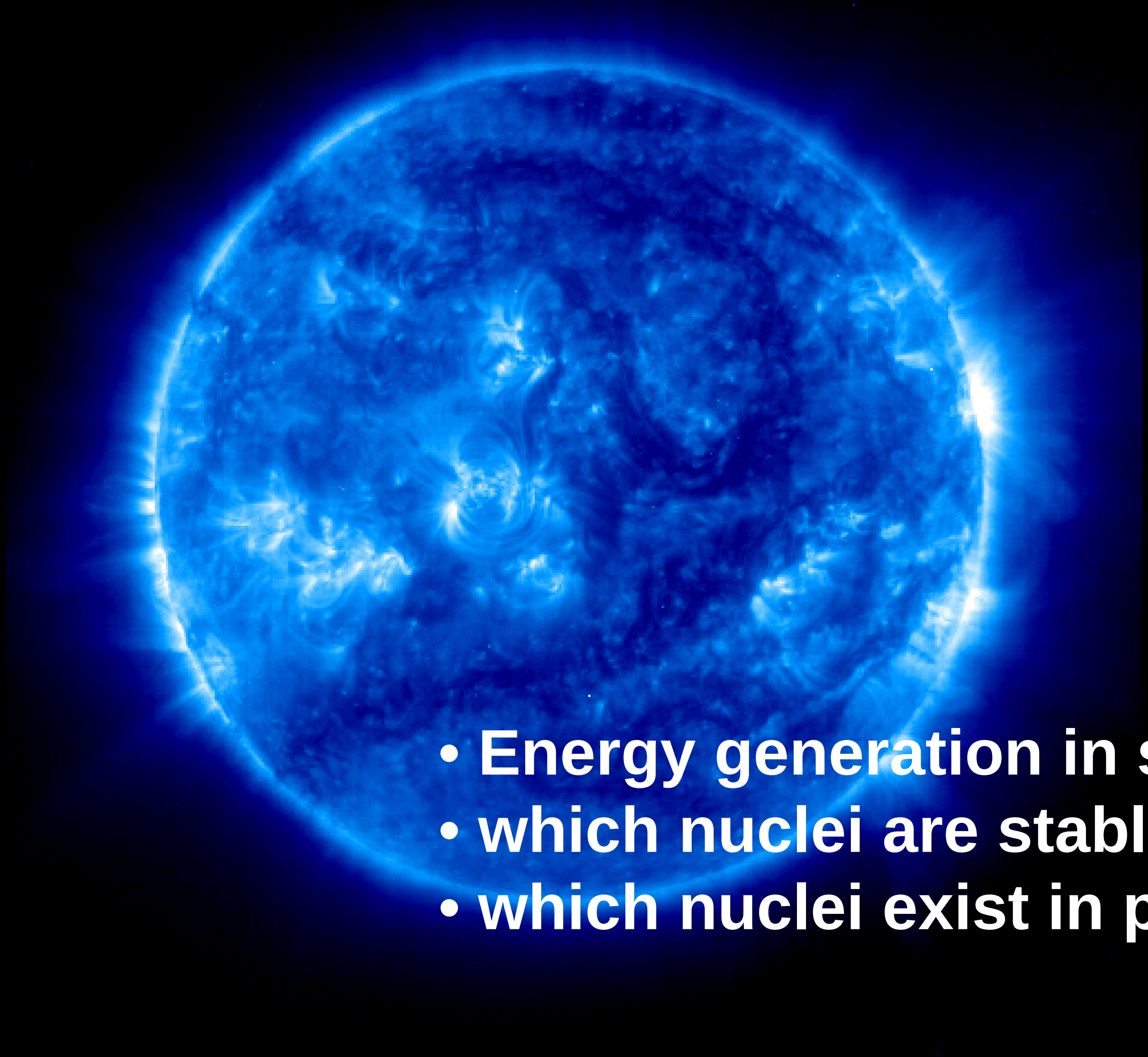
The mass of a nucleus



- **Energy generation in stars**
- **which nuclei are stable**
- **which nuclei exist in principle**

