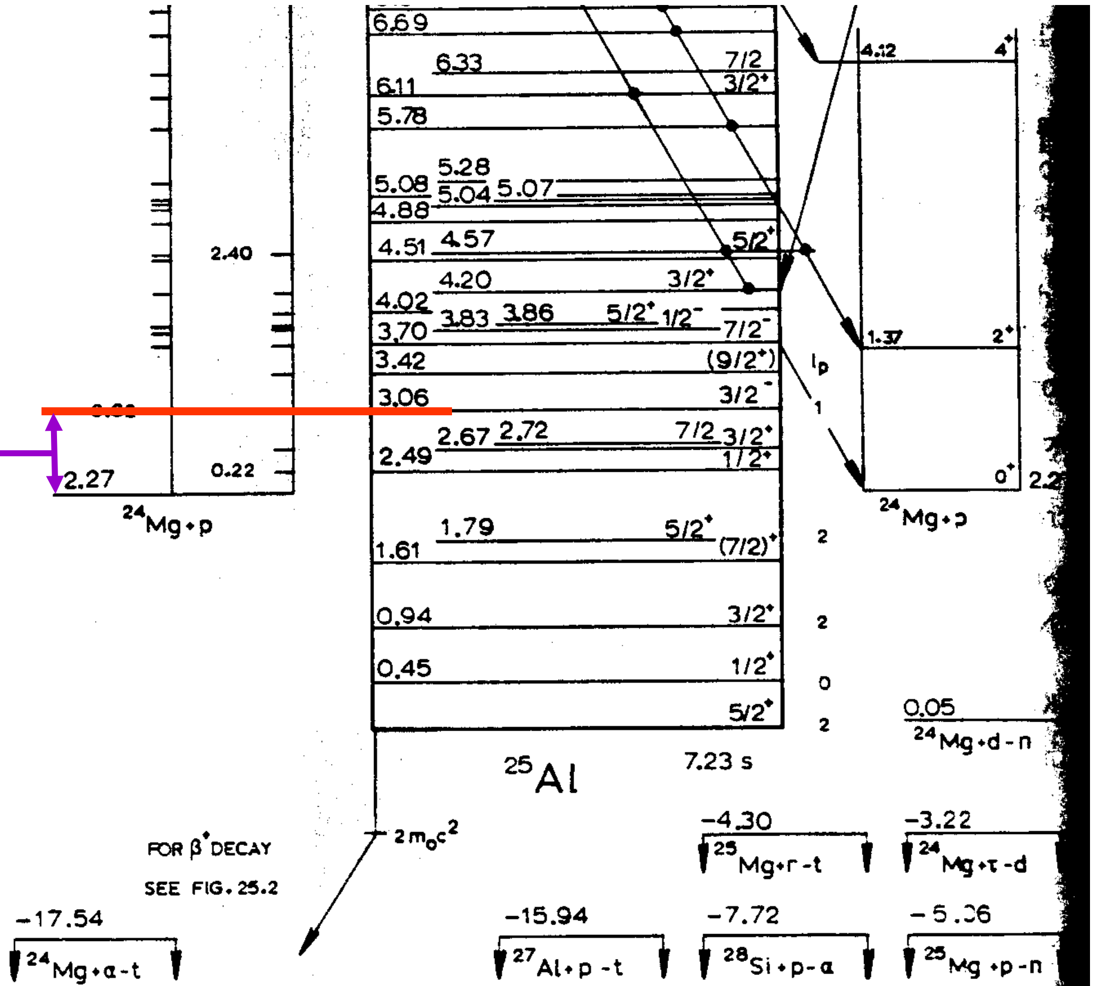


Fig. 25.4. Energy levels of ^{25}Al.

Angular momentum and Parity Conservation:

^{24}Mg: 0^+
p: $1/2^+$

So s-wave protons can populate $1/2^+$ resonances

p-wave protons can populate $1/2^-$, $3/2^-$ resonances

So the 823 keV resonance with $3/2^-$ is a p-wave resonance

Energy dependence of width's

Partial and total widths depend sensitively on the decay energy. Therefore:

- widths depend sensitively on the excitation energy of the state
- widths for a given state are a function of energy !
  (they are NOT constants in the Breit Wigner Formula)

Particle widths:

$$\Gamma_1 = 2P_l(E_1)\gamma^2$$

* - see note below

Main energy dependence (can be calculated) → $P_l(E_1)$

"reduced width" Contains the nuclear physics → γ^2

Photon widths:

$$\Gamma_\gamma = B(l)\, E_\gamma^{2l+1}$$

Reduced matrix element → $B(l)$

For particle capture: $E_1 = E_r$

$$E_\gamma = Q + E_r$$

For other cases: $E_1 = E_r$

$$E_2 = S_2 + E_r$$

Typically $E_r \ll Q$ and mostly also $E_r \ll S_2$ and therefore in many cases:

- $\Gamma_{\text{incoming particle}}$ has strong dependence on E_r (especially if it is a charged particle !)
- $\Gamma_{\text{outgoing particle}}$ has only weak dependence on E_r

So, for capture of particle 1, the main energy dependence of the cross section comes from λ^2 and Γ_1

Particle partial widths have the same (approximate) energy dependence than the "Penetrability" factor that we discussed in terms of the direct reaction mechanism.

Partial widths: For example theoretical calculations (Herndl et al. PRC52(95)1078)

TABLE V. Nonresonant direct capture transitions and the astrophysical S factors; resonance energies, γ widths, proton widths, and resonance strengths for $^{32}Cl(p,\gamma)^{33}Ar$.

