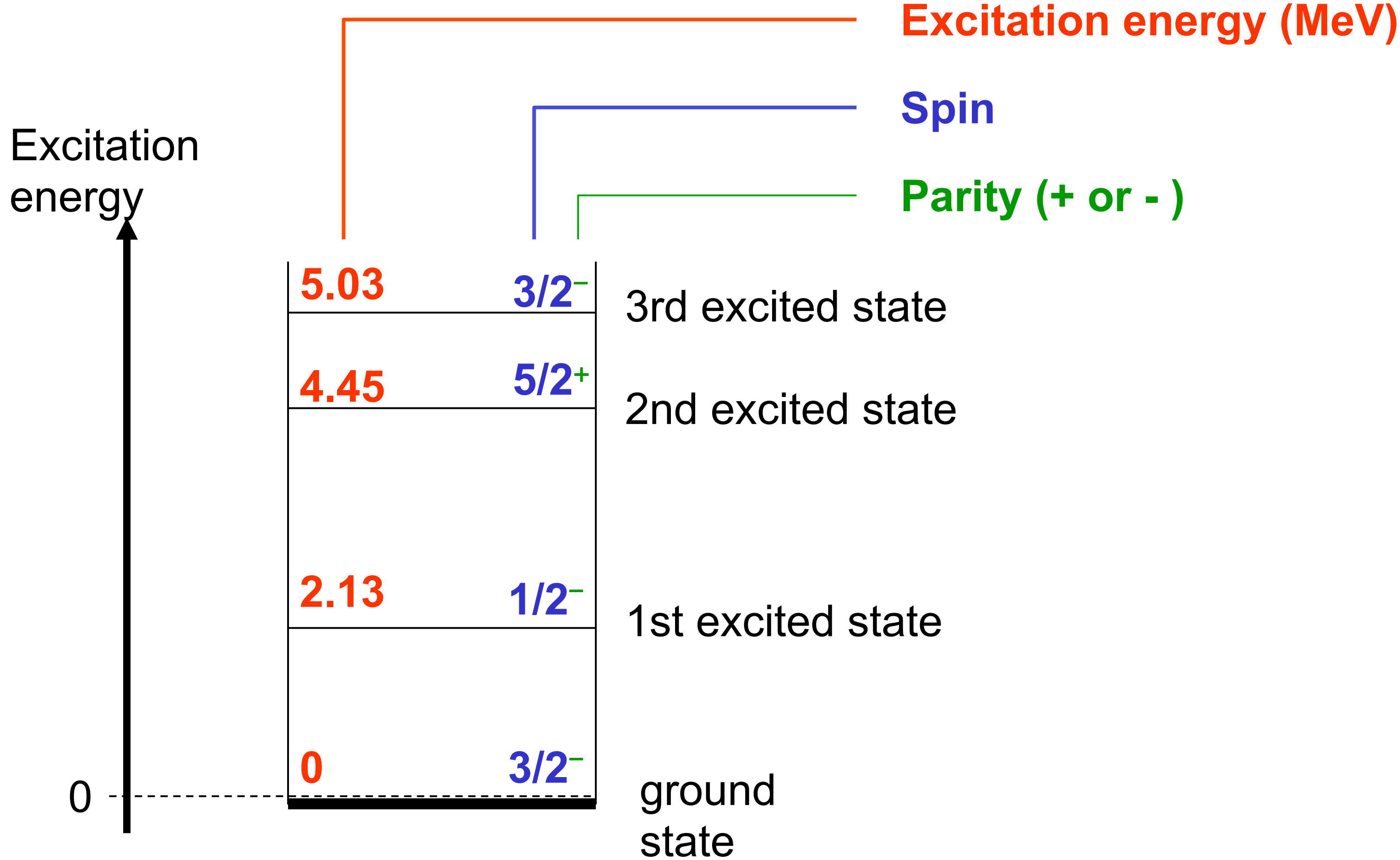
Nuclear properties that are relevant for reaction rates:

Nucleons in the nucleus can only have discrete energies. Therefore, the nucleus as a whole can be excited into discrete energy levels (excited states)



Each state is characterized by:

- energy (mass)
- spin
- parity
- lifetimes against γ,p,n, and α emission

The lifetime is usually given as a width as it corresponds to a width in the excitation energy of the state according to Heisenberg:

$$\Delta E \cdot \Delta t = \hbar$$

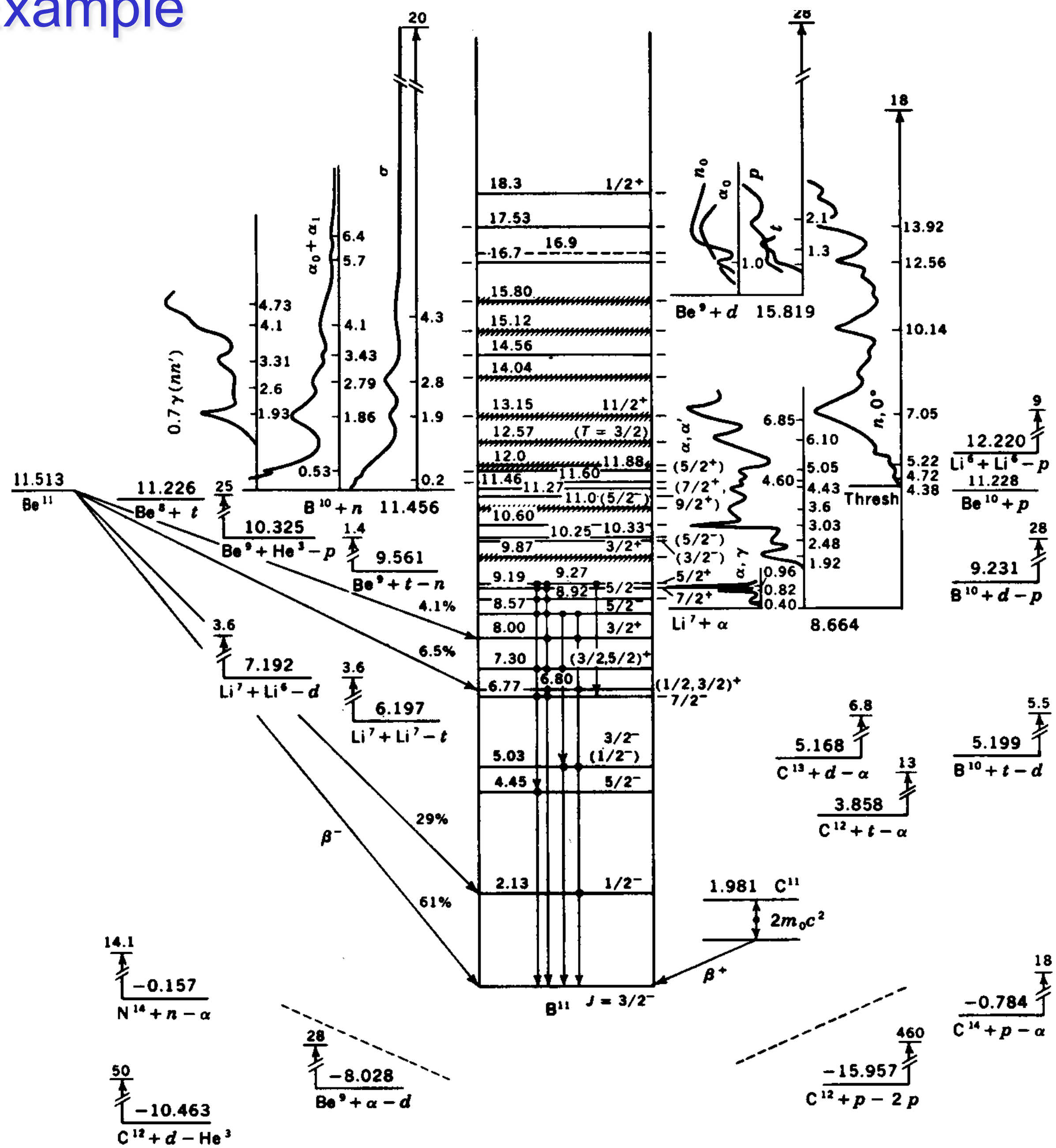
therefore, a lifetime τ corresponds to a width Γ:

$$\Gamma = \frac{\hbar}{\tau}$$

the lifetime against the individual "channels" for γ,p,n, and α emission are usually given as **partial widths**

Γ_{γ}, Γ_{p}, Γ_{n}, and Γ_{α} with $\Gamma = \sum \Gamma_i$

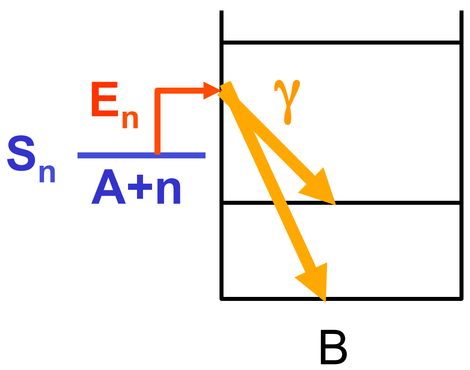
A Real Example



Basic reaction mechanisms involving strong or electromagnetic interaction:

Example: neutron capture A + n -> B + γ

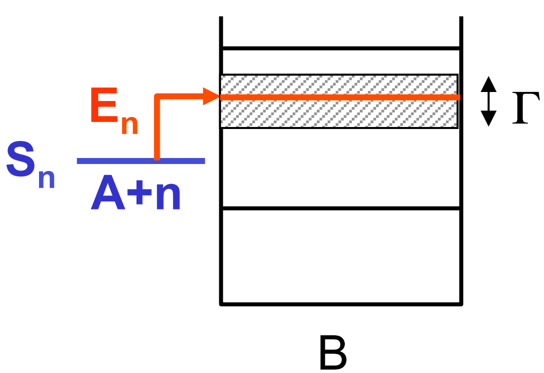
I. Direct reactions (for example, direct capture)



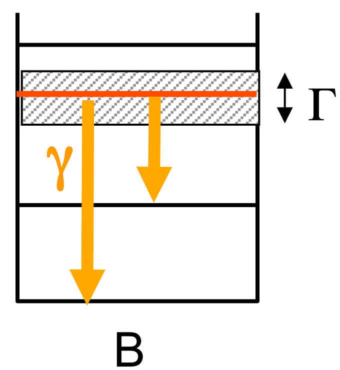
direct transition into bound states

II. Resonant reactions (for example, resonant capture)

Step 1: Coumpound nucleus formation
(in an unbound state)

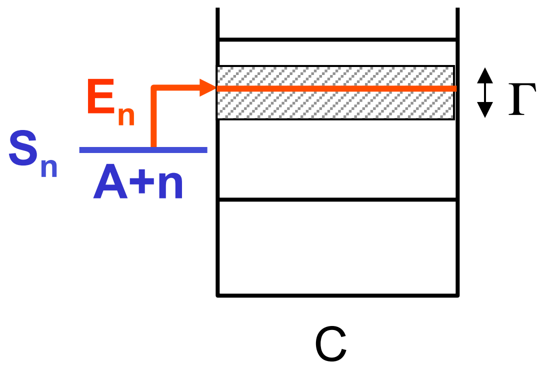


Step 2: Coumpound nucleus decay

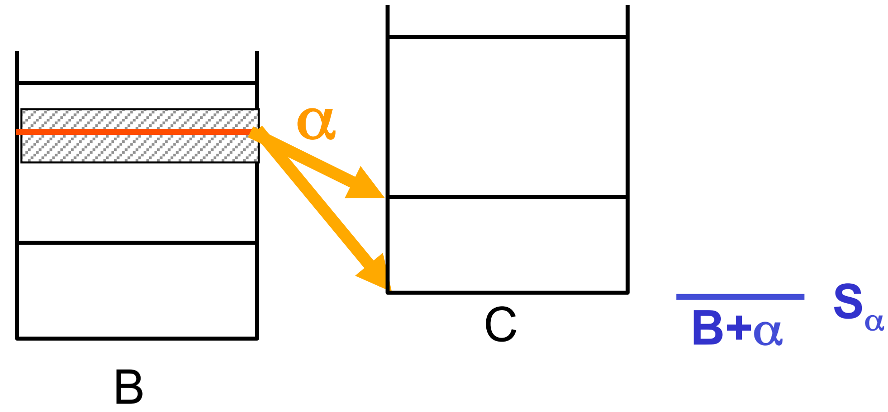


or a resonant A(n,α)B reaction:

Step 1: Compound nucleus formation
(in an unbound state)



Step 2: Compound nucleus decay



For resonant reactions, E_n has to "match" an excited state (but all excited states have a width and there is always some cross section through tails)

But enhanced cross section for $E_n \sim E_x - S_n$

more later …

Direct reactions - for example direct capture:

a + A -> B + γ

Direct transition from initial state |a+A> to final state <f| (some state in B)

$$\sigma \propto \pi\lambda_a^2 \cdot \left|\langle f | H | a + A \rangle\right|^2 \cdot P_l(E)$$

geometrical factor (deBroglie wave length of projectile - "size" of projectile)

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

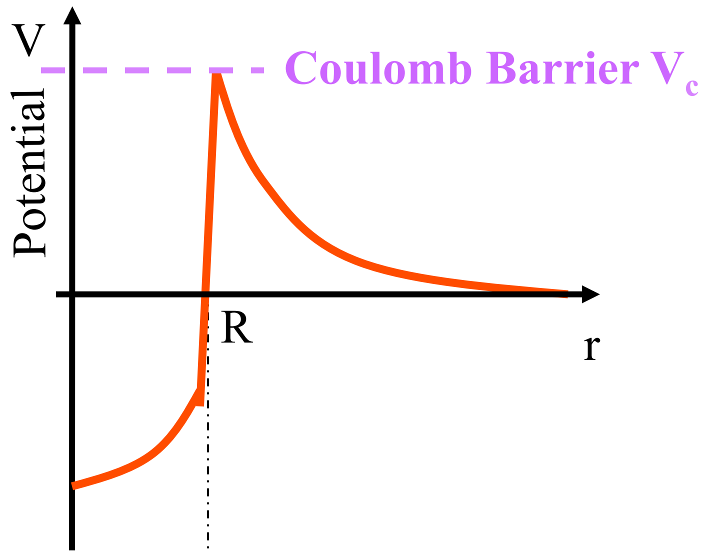
Interaction matrix element

Penetrability: probability for projectile to reach the target nucleus for interaction.
Depends on projectile Angular momentum l and Energy E

$$\sigma \propto \frac{1}{E} \cdot \left|\langle f | H | a + A \rangle\right|^2 \cdot P_l(E) \qquad \text{III.25}$$