SOHO, 171A Fe emission line

Nucleons

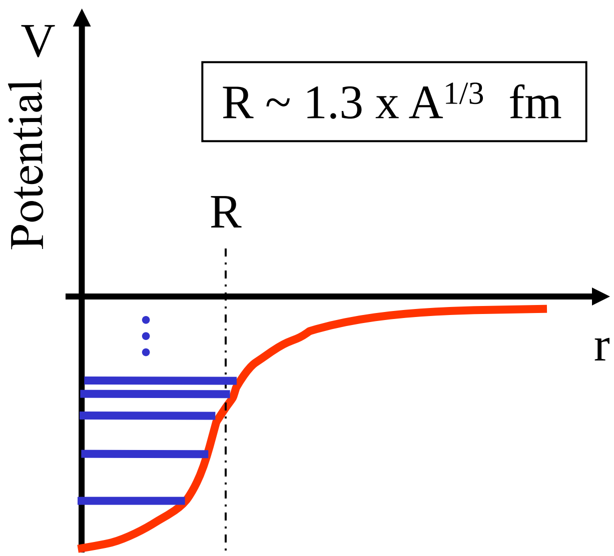
	Mass	Spin	Charge
Proton	938.272 MeV/c^2	1/2	+1e
Neutron	939.565 MeV/c^2	1/2	0

size: ~1 fm

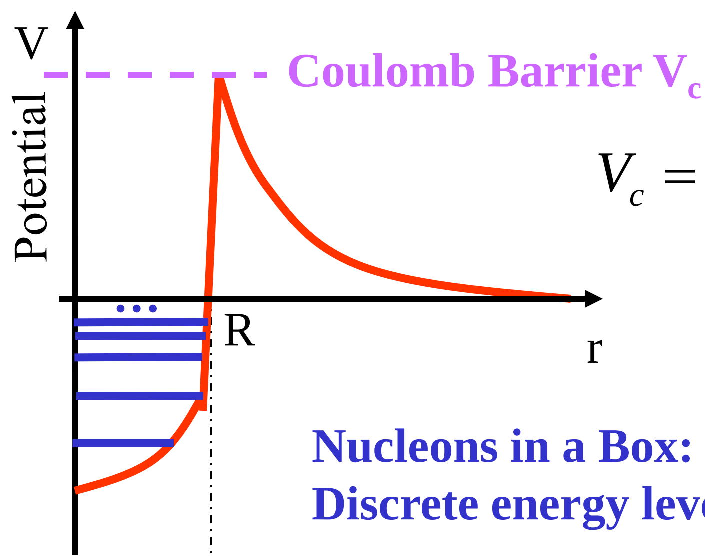
Nuclei

nucleons attract each other via the strong force (range ~ 1 fm)
a bunch of nucleons bound together create a potential for an additional :

neutron



proton
(or any other charged particle)



$$V_c = \frac{Z_1 Z_2 e^2}{R}$$

Nucleons in a Box:
Discrete energy levels in nucleus

→ Nucleons are bound by attractive force. Therefore, mass of nucleus is smaller than the total mass of the nucleons by the binding energy dm=B/c^2