$^{32}Cl(p,\gamma)^{33}Ar$	$Q = 3.34$ MeV				
E_x	J^π	ℓ_i	$n\ell_f$	C^2S_f	$S(E_0)$ (MeV b)
0.00	$\frac{1}{2}^+_1$	p	$2s_{1/2}$	0.080	7.00×10^{-3}
		p	$1d_{3/2}$	0.672	6.14×10^{-3}
1.34	$\frac{3}{2}^+_1$	p	$1d_{3/2}$	0.185	2.62×10^{-3}
1.79	$\frac{5}{2}^+_1$	p	$1d_{3/2}$	0.145	2.74×10^{-3}
2.47	$\frac{3}{2}^+_2$	p	$2s_{1/2}$	0.031	6.16×10^{-3}
		p	$1d_{3/2}$	0.167	1.67×10^{-3}
3.15	$\frac{3}{2}^+_3$	p	$2s_{1/2}$	0.068	1.46×10^{-2}
		p	$1d_{3/2}$	0.516	3.01×10^{-3}
E_x	E_p	J^π	Γ_γ (eV)	Γ_p (eV)	$\omega\gamma$ (eV)
3.43	0.09	$\frac{5}{2}^+_2$	1.77×10^{-2}	8.7×10^{-18}	8.7×10^{-18}
3.56	0.22	$\frac{7}{2}^+_2$	1.94×10^{-3}	1.13×10^{-9}	1.51×10^{-9}
3.97	0.63	$\frac{5}{2}^+_3$	1.54×10^{-2}	2.22×10^{-2}	9.09×10^{-3}
4.19	0.85	$\frac{1}{2}^+_2$	1.54×10^{-1}	46.74	5.12×10^{-2}
4.73	1.39	$\frac{3}{2}^+_4$	8.48×10^{-2}	100.3	5.65×10^{-2}

Direct

S_p=3.34 MeV

Res.

Weak changes in gamma width

Strong energy dependence of proton width

In principle one has to integrate over the Breit-Wigner formula (recall $\Gamma(\mathrm{E})$) to obtain the stellar reaction rate contribution from a resonance.

There are however 2 simplifying cases:

10.1. Rate of reaction through the wing of a broad resonance

Broad means: broader than the relevant energy window for the given temperature (Gamov window for charged particle rates)

In this case resonances outside the energy window for the reaction can contribute as well – in fact there is in principle a contribution from the wings of all resonances.

Assume Γ_2= const, Γ=const and use simplified

$$\sigma(E) = \pi \lambda\!\!\!^{-2} \Gamma_1(E) \omega \frac{\Gamma_2}{(E-E_r)^2 + (\Gamma/2)^2}$$

Example:

$^{12}C(p,\gamma)$

Proceeds mainly through tail of 0.46 MeV resonance

need cross section here !

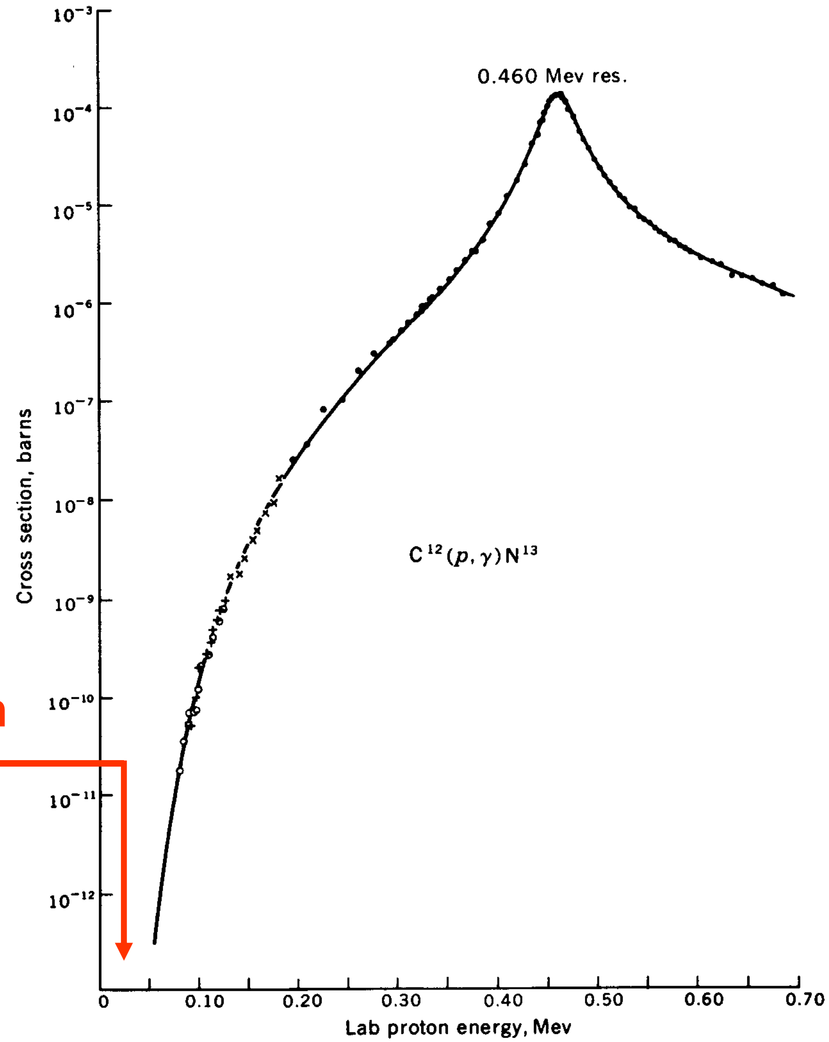
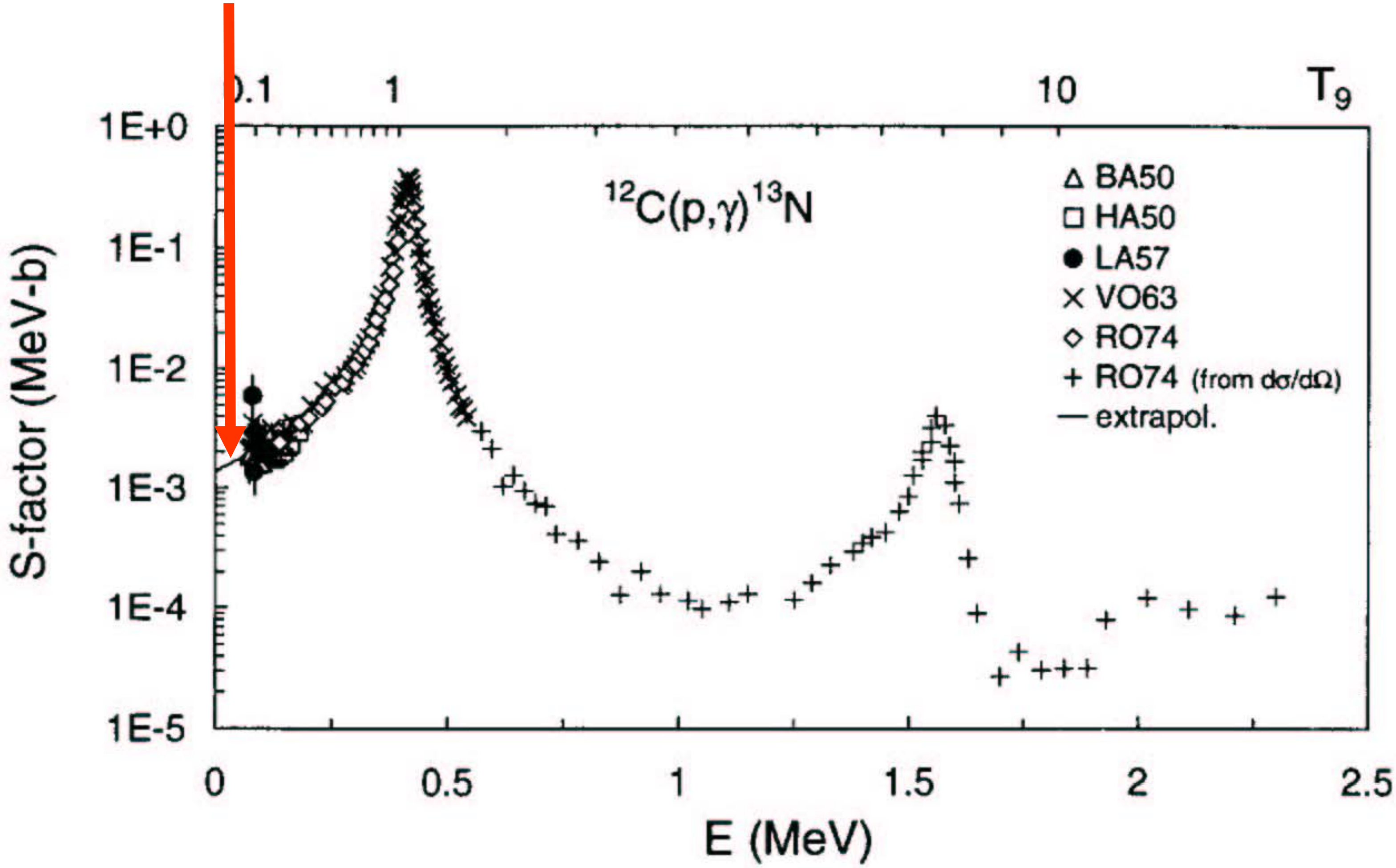