Penetrability: 2 effects that can strongly reduce penetrability:

1. Coulomb barrier



for a projectile with Z_2 and a nucleus with Z_1

$$V_c = \frac{Z_1 Z_2 e^2}{R}$$

or

$$V_c\,[\mathrm{MeV}] = 1.44\,\frac{Z_1 Z_2}{R\,[\mathrm{fm}]} \approx 1.2\,\frac{Z_1 Z_2}{(A_1^{1/3} + A_2^{1/3})}$$

Example: $^{12}C(p,\gamma)$ V_C= 3 MeV

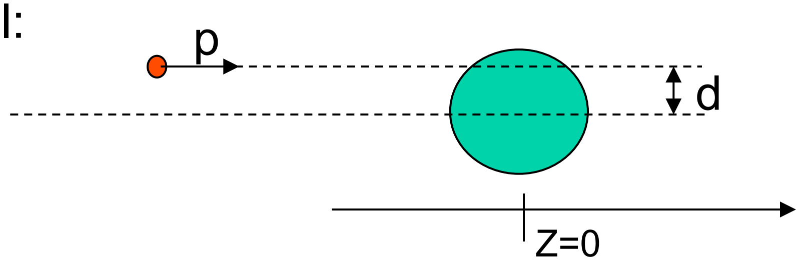
Typical particle energies in astrophysics are kT=1-100 keV !

Therefore, all charged particle reaction rates in nuclear astrophysics occur way below the Coulomb barrier – fusion is only possible through tunneling

2. Angular momentum barrier

Incident particles can have orbital angular momentum L

Classical:



Momentum p
Impact parameter d

$$L = pd$$

In quantum mechanics the angular momentum of an incident particle can have discrete values:

$$L = \sqrt{l(l+1)}\,\hbar$$

With

$l = 0$ s-wave
$l = 1$ p-wave
$l = 2$ d-wave
...

And parity of the wave function: $(-1)^l$

For radial motion (with respect to the center of the nucleus), angular momentum conservation (central potential !) leads to an energy barrier for non zero angular momentum.

Classically, one needs the radial kinetic energy to overcome the central potential, but if d != 0 then there is an increasing amount of "non radial kinetic energy", which one needs to supply as well (at z=0 for example, K_r=0, but of course K != 0)
In other words: only part of kinetic energy is radial so need higher energy

Energy E of a particle with angular momentum L (still classical)

$$E = \frac{L^2}{2mr^2}$$

Similar here in quantum mechanics:

$$V_l = \frac{l(l+1)\hbar^2}{2\mu r^2}$$

μ : reduced mass of projectile-target system

Peaks again at nuclear radius (like Coulomb barrier)

Or in MeV using the nuclear radius and mass numbers of projectile A_1 and target A_2:

$$V_l\,[\mathrm{MeV}] = 12\frac{l(l+1)}{\left(\frac{A_1 A_2}{A_1 + A_2}\right)(A_1^{1/3} + A_2^{1/3})}$$

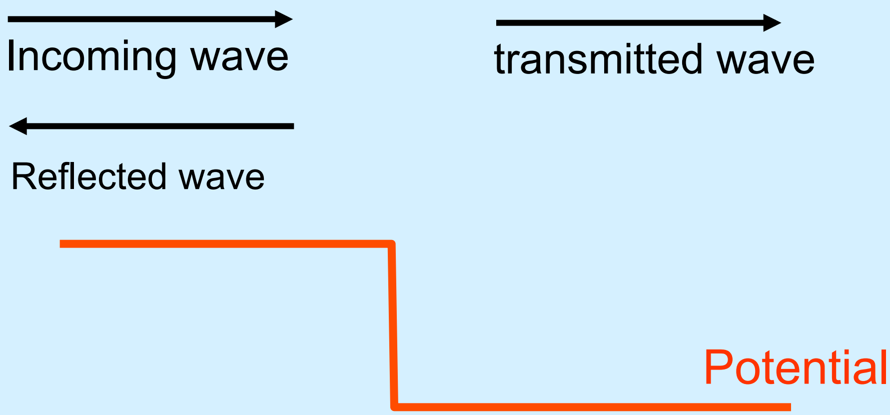
Direct reactions – the simplest case: s-wave neutron capture

No Coulomb or angular momentum barriers: $V_l=0$

$V_C=0$

s-wave capture therefore always dominates at low energies

But, change in potential still causes reflection – even without a barrier
Recall basic quantum mechanics:



Transmission proportional to $\sqrt{E}$

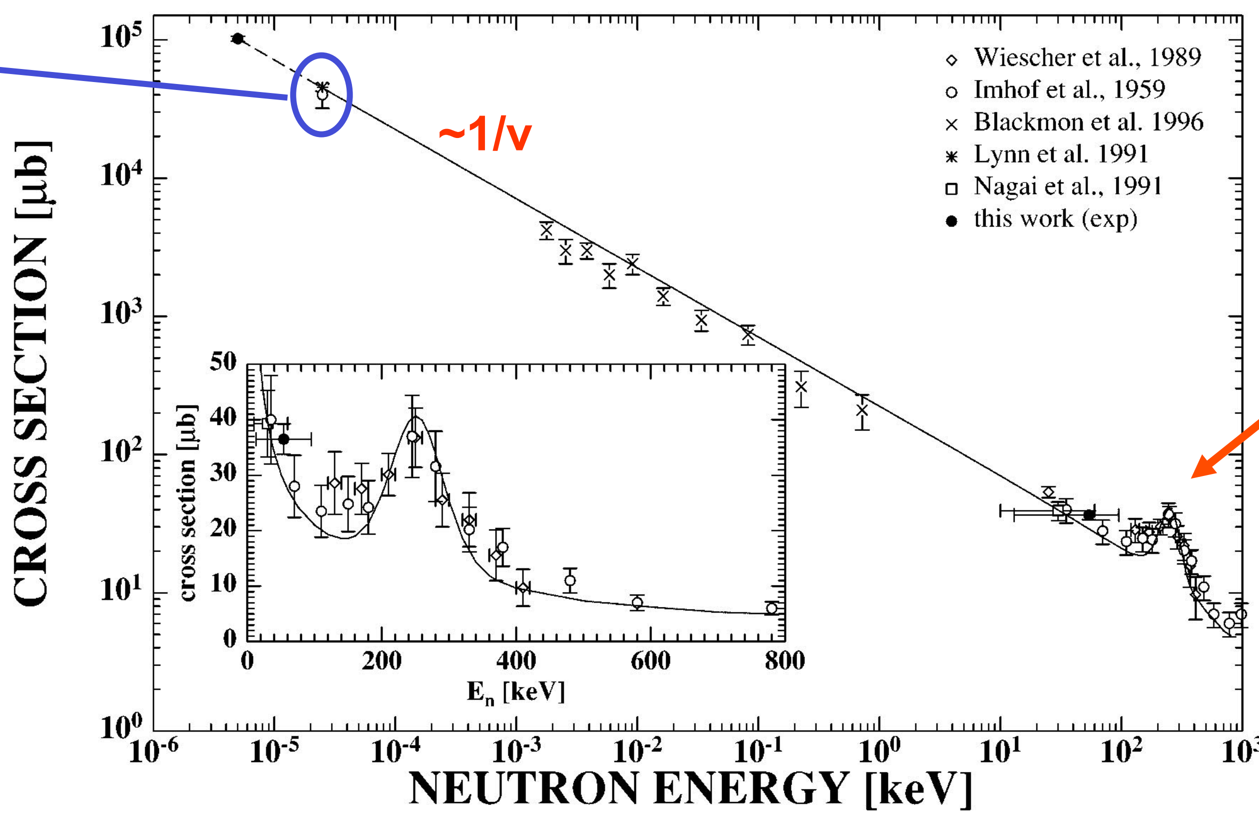
Therefore, for direct s-wave neutron capture:

Penetrability $P_l(E) \propto \sqrt{E}$

Cross section (use Eq. III.19): $\sigma \propto \frac{1}{\sqrt{E}}$ Or $\sigma \propto \frac{1}{v}$

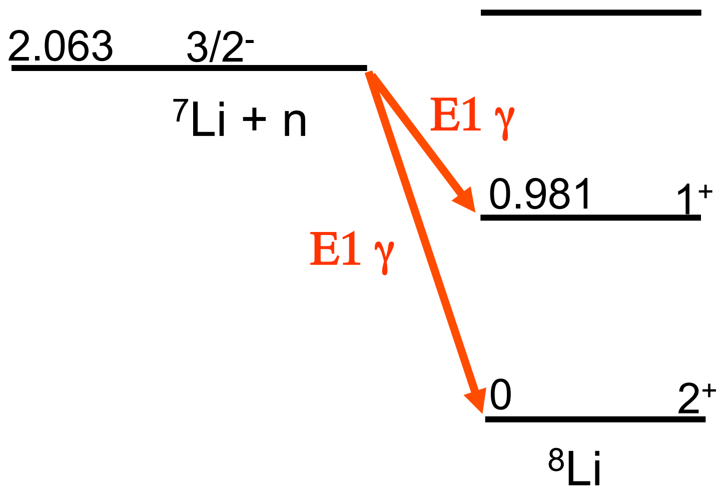
Example: ^{7}Li(n,γ)

thermal cross section $<\sigma>$=45.4 mb (see Pg. 27)



Deviation from 1/v due to resonant contribution

Why s-wave dominated ? Level scheme:



Angular momentum and parity conservation:

Entrance channel ^{7}Li + n : $3/2^- + 1/2^+ + l^{(-1)^l} = 1^-, 2^-$ (l=0 for s-wave)

Exit channel ^{8}Li + γ : 2^+ + ? (photon spin/parity)

Recall: Photon angular momentum/parity depend on multiploarity:

For angular momentum L (=multipolarity) electric transition EL parity $(-1)^L$
magnetic transition ML parity $(-1)^{L+1}$

Also recall:

E.M. Transition strength increases:

- for lower L
- for E over M
- for higher energy $\propto E_\gamma^{2L+1}$

Entrance channel ^{7}Li + n : $3/2^- + 1/2^+ + l^{(-1)} = 1^-, 2^-$ (l=0 for s-wave)

Exit channel ^{8}Li + γ : $2^+ + 1^-$ $= 1^-, 2^-, 3^-$

E1 photon lowest EL that allows to fulfill conservation laws

↑ match possible

Same for 1^+ state

→ "At low energies ^{7}Li(n,γ) is dominated by (direct) s-wave E1 capture".

Stellar reaction rate for s-wave neutron capture:

Because $\sigma \propto \frac{1}{v}$ $\longrightarrow$ $\sigma v = \mathrm{const} = <\sigma \mathrm{v}>$

Direct reactions – neutron captures with higher orbital angular momentum

For neutron capture, the only barrier is the angular momentum barrier

The penetrability scales with

$$P_l(E) \propto E^{1/2+l}$$

and therefore the cross section (Eq III.19)

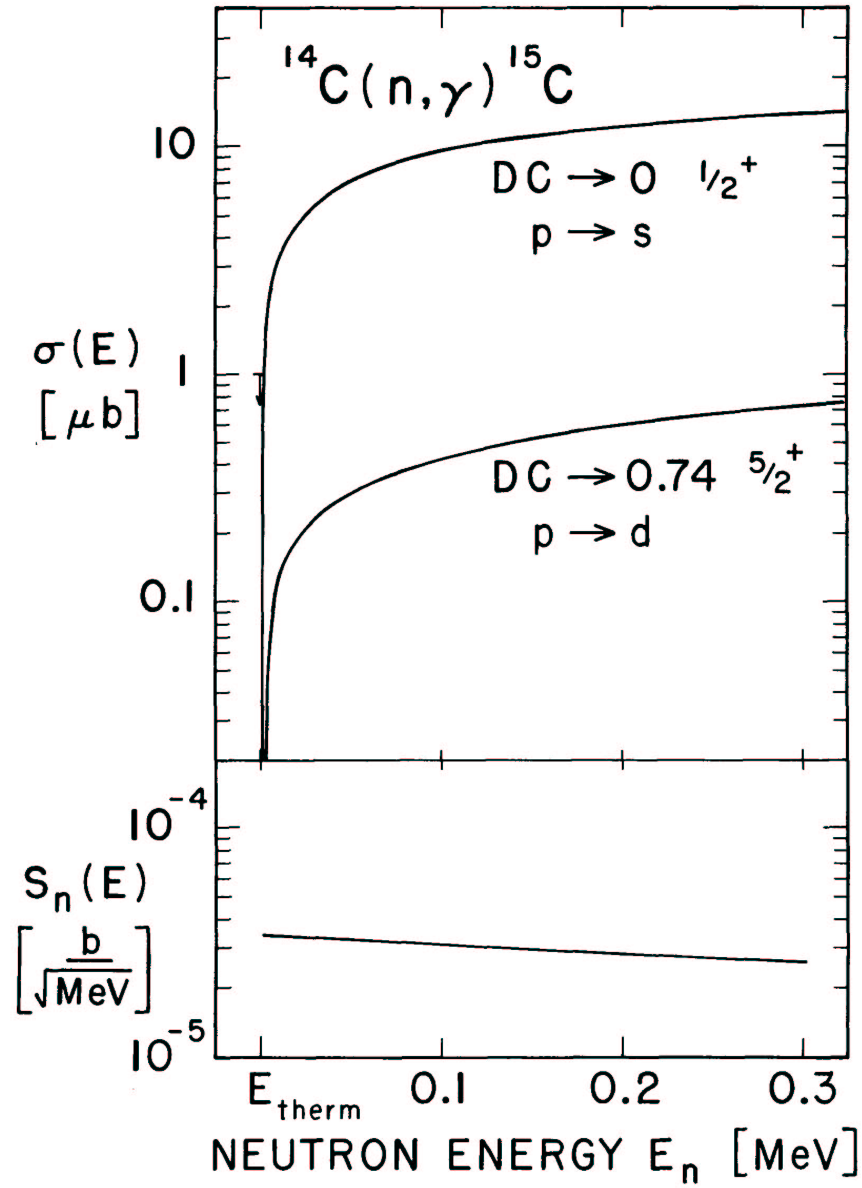
$$\sigma \propto E^{l-1/2}$$

for **l>0** cross section decreases with decreasing energy (as there is a barrier present)

