Simulation of Beam and Plasma Systems
Homework 7

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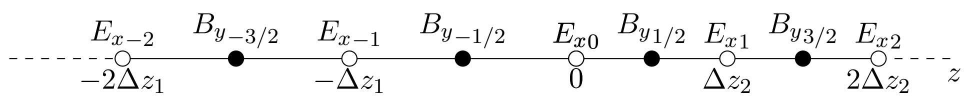
Problem 1 - Reflection between two grids with different resolution

We consider a grid which has a different resolution for $z > 0$ and $z < 0$:



In the area $z < 0$, the discretized Maxwell equations are:

$$\frac{B_{y\,\ell+1/2}^{\,n+1/2} - B_{y\,\ell+1/2}^{\,n-1/2}}{\Delta t} = -\left(\frac{E_{x\,\ell+1}^{\,n} - E_{x\,\ell}^{\,n}}{\Delta z_1}\right) \qquad \text{for any } \ell < 0 \tag{1}$$

$$\frac{E_{x\,\ell}^{\,n+1} - E_{x\,\ell}^{\,n}}{c^2\Delta t} = -\left(\frac{B_{y\,\ell+1/2}^{\,n+1/2} - B_{y\,\ell-1/2}^{\,n+1/2}}{\Delta z_1}\right) \qquad \text{for any } \ell < 0 \tag{2}$$

and in the area $z > 0$, the discretized Maxwell equations are:

$$\frac{B_{y\,\ell+1/2}^{\,n+1/2} - B_{y\,\ell+1/2}^{\,n-1/2}}{\Delta t} = -\left(\frac{E_{x\,\ell+1}^{\,n} - E_{x\,\ell}^{\,n}}{\Delta z_2}\right) \qquad \text{for any } \ell \geq 0 \tag{3}$$

$$\frac{E_{x\,\ell}^{\,n+1} - E_{x\,\ell}^{\,n}}{c^2\Delta t} = -\left(\frac{B_{y\,\ell+1/2}^{\,n+1/2} - B_{y\,\ell-1/2}^{\,n+1/2}}{\Delta z_2}\right) \qquad \text{for any } \ell > 0 \tag{4}$$

and finally, at the boundary between the two domains ($\ell = 0$) the equation for the field E_x is:

$$\frac{E_{x\,0}^{\,n+1} - E_{x\,0}^{\,n}}{c^2\Delta t} = -\left(\frac{B_{y\,1/2}^{\,n+1/2} - B_{y\,-1/2}^{\,n+1/2}}{\frac{\Delta z_1 + \Delta z_2}{2}}\right) \qquad \text{for any } \ell > 0 \tag{5}$$

We will look for a solutions of these equations, in the form:

$$E_{x\,\ell}^{\,n} = E e^{ik_1\Delta z_1 \ell - i\omega n\Delta t} - R E e^{-ik_1\Delta z_1 \ell - i\omega n\Delta t} \qquad \text{for any } \ell \leq 0 \tag{6}$$

$$E_{x\,\ell}^{\,n} = T E e^{ik_2\Delta z_2 \ell - i\omega n\Delta t} \qquad \text{for any } \ell \geq 0 \tag{7}$$

$$B_{y\,\ell+1/2}^{\,n+1/2} = \frac{E}{c} e^{ik_1\Delta z_1(\ell+1/2) - i\omega(n+1/2)\Delta t} + R\frac{E}{c} e^{-ik_1\Delta z_1(\ell+1/2) - i\omega(n+1/2)\Delta t} \qquad \text{for any } \ell < 0 \tag{8}$$

$$B_{y\,\ell+1/2}^{\,n+1/2} = T\frac{E}{c} e^{ik_2\Delta z_2(\ell+1/2) - i\omega(n+1/2)\Delta t} \qquad \text{for any } \ell \geq 0 \tag{9}$$

where R and T are unknown complex coefficients.