Fig. 4-4 The measured cross section for the reaction $C^{12}(p,\gamma)N^{13}$ as a function of laboratory proton energy. A four-parameter theoretical curve has been fitted to the experimental points. An extrapolation to $E_p = 0.025$ Mev, which is an interesting energy for this reaction in astrophysics, appears treacherous. *(Courtesy of W. A. Fowler and J. L. Vogl.)*

Need rate
about here



Note:

$$\sigma(E) = \pi\bar{\lambda}^2\Gamma_1(E)\frac{\Gamma_2}{(E-E_r)^2+(\Gamma/2)^2}$$

Same energy dependence than direct reaction

For E << E_r very weak energy dependence

Far from the resonance the contribution from wings has a similar energy dependence than the direct reaction mechanism.

In particular, for s-wave neutron capture there is often a 1/v contribution at thermal energies through the tails of higher lying s-wave resonances.

Therefore, resonant tail contributions and direct contributions to the reaction rate can be parametrized in the same way (for example S-factor)
Tails and DC are often mixed up in the literature.

Though they look the same, direct and resonant tail contributions are different things:

- in direct reactions, no compound nucleus forms
- resonance contributions can be determined from resonance properties measured at the resonance, far away from the relevant energy range (but need to consider interference !)

Rate of reaction through a narrow resonance

Narrow means: $\Gamma << \Delta E$

In this case, the resonance energy must be "near" the relevant energy range ΔE to contribute to the stellar reaction rate.

Recall:

$$<\sigma v> = \sqrt{\frac{8}{\pi\mu}} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) E \, e^{-\frac{E}{kT}} dE$$

and

$$\sigma(E) = \pi ƛ^2 \, \omega \frac{\Gamma_1(E)\Gamma_2(E)}{(E - E_r)^2 + (\Gamma(E)/2)^2}$$

For a **narrow** resonance assume:

M.B. distribution $\Phi(E) \propto E \, e^{-\frac{E}{kT}}$ constant over resonance $\Phi(E) \approx \Phi(E_r)$

All widths $\Gamma(E)$ constant over resonance $\Gamma_i(E) \approx \Gamma_i(E_r)$

$ƛ^2$ constant over resonance

Then one can carry out the integration analytically and finds:

For the contribution of a single narrow resonance to the stellar reaction rate:

$$N_A < \sigma v > = 1.54 \cdot 10^{11} (AT_9)^{-3/2} \omega\gamma \, [\text{MeV}] \, e^{\frac{-11.605 \, E_r [\text{MeV}]}{T_9}} \quad \frac{\text{cm}^3}{\text{s mole}}$$

III.68

The rate is entirely determined by the “resonance strength” $\omega\gamma$

$$\omega\gamma = \frac{2J_r + 1}{(2J_1 + 1)(2J_T + 1)} \frac{\Gamma_1 \Gamma_2}{\Gamma}$$