Nuclear Masses and Binding Energy

Energy that is released when a nucleus is assembled from neutrons and protons

$$m(Z,N) = Zm_p + Nm_n - B/c^2$$

m_p = proton mass, m_n = neutron mass, m(Z,N) = mass of nucleus with Z,N

- B>0
- With B the mass of the nucleus is determined.
- B is very roughly ~A

Masses are usually tabulated as **atomic masses**

$$m = m_{nuc} + Z\,m_e - B_e$$

Nuclear Mass
~ 1 GeV/A

Electron Mass
511 keV/Z

Electron Binding Energy
13.6 eV (H)
to 116 keV (K-shell U) / Z

Most tables give atomic mass excess Δ in MeV: $m = Am_u + \Delta / c^2$

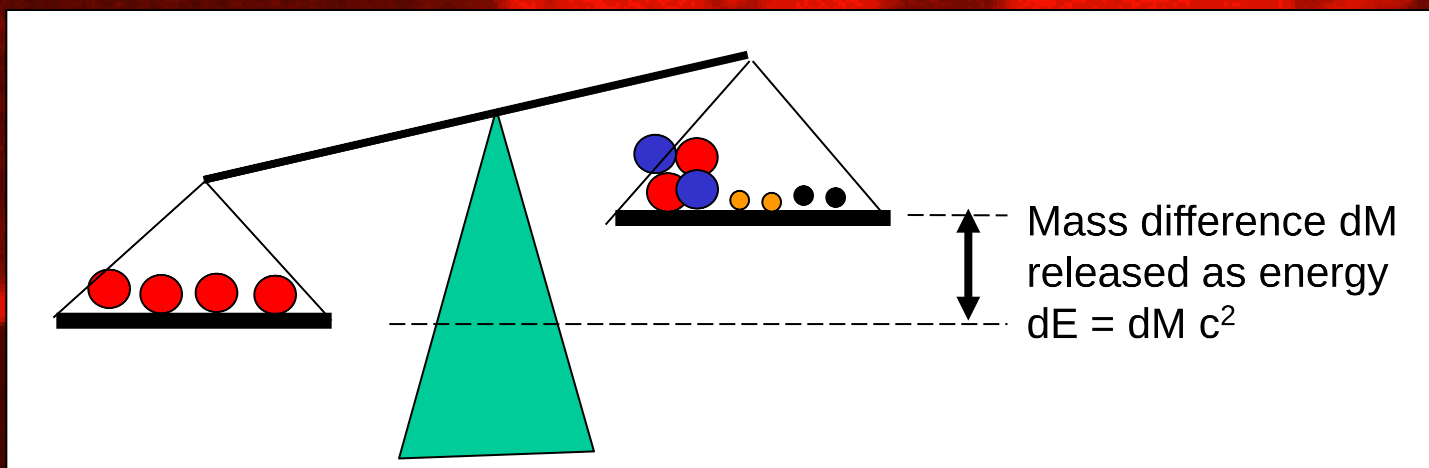
(so for ^{12}C: Δ=0) (see nuclear wallet cards for a table)

Q-value

Energy released in a nuclear reaction (>0 if energy is released, <0 if energy is use

Example: The sun is powered by the fusion of hydrogen into helium:

$$4p \rightarrow {}^{4}He + 2\,e^{+} + 2\nu_e$$



$$Q/c^2 = 4m_{nuc\ p} - m_{nuc\ 4He} - 2m_e - 2m_\nu$$

(using nuclear masses !)

In practice one often uses mass excess Δ and atomic masses.

Q-value with mass excess Δ

As A is always conserved in nuclear reactions the mass excess Δ can always be used instead of the masses (the Am_u term cancels)

(as nucleon masses cancel on both sides, its really the binding energies that entirely determine the Q-values !)

Q-value with atomic masses:

If Z is conserved (no weak interaction) atomic masses can be used instead of nuclear masses (Zme and most of the electron binding energy cancels)

Otherwise: For each positron emitted subtract $2m_e/c^2$= 1.022 MeV from the Q-value

Example: $4p \rightarrow {}^4He + 2\,e^+ + 2\nu_e$ Z changes an 2 positrons are emitted

$$Q/c^2 = 4m_H - m_{He} - 4m_e$$ With atomic masses

$$Q/c^2 = 4\Delta_H - \Delta_{He} - 4m_e$$ With atomic mass excess

How can we calculate that?

Find masses in nuclear wallet cards at http://www.nndc.bnl.gov/wallet/

Nuclear Wallet Cards

Nuclide Z	El	A	Jπ	Δ (MeV)	T½, Γ, or Abundance	Decay Mode
0	**n**	1	1/2+	8.071	10.183 m *17*	β−
1	**H**	1	1/2+	7.289	**99.9885% *70***	
		2	1+	13.136	**0.0115% *70***	
		3	1/2+	14.950	12.32 y *2*	β−
		4	2−	24.6		n
		5	(1/2+)	32.89	5.7 MeV *21*	2n
		6	(2−)	41.9	1.6 MeV *4*	n
		7	(1/2+)	47.9	29×10^{-23} y *7*	
2	**He**	3	1/2+	14.931	**0.000134% *3***	
		4	0+	2.425	**99.999866% *3***	
		5	3/2−	11.23	0.60 MeV *2*	α, n
		6	0+	17.592	801 ms *10*	β−
		7	(3/2)−	26.067	150 keV *20*	n
		8	0+	31.609	119.1 ms *12*	β−, β−n 16%
		9	1/2+	39.78		n
		10	0+	48.81	300 keV *200*	n
3	**Li**	3		29s	unbound	p?
		4	2−	25.3	6.03 MeV	p
		5	3/2−	11.68	≈1.5 MeV	p, α

Result for

$$Q/c^2 = 4\Delta_H - \Delta_{He} - 4m_e$$

= 4 x 7.289 MeV – 2.425 MeV – 4 x 0.511 MeV
= 24.687 MeV
This is the energy released per reaction

Note that for example inside the sun the 2 positrons annihilate with 2 electrons releasing an additional energy of $4m_e$
So if the positrons do not escape the total energy release is 26.731 MeV