Therefore, s-wave capture in general dominates at low energies, in particular at thermal energies. Higher l-capture usually plays only a role at higher energies. What "higher" energies means depends on case to case - sometimes s-wave is strongly suppressed because of angular momentum selection rules (as it would then require higher gamma-ray multipolarities)

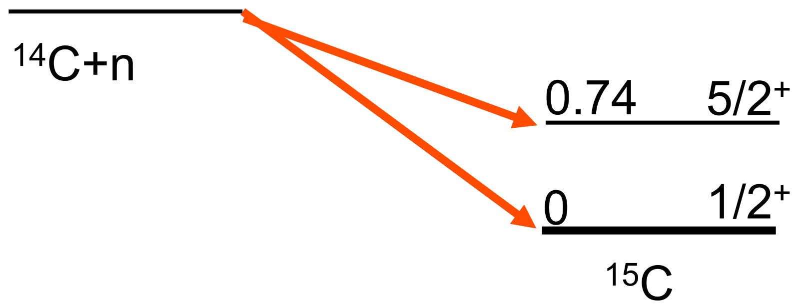
Example: p-wave capture in $^{14}C(n,\gamma)^{15}C$

$$\sigma \propto \sqrt{E}$$



(from Wiescher et al. ApJ 363 (1990) 340)

Why p-wave ?



Exit channel ($^{15}C + \gamma$)

	γ	total to $1/2^+$	total to $5/2^+$	
E1	**1^-**	**$1/2^-$ $3/2^-$**	**$3/2^-$ $5/2^-$ $7/2^-$**	**strongest !**
M1	1^+	$1/2^+$ $3/2^+$	$3/2^+$ $5/2^+$ $7/2^+$	
E2	2^+	$3/2^+$ $5/2^+$	$1/2^+$ $3/2^+$ $5/2^+$ $7/2^+$ $9/2^+$	

Entrance channel:

					strongest possible Exit multipole	
	l^π	^{14}C	n	total	into $1/2^+$	into $5/2^+$
s-wave	0^+	0^+	$1/2^+$	$1/2^+$	M1	E2
p-wave	1^-	0^+	$1/2^+$	$1/2^-$ $3/2^-$	**E1**	**E1**

despite of higher barrier, for relevant energies (1-100 keV) p-wave E1 dominates. At low energies, for example thermal neutrons, s-wave still dominates. But here for example, the thermal cross section is exceptionally low (<1μb limit known)

Charged particle induced direct reactions

Cross section and S-factor definition

(for example proton capture - such as $^{12}C(p,\gamma)$ in CN cycle)

incoming projectile Z_1 A_1 (for example proton or α particle)
target nucleus Z_2 A_2

again

$$\sigma \propto \frac{1}{E} \cdot P_l(E) \cdot \left| \langle f | H | a + A \rangle \right|^2$$

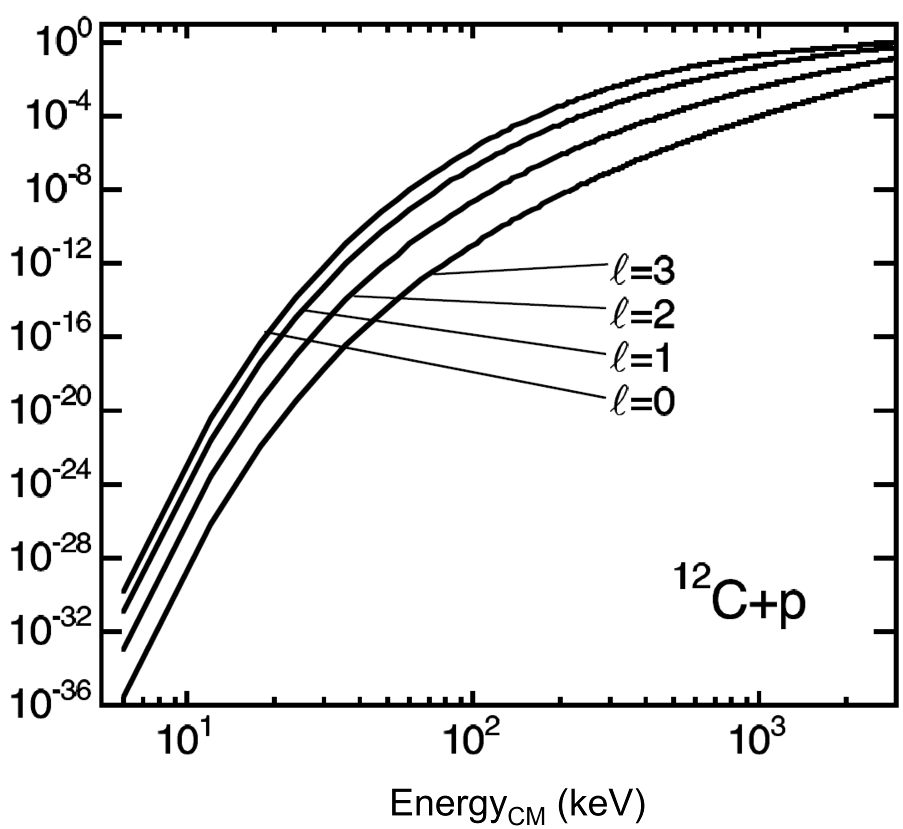
but now incoming particle has to overcome Coulomb barrier. Therefore

$$P_l(E) \propto \mathrm{e}^{-2\pi\eta} \quad \text{with} \quad \eta = \sqrt{\frac{\mu}{2E}} \frac{Z_1 Z_2 \mathrm{e}^2}{\hbar}$$