III.68a

Which in turn depends mainly on the total and partial widths of the resonance at resonance energies.

$$\omega\gamma = \frac{2J_r + 1}{(2J_1 + 1)(2J_T + 1)} \frac{\Gamma_1 \Gamma_2}{\Gamma}$$ III.68a

Often $\Gamma = \Gamma_1 + \Gamma_2$ Then for $\Gamma_1 << \Gamma_2 \longrightarrow \Gamma \approx \Gamma_2 \longrightarrow \frac{\Gamma_1 \Gamma_2}{\Gamma} \approx \Gamma_1$

$\Gamma_2 << \Gamma_1 \longrightarrow \Gamma \approx \Gamma_1 \longrightarrow \frac{\Gamma_1 \Gamma_2}{\Gamma} \approx \Gamma_2$

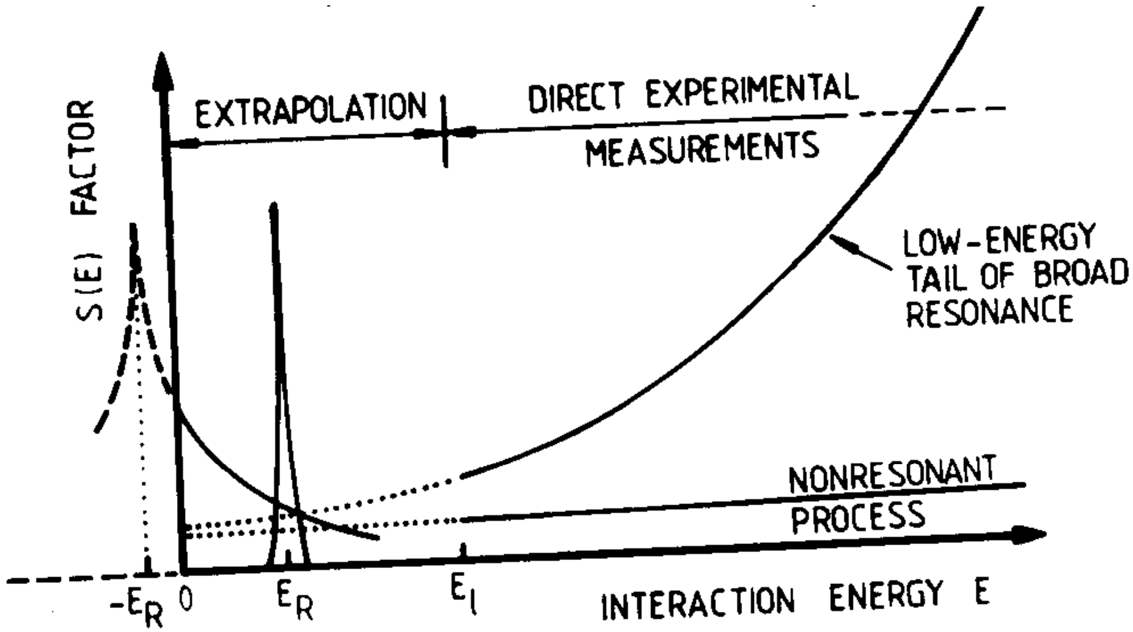
And reaction rate is determined by the smaller one of the widths !

Summary:

The stellar reaction rate of a nuclear reaction is determined by the sum of

- sum of direct transitions to the various bound states
- sum of all narrow resonances in the relevant energy window
- tail contribution from higher lying resonances

Or as equation: $$< \sigma v > = \sum < \sigma v >_{\mathrm{DC->state\ i}} + \sum < \sigma v >_{\mathrm{Res;\ i}} + < \sigma v >_{\mathrm{tails}}$$



(Rolfs & Rodney)

Caution: Interference effects are possible (constructive or destructive addition) among

- Overlapping resonances with same quantum numbers
- Same wave direct capture and resonances

Again as example: (Herndl et al. PRC52(95)1078)

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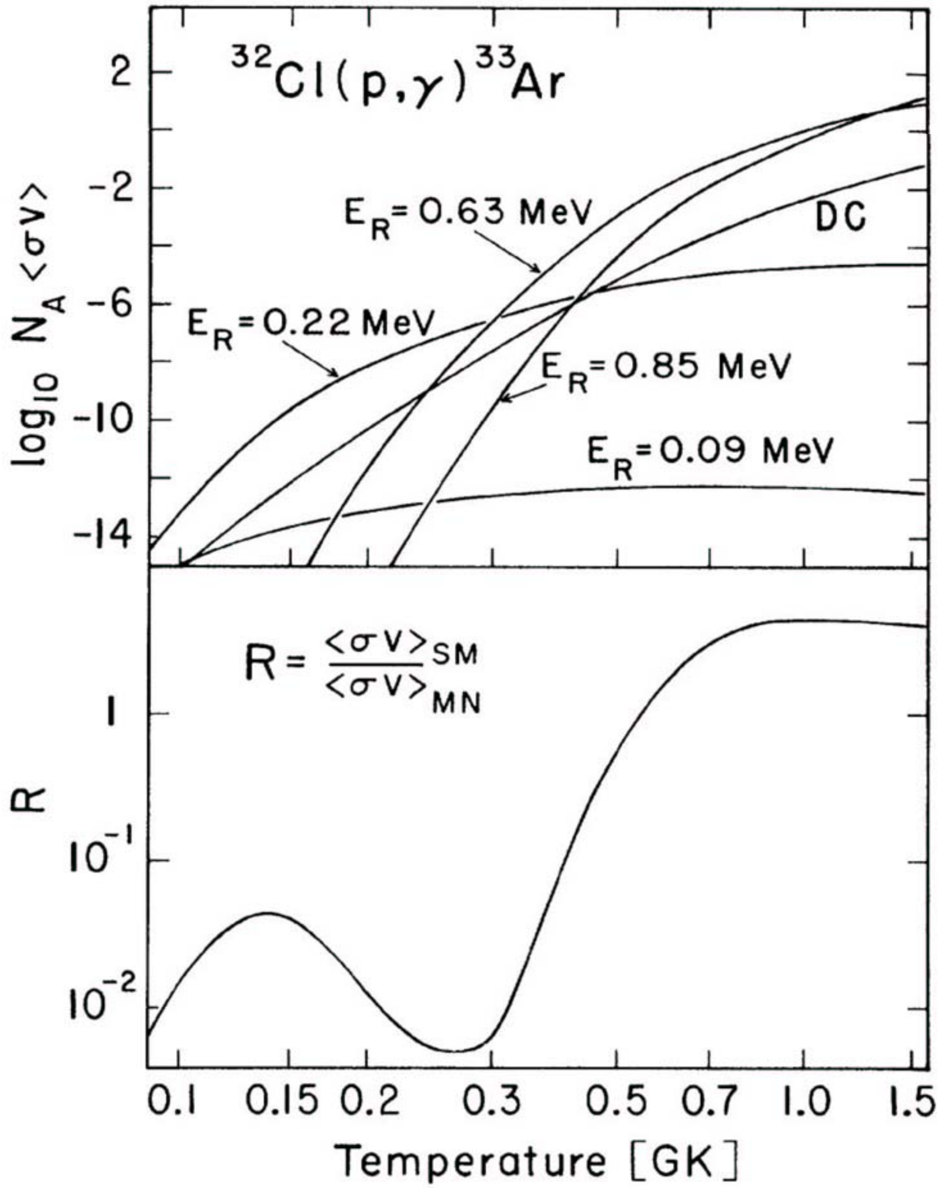
Direct