New atomic mass data center for experimental mass evaluations: http://ribll.impcas.ac.cn/ame/
Mass data sets for nuclear astrophysics: https://groups.nscl.msu.edu/jina/nucdatalib/

The liquid drop mass model for the binding energy: (Weizaecker Formula)
(assumes incompressible fluid (volume ~ A) and sharp surface)

$$B(Z,A) = a_V A$$

Volume Term (each nucleon gets bound by about same energy)

$$- a_s A^{2/3}$$

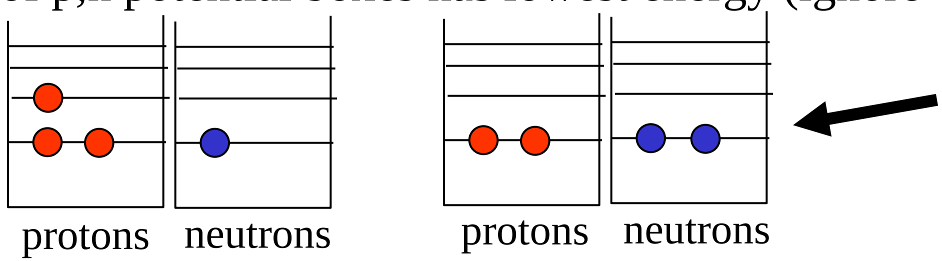
Surface Term ~ surface area (Surface nucleons less bound)

$$- a_C \frac{Z^2}{A^{1/3}}$$

Coulomb term. Coulomb repulsion leads to reduction
uniformly charged sphere has E=3/5 Q^2/R

$$- a_A \frac{(Z - A/2)^2}{A}$$

Asymmetry term: Pauli principle to protons: symmetric filling of p,n potential boxes has lowest energy (ignore Coulomb)



lower total energy = more bound

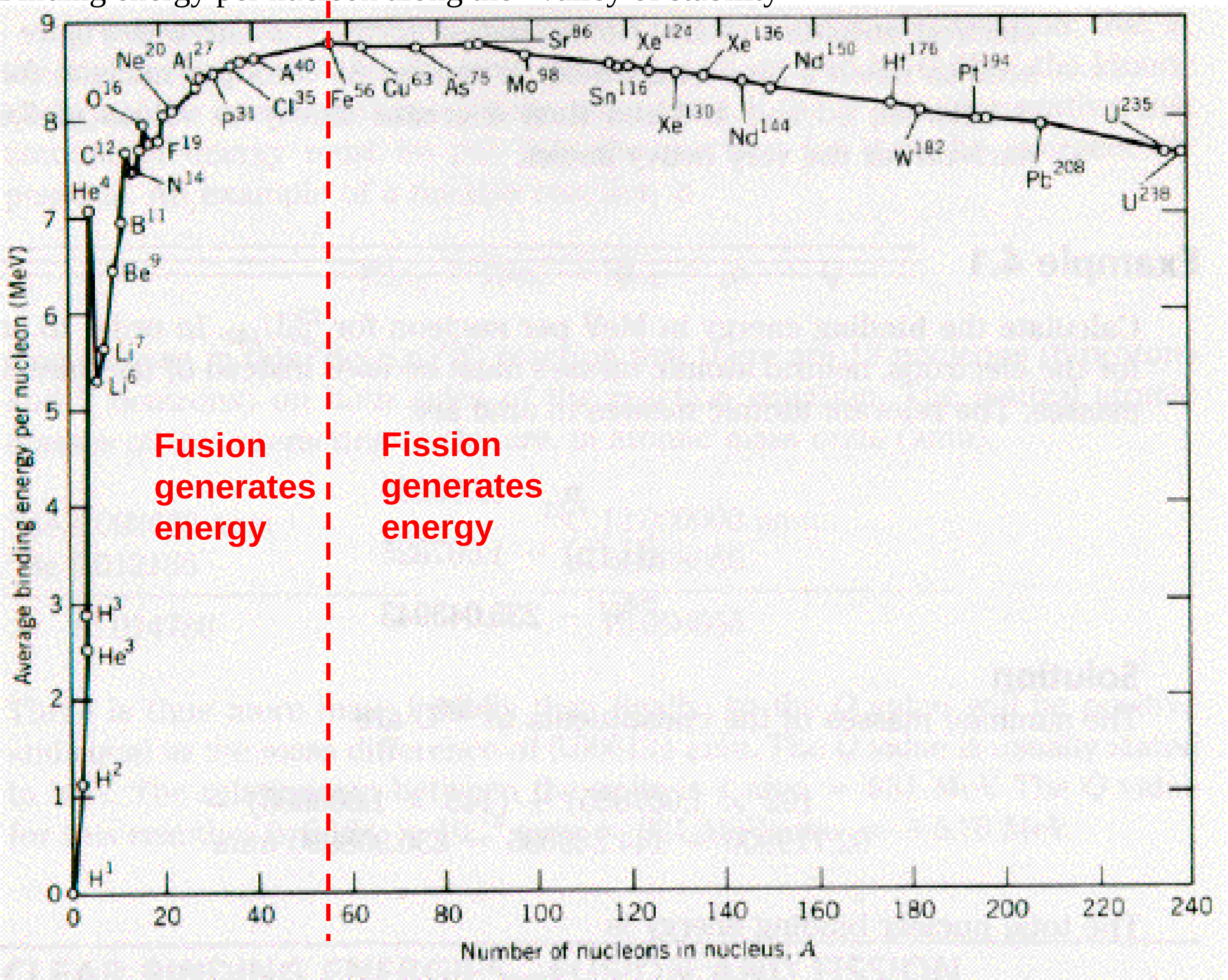
and in addition: p-n more bound than p-p or n-n (S=1,T=0 more bound than S=0,T=1)

