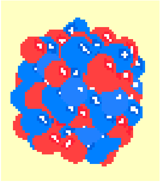
The nucleus



The atomic nucleus consists of protons and neutrons

Protons and Neutrons are therefore called nucleons

A nucleus is characterized by:

- A: Mass Number = number of nucleons
- Z: Charge Number = number of protons
- N: Neutron Number

Determines the Element

Determines the Isotope

Of course A=Z+N

Usual notation:

Mass number A

$^{12}\mathrm{C}$

Element symbol – defined by charge number
C is Carbon and Z=6

So this nucleus is made of 6 protons and 6 neutrons

Abundance of a nucleus

How can we describe the relative abundances of nuclei of different species and their evolution in a given sample (say, a star, or the Universe) ?

1) Number density

We could use the number density n_i = number of nuclei of species i per cm^3

Disadvantage: tracks not only nuclear processes that create or destroy nuclei, but also density changes, for example due to compression or expansion of the material.

→ not useful as characterization of composition

2) Mass fraction

Mass fraction X_i is fraction of total mass of sample that is made up by nucleus of species i

$$n_i = \frac{X_i \rho}{m_i}$$

ρ : mass density (g/cm^3)

m_i mass of atom of species i

Or in terms of moles:

$$n_i = \frac{X_i \, \rho \, N_A}{m_{i\,mole}}$$

$m_{i\,mole}$ *(atomic)* mole mass of species i

Of course:

$$\sum_i X_i = 1$$

Disadvantage: depends on mass of nucleus (for equal numbers of particles a heavier nucleus has a larger mass fraction)

3) Abundance

reactions mostly affect particle numbers, therefore sometimes one wants a quantity that is a measure for the number of particles (if two species have the same number density one wants the "abundance" to be the same regardless of mass)

call this abundance Y_i

$$n_i = \frac{X_i}{m_{i\,mole}} \rho N_A$$

so $\boxed{n_i = Y_i \rho N_A}$

The abundance Y is proportional to number density but changes only if the nuclear species gets destroyed or produced. Changes in density are factored out.

$$\boxed{Y = \frac{X}{m_{mole}}}$$

Unit: in principle mole/g

So abundance Y is the number of moles of a species per gram matter (sometimes called mole fraction BUT does not sum to 1 !!!)

Common approximation: $m_{mole} = A$ With nuclear mass number A

- Neglects nuclear binding energy
- Neglects electron masses (and binding energies)
- ONLY VALID IN CGS UNITS

With that:

$$\boxed{Y = \frac{X}{A}}$$

And typically Y is considered unit less

$n_i = Y_i \rho N_A$ Still works out unit wise as $N_A=1/m_u$ in CGS units

Some useful quantities and relations

of course $\sum_i X_i = 1$ but, as Y=X/A < X $\sum_i Y_i < 1$

Abundance is not a fraction !

- Mean molecular weight μ_i

  = average mass number = $\frac{\sum_i A_i Y_i}{\sum_i Y_i} = \frac{1}{\sum_i Y_i}$ or $\boxed{\mu_i = \frac{1}{\sum_i Y_i}}$

- Electron Abundance Y_e

  As matter is electrically neutral, for each nucleus with charge number Z there are Z electrons:

$$Y_e = \sum_i Z_i Y_i$$

and as with nuclei,electron density $n_e = \rho \mathrm{N}_\mathrm{A} Y_e$

can also write: $Y_e = \frac{\sum_i Z_i Y_i}{\sum_i A_i Y_i}$

prop. to number of protons

prop. to number of nucleons

So Y_e is ratio of protons to nucleons in sample
(counting all protons including the ones contained in nuclei
- not just free protons as described by the "proton abundance")

some special cases:

For 100% hydrogen: $Y_e=1$

For equal number of protons and neutrons (N=Z nuclei): $Y_e=0.5$

For pure neutron gas: $Y_e=0$