S_p=3.34 MeV

Res.

Resonance strengths



Gamov Window:

0.1 GK: 130-220 keV

0.5 GK: 330-670 keV

1 GK: 500-1100 keV

But note: Gamov window has been defined for direct reaction energy dependence !

The Gamow window moves to higher energies with increasing temperature – therefore different resonances play a role at different temperatures.

Some other remarks:

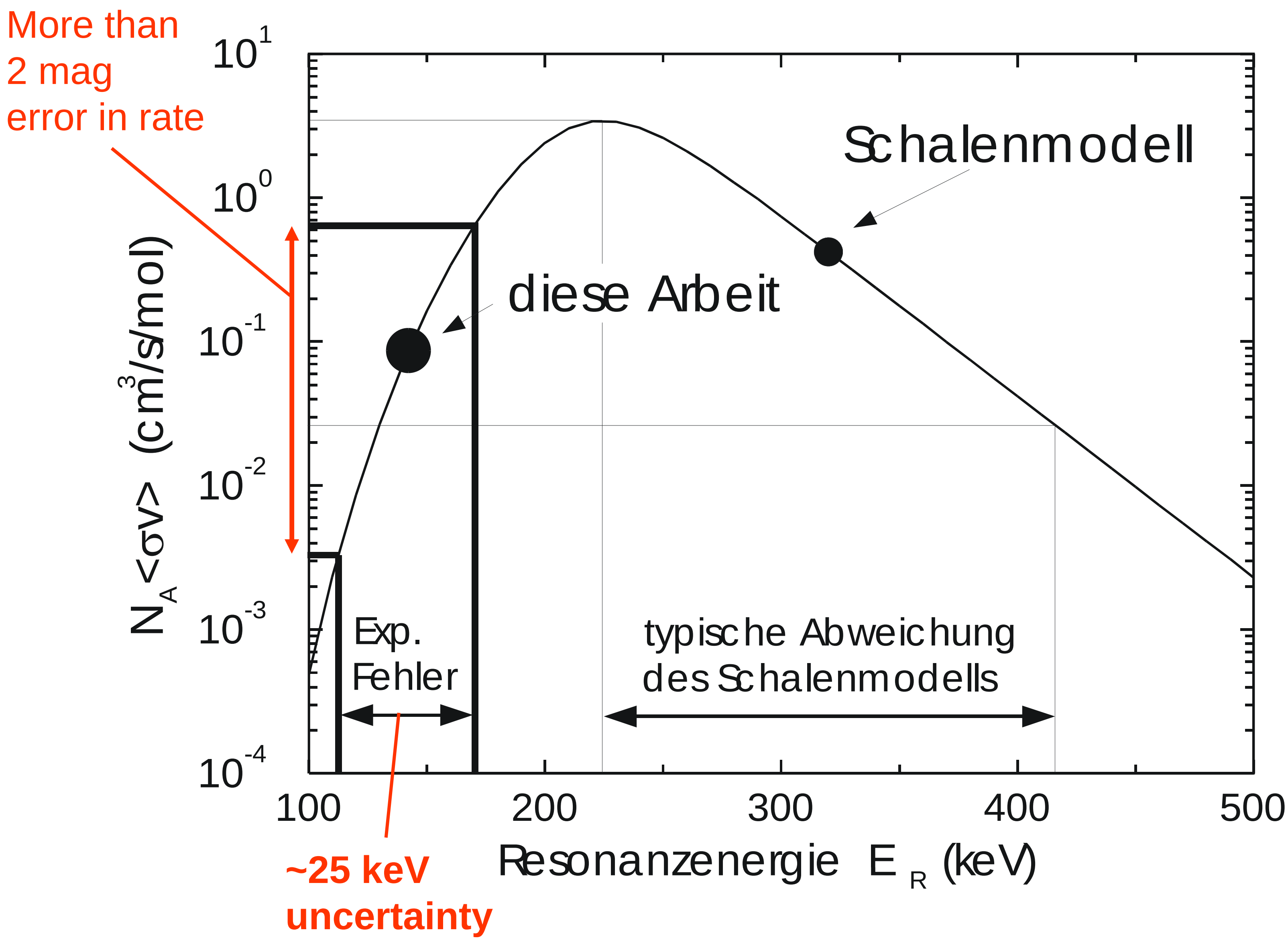
- If a resonance is in or near the Gamov window it tends to dominate the reaction rate by orders of magnitude

- As the level density increases with excitation energy in nuclei, higher temperature rates tend to be dominated by resonances, lower temperature rates by direct reactions.