$$+ a_p A^{-1/2} \begin{cases} \text{x } 1 & \text{ee} \\ \text{x } 0 & \text{oe/eo} \\ \text{x } (-1) & \text{oo} \end{cases}$$

Pairing term: even number of like nucleons favoured

(e=even, o=odd referring to Z, N respectively)

Binding energy per nucleon along the “valley of stability”

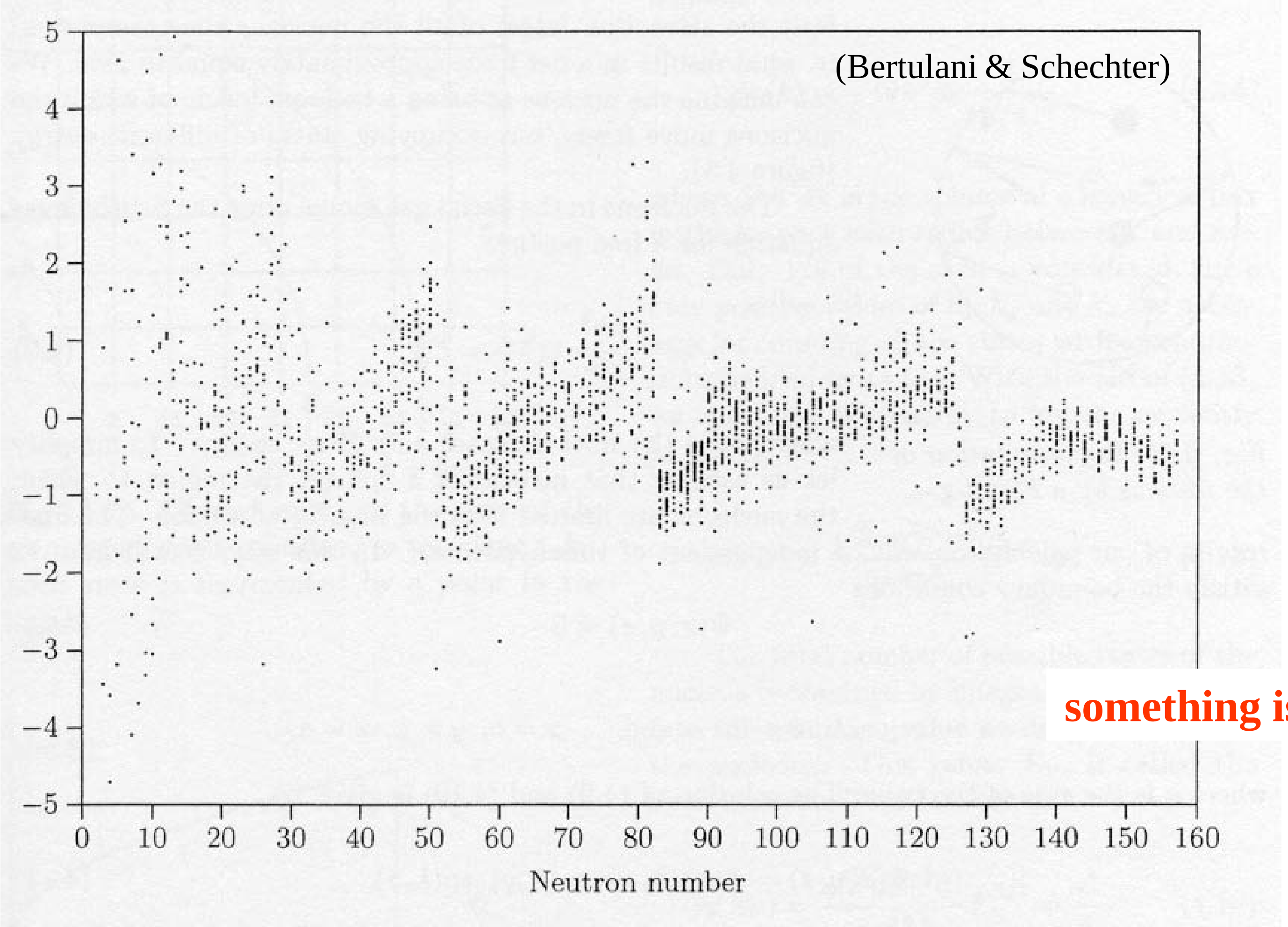


Best fit values (from A.H. Wapstra, Handbuch der Physik 38 (1958) 1)

in MeV/c^2

a_V	a_S	a_C	a_A	a_P
15.85	18.34	0.71	92.86	11.46

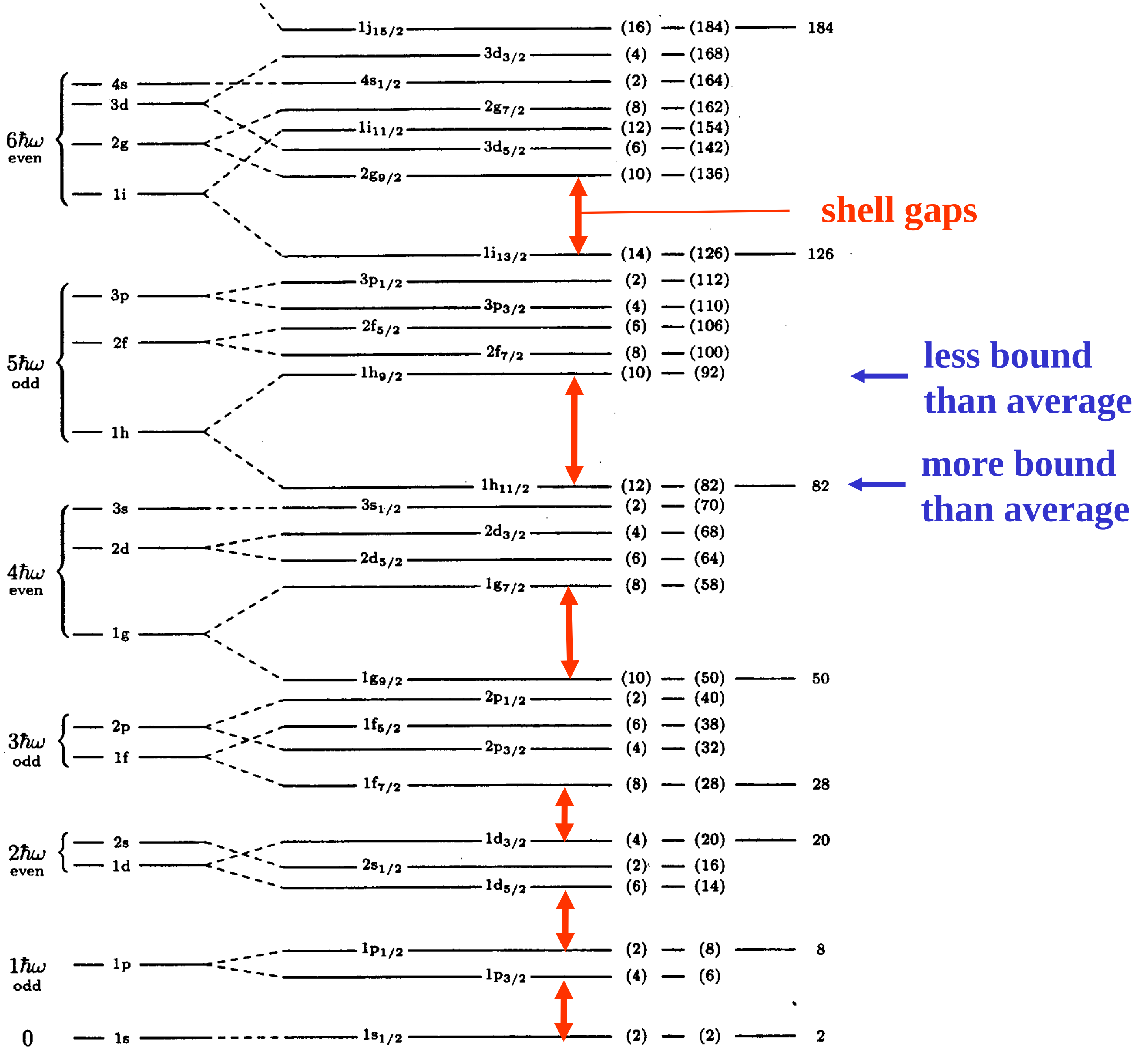
Deviation (in MeV) to experimental masses:



something is missing !

Shell model:
(single nucleon
energy levels)

are not evenly spaced



need to add
shell correction term
S(Z,N)

Magic numbers

Modern mass models

Global mass models – 2 basic philosophies:

1) Microscopic – Macroscopic mass models

Macroscopic part: liquid drop, droplet, or refinements thereof)

Microscopic part: shell correction, pairing correction, refinement of surface term accounting for finite range of nuclear force …

2) Microscopic mass models

based on some (parametrized) nucleon-nucleon interaction

Problem: not very accurate due to limitations of current microscopic theories

Solution: Fit parameters of interaction specifically to masses to obtain a mass model

Local mass "models":

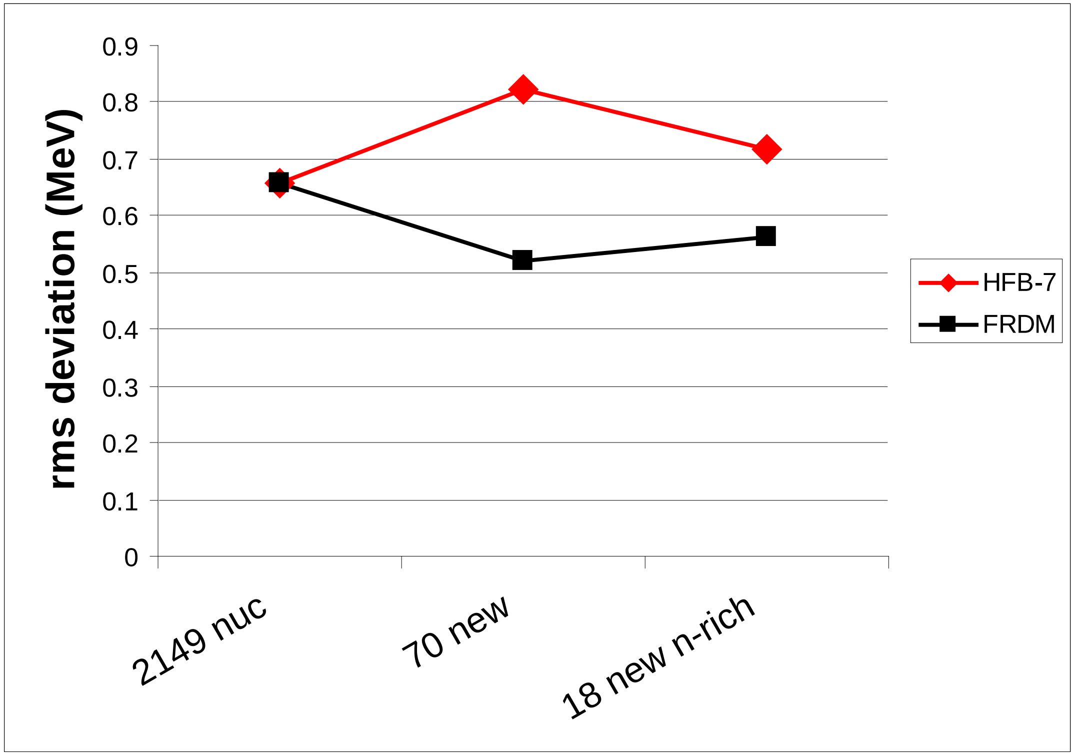
- extrapolations based on neighboring masses (Atomic Mass Evaluation)
- mirror symmetry: Coulomb shifts, IMME
- Garvey-Kelson …

Mass measurements have sufficiently progressed so that global mass models are only needed for very neutron rich nuclei (r-process, neutron star crusts)

Modern mass models – how well are they doing?