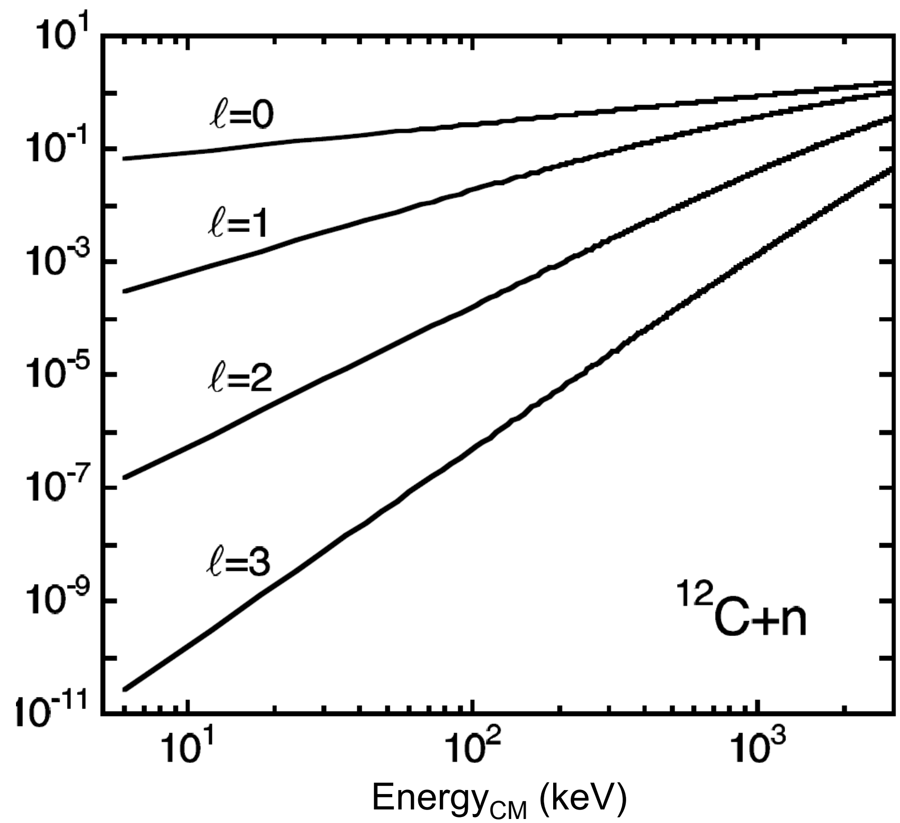
(from basic quantum mechanical barrier transmission coefficient)

Penetrability factor $P_l(E)$ example

Charged particle (proton)



Neutron



(from Iliadis “Nuclear Physics of Stars”)

The concept of the astrophysical S-factor (for n-capture)

recall:

$$\sigma \propto \underbrace{\frac{1}{E}\cdot P_l(E)}_{\text{"trivial" strong energy dependence}} \cdot \underbrace{\left|\left\langle f \middle| H \middle| a + A \right\rangle\right|^2}_{\text{"real" nuclear physics, weak energy dependence (for direct reactions !)}} \qquad \text{III.25}$$

"trivial" strong energy dependence

"real" nuclear physics
weak energy dependence
(for direct reactions !)

S-factor concept: write cross section as

strong "trivial" energy dependence X weakly energy dependent S-factor

The S-factor can be
- easier graphed
- easier fitted and tabulated
- easier extrapolated
- and contains all the essential nuclear physics

Note: There is no "universally defined S-factor - the S-factor definition depends on the type of reaction and (for neutrons at least) on l-value

Here the main energy dependence of the cross section (for direct reactions !) is given by

$$\sigma \propto \frac{1}{E} e^{-\frac{b}{\sqrt{E}}}$$

$$b = 31.28 \cdot Z_1 Z_2 A^{1/2} \sqrt{\text{keV}}$$

$$A = \frac{A_1 A_2}{A_1 + A_2} = \frac{\mu}{m_U}$$

therefore the **S-factor for charged particle reactions is defined** via

$$\sigma = \frac{1}{E} e^{-b/\sqrt{E}} S(E)$$

typical unit for S(E): keV barn

So far this all assumed s-wave capture. However, the additional angular momentum barrier leads only to a roughly constant addition to this S-factor that strongly decreases with l

Therefore, the S-factor for charged particle reactions is defined independently of the orbital angular momentum