- As can be seen from Eq. III.68, the reaction rate is extremely sensitive to the resonance energy. For p-capture this is due to the exp(E_r/kT) term **AND** Γ_p(E) (Penetrability) !

As E_r=E_x-Q one needs accurate excitation energies **and** masses !

Example: only relevant resonance in $^{23}Al(p,g)^{24}Si$



(for a temperature of 0.4 GK and a density of 10^4 g/cm^3)

Complications in stellar environment

Beyond temperature and density, there are additional effects related to the extreme stellar environments that affect reaction rates.

In particular, experimental laboratory reaction rates need a (theoretical) correction to obtain the stellar reaction rates.

The most important two effects are:

1. Thermally excited target

At the high stellar temperatures photons can excite the target. Reactions on excited target nuclei can have different angular momentum and parity selection rules and have a somewhat different Q-value.

2. Electron screening

Atoms are fully ionized in a stellar environment, but the electron gas still shields the nucleus and affects the effective Coulomb barrier.

Reactions measured in the laboratory are also screened by the atomic electrons, but the screening effect is different.

11.1. Thermally excited target nuclei

Ratio of nuclei in a thermally populated excited state to nuclei in the ground state is given by the Saha Equation:

$$\frac{n_{\mathrm{ex}}}{n_{\mathrm{gs}}} = \frac{g_{\mathrm{ex}}}{g_{\mathrm{gs}}} e^{-\frac{E_x}{kT}} \qquad\qquad g = (2J+1)$$

Ratios of order 1 for E_x~kT

In nuclear astrophysics, kT=1-100 keV, which is small compared to typical level spacing in nuclei at low energies (~ MeV).

-> usually only a very small correction, but can play a role in select cases if:

- a low lying (~100 keV) excited state exists in the target nucleus
- temperatures are high
- the populated state has a very different rate
  (for example due to very different angular momentum or parity
  or if the reaction is close to threshold and the slight increase in
  Q-value 'tips the scale' to open up a new reaction channel)

The correction for this effect has to be calculated. NACRE, for example, gives a correction.

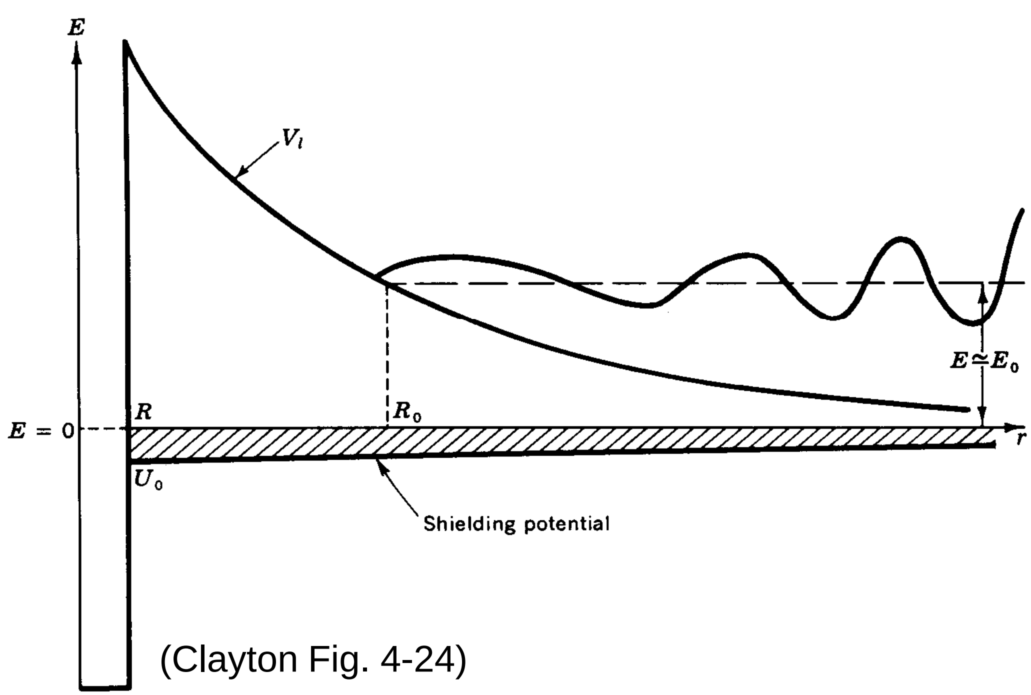
11.2. Electron screening

The nuclei in an astrophysical plasma undergoing nuclear reactions are fully ionized.

However, they are immersed in a dense electron gas, which leads to some shielding of the Coulomb repulsion between projectile and target for charged particle reactions.

Charged particle reaction rates are therefore enhanced in a stellar plasma, compared to reaction rates for bare nuclei.

The Enhancement depends on the stellar conditions



(Clayton Fig. 4-24)

$$V(r) = \frac{Z_1 Z_2 e^2}{r} + U(r)$$

Bare nucleus Coulomb

Extra Screening potential

(attractive so <0)

In general define screening factor f:

$$< \sigma v >_{\text{screened}} = f < \sigma v >_{\text{bare}}$$

11.2.1. Case 1: Weak Screening

Definition of weak screening regime:

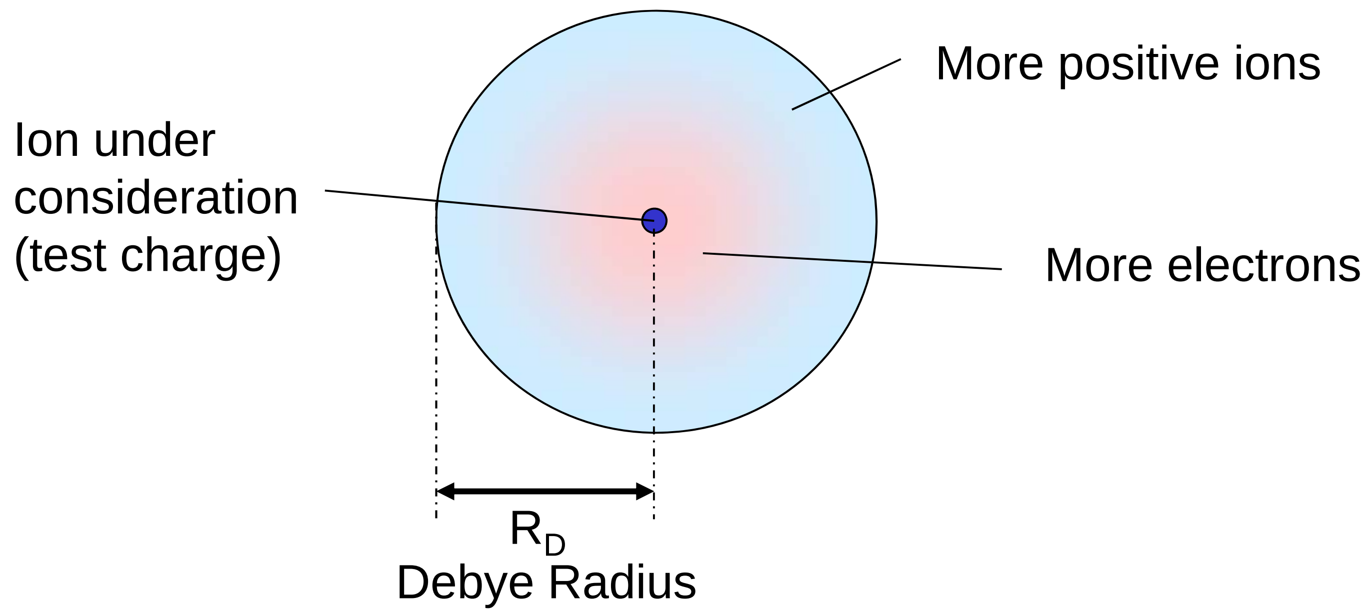
Average Coulomb energy between ions << thermal Energy

$$\frac{e^2 Z^2}{n^{-1/3}} << kT$$ (for a single dominating species)

Means:
- high temperature
- low density

(typical for example for stellar hydrogen burning)

For weak screening, each ion is surrounded by a sphere of ions and electrons that are somewhat polarized by the charge of the ion (Debeye Huckel treatment)



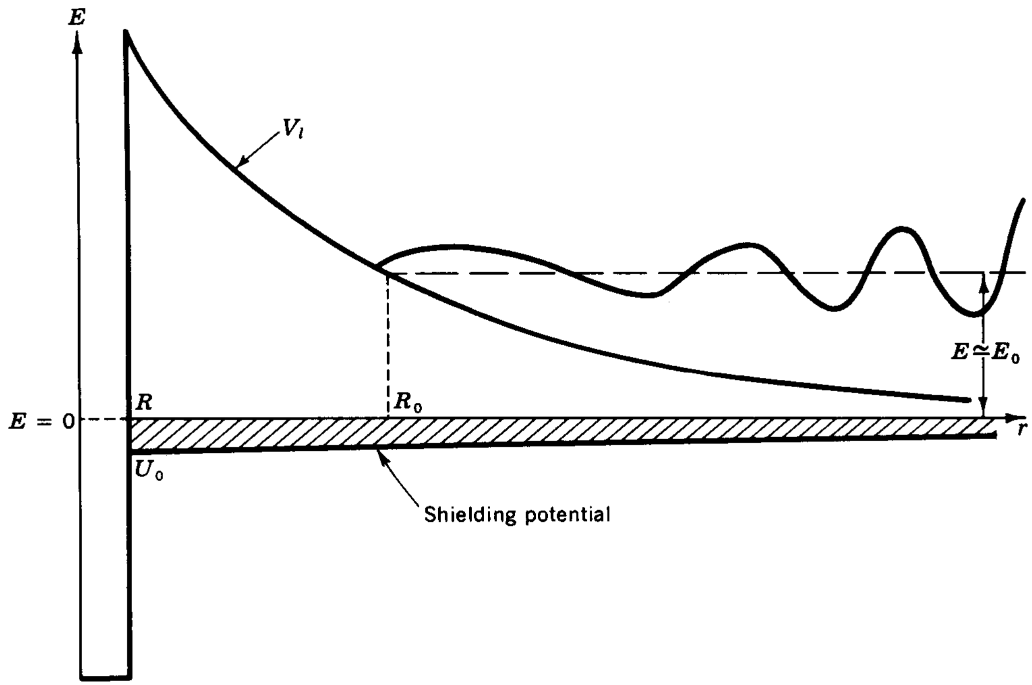
Exp: Quicker drop off due to screening

Then potential around ion $$V_1(r) = \frac{eZ}{r} \mathrm{e}^{-\mathrm{r}/\mathrm{R_D}}$$

With $$R_D = \sqrt{\frac{kT}{4\pi e^2 \rho N_A \xi^2}} \qquad \xi = \sqrt{\sum_i (Z_i^2 + Z_i \Theta_e) Y_i}$$

So for $r \gg R_D$ complete screening

But effect on barrier penetration and reaction rate only for potential between R and classical turning point R_0



In weak screening regime, $R_D >> (R_0\text{-}R)$

And therefore one can assume U(r) ~ const ~ U(0).

In other words, we can expand V(r) around r=0:

$$V_1(r) = \underbrace{\frac{eZ_1}{r}\left(1 - \frac{r}{R_D}\right.}_{\text{To first order}}\left.+ \frac{r^2}{2R_D^2} - \ldots\right)$$

So to first order, barrier for incoming projectile

$$V(r) = eZ_2 V_1(r) = \frac{e^2 Z_1 Z_2}{r} - \frac{e^2 Z_1 Z_2}{R_D}$$

Comparison with

$$V(r) = \frac{Z_1 Z_2 e^2}{r} + U(r) \qquad \text{III.80}$$

Yields for the screening potential:

$$U(r) = U(0) = U_0 = -\frac{e^2 Z_1 Z_2}{R_D} \qquad \text{III.80a}$$

Equations III.80 and III.80.a describe a corrected Coulomb barrier for the astrophysical environment.