Example: mic model: HFB series (Goriely, Pearson) currently at HFB-15 (2008)
mic-mac : Finite Range Droplet Model FRDM (Moller et al.) unchanged since 1993

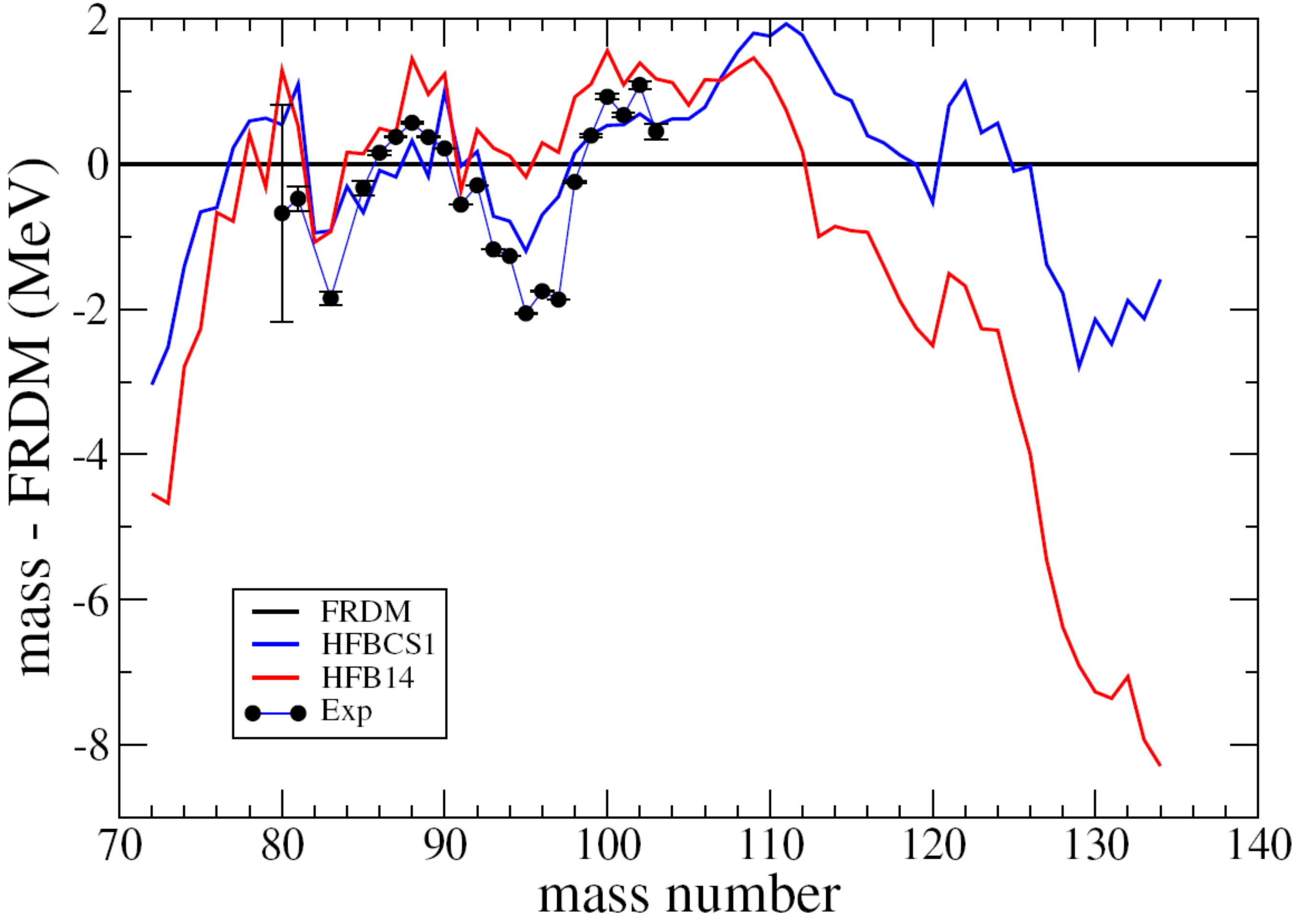
Compare rms deviations to experiment:



Important is not how well the model fits known masses, but how well it predicts unknown masses !

Modern mass models – how well are they doing?

Example: predicted masses for Zr isotopes



Modern mass models – how well are they doing?

What about mass differences?

Neutron capture Q-values for Zr isotopes
(neutron separation energy Sn)