Example:
$^{12}C(p,\gamma)$ cross section

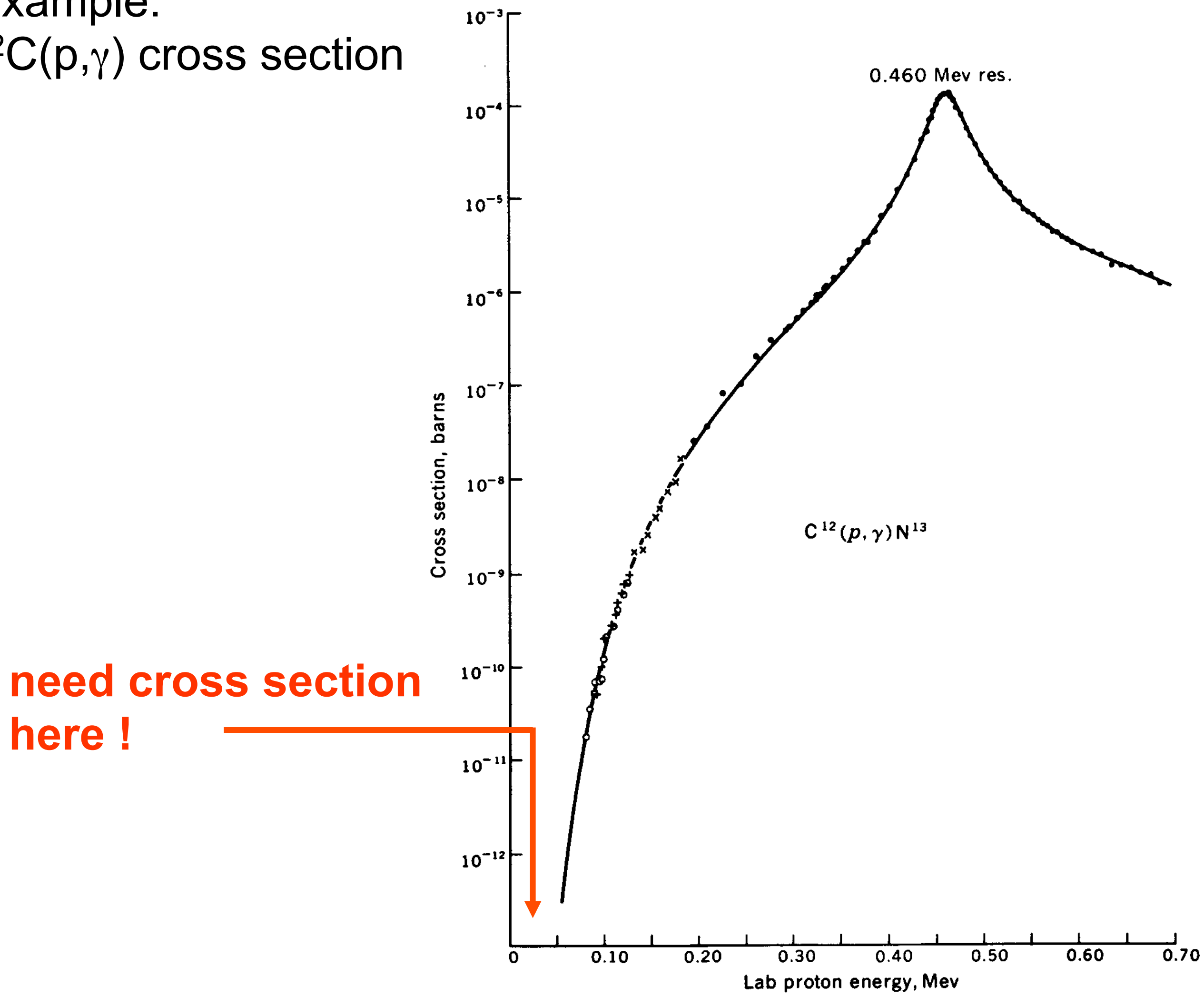
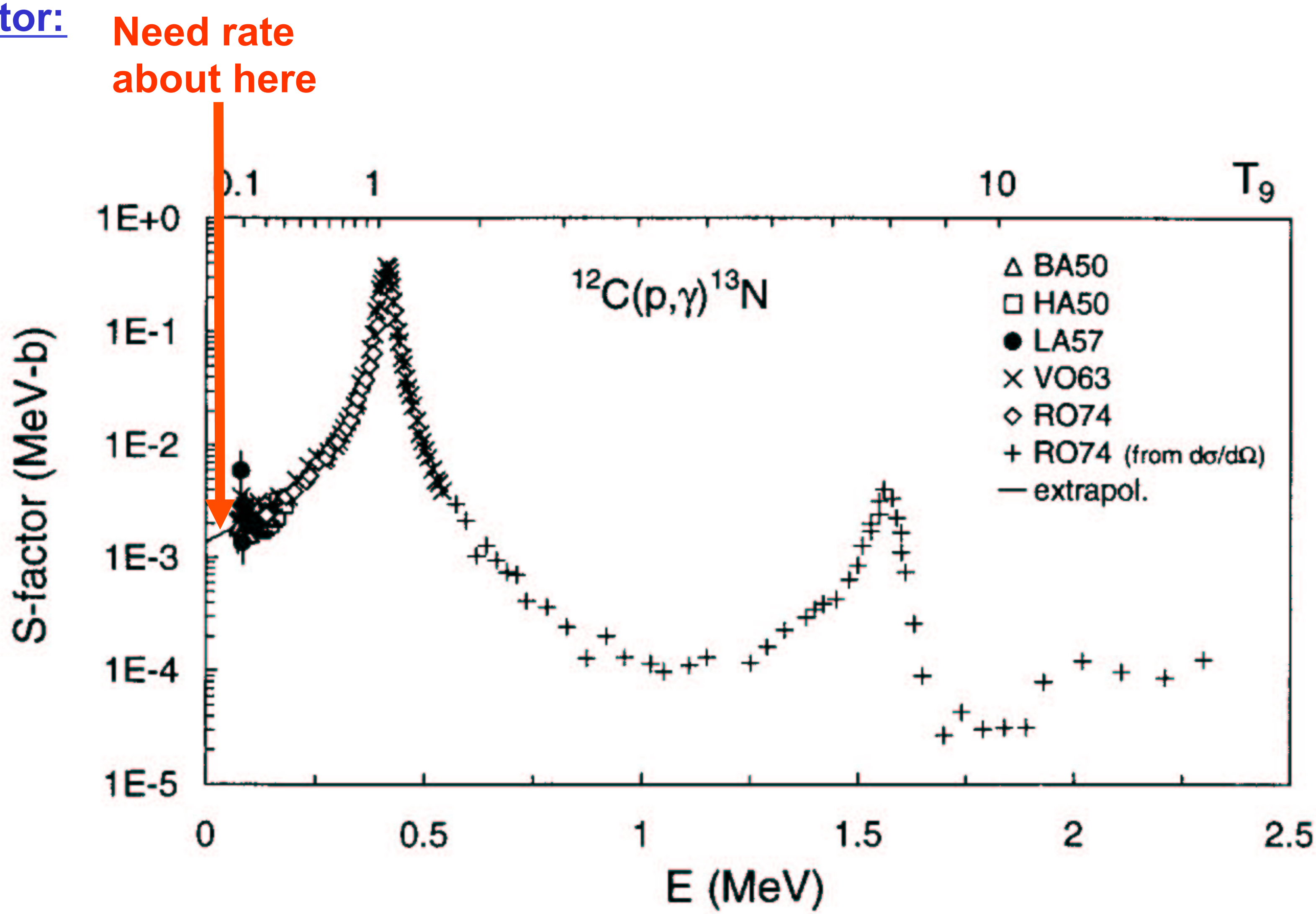


Fig. 4-4 The measured cross section for the reaction $C^{12}(p,\gamma)N^{13}$ as a function of laboratory proton energy. A four-parameter theoretical curve has been fitted to the experimental points. An extrapolation to $E_p = 0.025$ Mev, which is an interesting energy for this reaction in astrophysics, appears treacherous. *(Courtesy of W. A. Fowler and J. L. Vogl.)*

S-Factor:



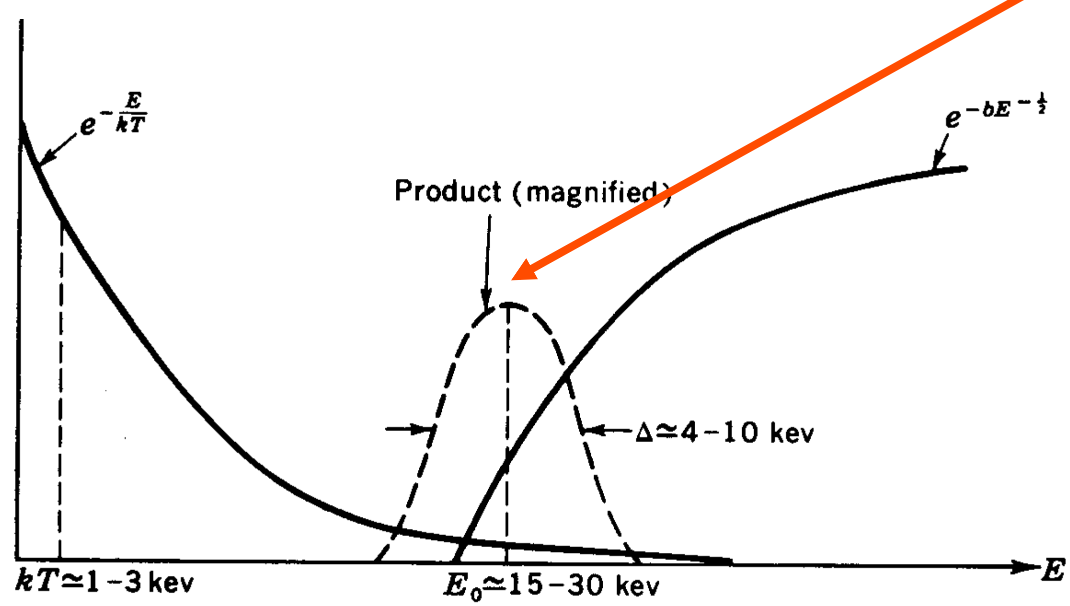
From the **NACRE compilation** of charged particle induced reaction rates on stable nuclei from H to Si (Angulo et al. Nucl. Phys. A 656 (1999) 3

9.3.2. Relevant cross section - Gamov Window

for charged particle reactions

$$< \sigma v > = \sqrt{\frac{8}{\pi\mu}} (kT)^{-3/2} \int \sigma(E) E \, e^{-\frac{E}{kT}} dE = \sqrt{\frac{8}{\pi\mu}} (kT)^{-3/2} \int S(E) e^{-\left(\frac{b}{\sqrt{E}} + \frac{E}{kT}\right)} dE$$

Gamov Peak



Note: relevant cross section in tail of M.B. distribution, much larger than kT (very different from n-capture !)