One can show, that the impact of the correction on the barrier penetrability and therefore on the astrophysical reaction rate can be approximated through a Screening factor f:

$$f = \mathrm{e}^{-\mathrm{U}_0/kT}$$

In weak screening U_0 << kT and therefore

$$f \approx 1 - \frac{U_0}{kT} \qquad U_0 = -\frac{e^2 Z_1 Z_2}{R_D}$$

Summary weak screening:

$$<\sigma v>_{\text{screened}} = f <\sigma v>_{\text{bare}}$$

$$f = 1 + 0.188\, Z_1 Z_2 \rho^{1/2} \xi T_6^{-3/2}$$

$$\xi = \sqrt{\sum_i (Z_i^2 + Z_i \Theta_e) Y_i}$$

11.2.2. Other cases:

Strong screening:

Average coulomb energy larger than kT – for high densities and low temperatures

Again simple formalism available, for example in Clayton

Intermediate screening:

Average Coulomb energy comparable to kT – more complicated but formalisms available in literature

11.2.3. Screening in Laboratory Experiments:

If one measures reaction rates in the laboratory, using atomic targets (always), then atomic electrons screen as well.

In the laboratory one measures screened reaction rates. BUT the screening is different from the screening in the stellar plasma.

- In the star it depends on temperature, density and composition
- In the lab it depends on the material (and temperature ?)

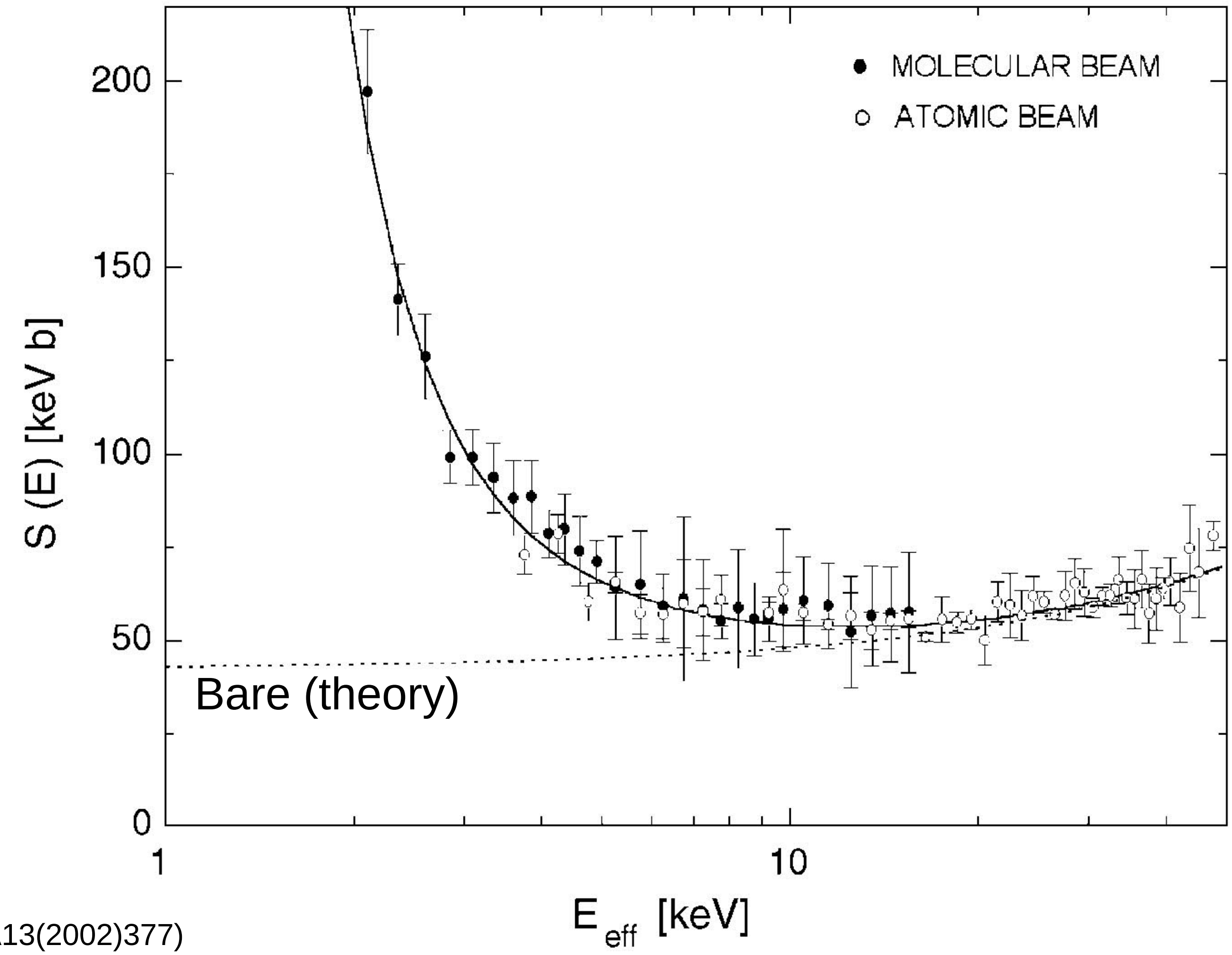
Measured reaction rates need to be corrected to obtain bare reaction rates. These are employed in stellar models that then include the formalism to calculate the screening correction in the astrophysical plasma.

In the laboratory, screening is described with screening potential U_e:

$$\frac{\sigma_{\text{screened}}}{\sigma_{\text{bare}}} = \frac{E}{E+U_e} e^{\frac{\pi\eta U_e}{E}} \qquad \eta = \frac{Z_1 Z_2 e^2}{h\nu}$$

Example:

d(d,p)t with
d-implanted
Ta target



(F. Raiola et al. Eur.Phys.J. A13(2002)377)