a) Show that the expressions in equations (6) to (9) are solutions of the discrete Maxwell equations for $z < 0$ and $z > 0$ (i.e. equations (1) to (4)), provided that:

$$\frac{1}{c\Delta t}\sin\left(\frac{\omega\Delta t}{2}\right) = \frac{1}{\Delta z_1}\sin\left(\frac{k_1\Delta z_1}{2}\right) \qquad \text{and} \qquad \frac{1}{c\Delta t}\sin\left(\frac{\omega\Delta t}{2}\right) = \frac{1}{\Delta z_2}\sin\left(\frac{k_2\Delta z_2}{2}\right) \tag{10}$$

b) From the fact that equation (6) and (7) are both valid for $\ell = 0$, deduce that

$$R + T = 1$$

c) By inserting the expressions (7) to (9) into (5), and by using the relation $R + T = 1$, show we obtain the following equation for R:

$$(e^{-i\omega\Delta t/2} - e^{i\omega\Delta t/2})(1 - R) = -\beta[(1 - R)e^{ik_2\Delta z_2/2} - e^{-ik_1\Delta z_1/2} - Re^{ik_1\Delta z_1/2}]$$

with $\beta = \frac{2c\Delta t}{\Delta z_1 + \Delta z_2}$. (Note that, from equations (7) to (9) and $R+T = 1$, one has $E_{x0}^{n+1} = E(1-R)e^{-i\omega\Delta t}$, $E_{x0}^{n} = E(1-R)$, $B_{y1/2}^{n+1/2} = \frac{E}{c}(1-R)e^{ik_2\Delta z_2/2-i\omega\Delta t/2}$, $B_{y-1/2}^{n+1/2} = \frac{E}{c}e^{ik_1\Delta z_1/2-i\omega\Delta t/2} + \frac{RE}{c}e^{-ik_1\Delta z_1/2-i\omega\Delta t/2}$.)

Conclude that the reflection coefficient $|R|$ is

$$|R| = \left|\frac{(e^{-i\omega\Delta t/2} - e^{i\omega\Delta t/2}) + \beta(e^{ik_2\Delta z_2/2} - e^{-ik_1\Delta z_1/2})}{(e^{-i\omega\Delta t/2} - e^{i\omega\Delta t/2}) + \beta(e^{ik_2\Delta z_2/2} + e^{ik_1\Delta z_1/2})}\right|$$

d) We wish to plot $|R|$ as a function ω. In order to do so, we first need to express $e^{ik_1\Delta z_1/2}$ and $e^{ik_2\Delta z_2/2}$ as a function of ω.

From equation (10), using $\sin(k_1\Delta z_1/2) = \frac{e^{ik_1\Delta z_1/2} - e^{-ik_1\Delta z_1/2}}{2i}$, show that $e^{ik_1 z_1/2}$ satisfies the equation

$$(e^{ik_1\Delta z_1/2})^2 - 2i\frac{\Delta z_1}{c\Delta t}\sin\left(\frac{\omega\Delta t}{2}\right)e^{ik_1\Delta z_1/2} - 1 = 0$$

and by remarking that this is a second-order polynomial equation, show that the expression of $e^{ik_1\Delta z_1/2}$ as a function of ω is:

$$e^{ik_1\Delta z_1/2} = i\frac{\Delta z_1}{c\Delta t}\sin\left(\frac{\omega\Delta t}{2}\right) + (1 - \left(\frac{\Delta z_1}{c\Delta t}\right)^2\sin^2\left(\frac{\omega\Delta t}{2}\right))^{1/2} \tag{11}$$

(where the sign + is chosen from knowing the solution for $\Delta z_1 = c\Delta t$)

e) Download the file from

`https://raw.githubusercontent.com/RemiLehe/uspas_exercise/master/plot_reflection.py`. This script plots $|R|$ as a function of ω (and of $\lambda/\Delta z_2$, where λ is the wavelength of the incident wave), using the above formula. In the case of the script, the resolution of the second grid is 5 times coarser than that of the first grid.

Run the script and interpret the evolution of the reflection coefficient: what happens when the wave is not resolved anymore by the second grid (i.e. when $\lambda < 3\Delta z_2$)?

f) What is the value of the coefficient $|R|$ when $\Delta z_2 = \Delta z_1$? Is this to be expected?

g) Download the file from

`https://raw.githubusercontent.com/RemiLehe/uspas_exercise/master/em_pic_1d_mr.py`.

The script simulates the propagation of electromagnetic fields in 1D on a succession of 2 grids that can be set a different resolutions and are linked by an algorithm selected by the user. The code prints the coefficients of reflection and transmission, and can perform scans on the wavelength and the method used to connect the grids.