The Gamov peak can be approximated with a Gaussian

$$e^{-\left(\frac{b}{\sqrt{E}}+\frac{E}{kT}\right)} \approx e^{-\left(\frac{3\mathrm{E}_0}{\mathrm{kT}}\right)} e^{-\left(\frac{\mathrm{E}-\mathrm{E}_0}{\Delta \mathrm{E}/2}\right)^2}$$

centered at same energy E_0 with width ΔE such that $d^2/dE^2|_{E0}$ is the same

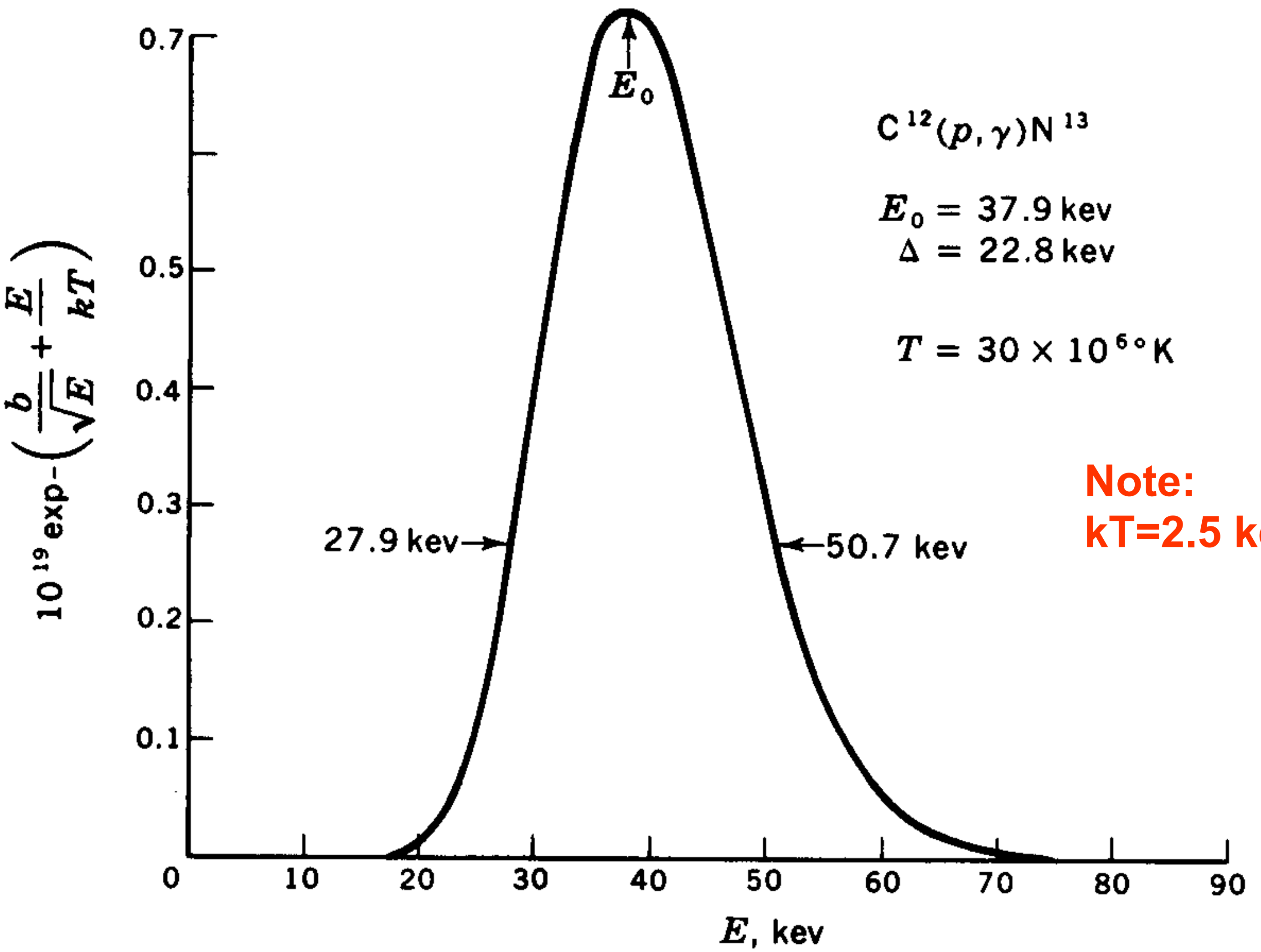
Then, the **Gamov window** or the range of relevant cross section can be easily calculated using:

$$E_0 = \left(\frac{bkT}{2}\right)^{3/2} = 0.12204\left(Z_1^2 Z_2^2 A\right)^{1/3} T_9^{2/3}\ \mathrm{MeV}$$

$$\Delta E = \frac{4}{\sqrt{3}}\sqrt{E_0 kT} = 0.23682\left(Z_1^2 Z_2^2 A\right)^{1/6} T_9^{5/6}\ \mathrm{MeV}$$

with A "reduced mass number" and T_9 the temperature in GK

Example:



Note:
kT=2.5 keV !

Fig. 4-7 The Gamow peak for the reaction $C^{12}(p,\gamma)N^{13}$ at $T = 30 \times 10^6$ °K. The curve is actually somewhat asymmetric about E_0, but it is nonetheless adequately approximated by a gaussian.

9.3.3. Reaction rate from S-factor

Often (for example with theoretical reaction rates) one approximates the rate calculation by assuming the S-factor is constant over the Gamov WIndow

$$S(E)=S(E_0)$$

then one finds the useful equation:

$$N_A < \sigma v >= 7.83 \cdot 10^9 \left(\frac{Z_1 Z_2}{A T_9^{\,2}} \right)^{1/3} S(E_0)[\text{MeV barn}]\ e^{-4.2487 \left(\frac{Z_1^2 Z_2^2 A}{T_9} \right)^{1/3}}$$

Equation III.53

(A reduced mass number !)

better (and this is often done for experimental data) one expands S(E) around E=0 as powers of E to second order:

$$S(E) = S(0) + ES'(0) + \frac{1}{2} ES''(0)$$

If one integrates this over the Gamov window, one finds that one can use Equation III.46 when **replacing $S(E_0)$ with the effective S-factor S_{eff}**

$$S_{eff} = S(0)\left[1 + \frac{5}{12\tau} + \frac{S'(0)}{S(0)}\left(E_0 + \frac{35}{36}kT\right) + \frac{1}{2}\frac{S''(0)}{S(0)}\left(E_0^2 + \frac{89}{36}E_0 kT\right)\right]$$

with $\tau = \frac{3E_0}{kT}$ and E_0 as location of the Gamov Window (see Pg. 51)