Simulation of Beam and Plasma Systems
Homework 3

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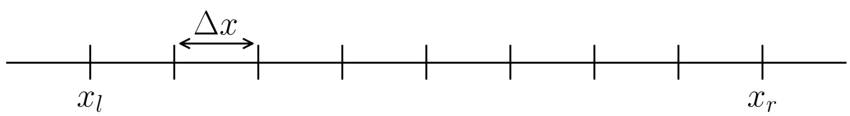
Problem 1 - Numerical field solutions 1

In 1D electrostatics, on a uniform grid



the Poisson equation is

$$\frac{\partial^2}{\partial x^2}\phi = \frac{-\rho}{\epsilon_0},$$

where

$$\begin{aligned}\phi &= \phi(x)\\ \rho &= \rho(x)\end{aligned}$$

and

$$\begin{aligned} x_j &= x_l + \Delta x * j, \qquad \Delta x = \frac{x_r - x_l}{n_x} \\ j &= 0, 1, 2, \cdots, n_x, \qquad n_x + 1 = \text{number of grid points.}\end{aligned}$$

In class we defined a discrete Fourier transform for the 1D potential:

$$\tilde{\phi}_n = \Delta x \sum_{j=0}^{n_x} e^{\frac{i2\pi nj}{n_x+1}} \phi_j, \qquad n = 0, 1, 2, \cdots, n_x$$

a) Show that the transform can be inverted exactly with

$$\phi_j = \frac{1}{(n_x+1)\Delta x} \sum_{n=0}^{n_x} e^{\frac{-i2\pi nj}{n_x+1}} \tilde{\phi}_n$$

b) Define a gridded electric field

$$E_{xj} = \frac{-(\phi_{j+1} - \phi_{j-1})}{2\Delta x}$$

and show that

$$\tilde{E}_{xn} = i\hat{\kappa}_n \tilde{\phi}_n$$

where

$$\hat{\kappa}_n = k_n \left[\frac{\sin(k_n \Delta x)}{k_n \Delta x} \right]$$

and

$$k_n = \frac{2\pi n}{(n_x + 1)\Delta x}.$$

Here

$$\tilde{E}_{xn} = \Delta x \sum_{j=0}^{n_x} e^{\frac{i2\pi nj}{n_x+1}} E_{xj}.$$

Problem 2 - Numerical field solutions 2

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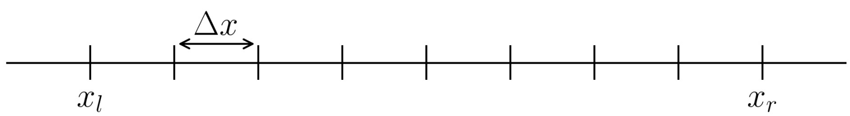

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a) In class we setup an explicit matrix discretization of this problem for Dirichlet boundary conditions with $\phi(x_l) = V_l$ and $\phi(x_r) = V_l$ where source terms were isolated on the right-hand side of the resulting matrix equation. Repeat this construction using

$$\left.\frac{\partial^2 f(x)}{\partial x^2}\right|_j = \frac{f_{j+1} - 2f_j + f_{j-1}}{\delta x^2} + \mathcal{O}(\Delta x^2)$$

for Neumann boundary conditions with

$$\left.\frac{\partial \phi}{\partial x}\right|_{x_l} = G_l.$$

b) Could we also specify

$$\left.\frac{\partial \phi}{\partial x}\right|_{x_r} = G_r$$

and have a solution? Why?

Problem 3 - Graphical User Interfaces (GUIs)

a) It is a well-recognized rule of thumb for user interfaces, that one should only support the 80% most common use cases for the community of users. Do you agree or not? Write 1 or 2 paragraphs to explain your answer, considering both software sustainability and ease of use.

b) Use Sirepo/elegant at https://uspas-sirepo.radiasoft.org to propagate a charged particle beam through a drift length. Specify a 100 MeV electron beam with 1 nC of charge. Allow it to drift for 100 m. Vary the energy, the charge and the drift length (by factors of 2 or 3 or as much as 10). Generate relevant plots that show how the beam dynamics is changing. Write a sentence or two for each plot, or each logically connected set of plots, in order to clearly explain what is being shown and how the parameter variations are changing the beam dynamics.