Set l_scan=0, l_method=1, Nz=300, lw=25., and run. Repeat for lw=15., lw=10. and lw=5. Repeat for method=2 and method=3. Observe how for all the tested methods, the reflection is total for wavelengths that are below the cutoff of the coarser grid.

Then, set l_scan=1, Nz=500, and run the script. After some time, the run concludes and you have a file coef_refl.pdf that you may open. The plot shows the coefficient of reflection versus incident wavelength, for three tested methods to connect the two grids, from analysis (solid curves, for methods 1 and 2), and from simulations (crosses, circles and x for methods 1, 2 and 3). Observe that the simulations confirm the theoretical predictions. Also observe that the different algorithms to connect the grids result in widely different coefficient of reflection at long wavelength.

Finally, repeat the scan with Nz=100. What happened to the agreement between theory and simulations? Can you explain why?

Problem 2 - Diagnostics in Simulations

Download the file `https://people.nscl.msu.edu/~lund/uspas/sbp_2018/lec_intro/05.diag_examples/xy-quad-mag-mg-diag.py` from the course website.

The script in the directory is setup with some basic parameters and diagnostics:

- rms matched waterbag distribution with zero centroid.
- 100 steps per lattice period axial advance, with transverse symmetry options on the field solver off.
- Diagnostic setup for x-y, x-x', y-y' and x'-y' snapshot distribution projection plots and beam centroid (X,Y), envelope (r_x,r_y), and emittance (ε_x, ε_y) history evolution plots.

Use this script and diagnostics to carry out the following:

a) Run the simulation first for a Waterbag initial distribution. Repeat runs for Thermal, Semi-Gaussian, and KV initial distributions. Comment on the differences and similarities between the results over 10 lattice periods. Would you expect to see much differences in the snapshot and history diagnostics? Would differences of this type be observable in laboratory diagnostics?

b) If the intial KV distribution is an exact "equilibrium" of Vlasov's equation that repeats exactly every lattice period, why is the distribution being observed to evolve? Should one expect the beam edge to be resolved?

c) Run the initial KV distribution case with 30 lattice periods rather than 10 lattice periods and comment on emittance growth and phase-space distortions that develop by the end of the advance. Is the beam stable or unstable? If the KV beam could be initialized in the laboratory, would we expect to see this evolution or not?

d) Run the original simulation with an initial (10 lattice period advance) waterbag distribution including non-zero initial centroid amplitudes of $X = Y = 2$ mm and $X' = 0 = Y'$ and compare results to the corresponding simulation with zero centroid amplitude. Comment on any differences in centroid history and emittance evolution. Should the emittance evolution be expected to be the same?

Problem 3 - Coherent Synchrotron Radiation

Note: Begin with your linac and bunch compression beamline that you built yesterday and today.

a) You need to adjust how elegant is modeling the CSR. We have been using the 'Steady State' CSR model. Now let's change the model. Go to control and find the `alter_elements` command that says `item = STEADY_STATE, name = BEND?`. Change `Value` from 1 to 0 and save changes. Now when we run the simulation elegant will use a more sophisticated calculation of CSR that includes transient effects.

   1 Based on what you've learned what do you think will happen to the emittance at the end of the beam line? Will it be higher, lower, or unchanged?

   2 Now rerun the simulation. Look at `enx` at the end of the beam line again. What happened? Can you explain your observation?

b) Now let us turn on one more option in elegants CSR routines. Currently the electrons only experience CSR while they are in a dipole. However, while electrons will not produce radiation outside the dipole, any radiation they produced in an upstream dipole could continue to propagate with the electrons out into a drift space. Let us turn on `csrdrifts` to allow this effect. Go to `Control` and find the `alter_elements` command with `item = CSR, name = D_FODO` and change `Value` from 0 to 1. Now rerun the simulation.

   1 Now look at the `hist_chicane_end` and `hist_final` plots and plot **deltaFrequency** vs. **delta** in each one. Are they different? Why is this?

   2 Look at **tFrequency** vs. **t** in `hist_chicane_end` and hist_final. Are they different? Why or why not?

   3 Look at `enx`, what is the value of `enx`? Has it changed? Why would this be?