Simulation of Beam and Plasma Systems
Homework 2

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Problem 1 - Moment equations and conservation constraints

The non-relativistic Vlasov-equation is:

$$\left\{ \frac{\partial}{\partial t} + \boldsymbol{v} \cdot \frac{\partial}{\partial \boldsymbol{x}} + \frac{q}{m} \left[\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} \right] \cdot \frac{\partial}{\partial \boldsymbol{v}} \right\} f(\boldsymbol{x}, \boldsymbol{v}, t) = 0$$

Define a fluid density n and a fluid flow velocity $\mathbf{V}$ by

$$\begin{aligned} n(\boldsymbol{x}, t) &= \int d^3v\, f(\boldsymbol{x}, \boldsymbol{v}, t) \\ n(\boldsymbol{x}, t)\mathbf{V}(\boldsymbol{x}, t) &= \int d^3v\, \boldsymbol{v}\, f(\boldsymbol{x}, \boldsymbol{v}, t) \end{aligned}$$

a) Operate on the Vlasov equation with

$$\int d^3v \cdots$$

to derive the continuity equation:

$$\frac{\partial}{\partial t} n(\boldsymbol{x}, t) + \frac{\partial}{\partial \boldsymbol{x}} \cdot \left[n(\boldsymbol{x}, t)\mathbf{V}(\boldsymbol{x}, t) \right] = 0$$

b) Can the continuity equation be solved by <u>itself</u> if you specify the intial density field $n(\boldsymbol{x}, t = 0)$? Why?

c) Operate on the Vlasov equation with

$$\int d^3v\, \boldsymbol{v} \cdots$$

to derive the fluid force equation.

$$\begin{aligned} \frac{\partial}{\partial t}(n\mathbf{V}) + \nabla \cdot (n \langle \boldsymbol{v}\boldsymbol{v} \rangle_v) &= \frac{q}{m} n \left(\boldsymbol{E} + \mathbf{V} \times \boldsymbol{B} \right) \\ \text{with } \langle \boldsymbol{v}\boldsymbol{v} \rangle_v &\equiv \frac{\int d^3v\, \boldsymbol{v}\boldsymbol{v} f}{\int d^3v\, f} \end{aligned}$$

Defining a pressure tensor as

$$\begin{aligned} \underline{\underline{\mathrm{P}}} &= m \int d^3v \, (\boldsymbol{v} - \mathbf{V})(\boldsymbol{v} - \mathbf{V}) f(\boldsymbol{x}, \boldsymbol{v}, t) \\ &= mn \langle \boldsymbol{v}\boldsymbol{v} \rangle_v - mn\mathbf{V}\mathbf{V}, \end{aligned}$$

the fluid force equation can be expressed as

$$\frac{\partial}{\partial t}\mathbf{V} + \mathbf{V} \cdot \frac{\partial}{\partial \boldsymbol{x}}\mathbf{V} = \frac{q}{m}(\boldsymbol{E} + \mathbf{V} \times \boldsymbol{B}) - \frac{1}{mn}\frac{\partial}{\partial \boldsymbol{x}} \cdot \underline{\underline{\mathrm{P}}}.$$

This form is often used in fluid/plasma analysis.

d) If the continuity and force equation derived in parts a) and c) are analyzed, can they be solved in principle if you specify the initial density field $n(\boldsymbol{x}, t = 0)$ and the velocity field $\mathbf{V}(\boldsymbol{x}, t = 0)$? Why? Does the answer change if we assume a cold initial beam with $\underline{\underline{\mathrm{P}}} = 0$? Why?

e) Let $G(f)$ be some smooth, differentiable function of f, satisfying $G(f \to 0) = 0$. Show that

$$\int d^3x \int d^3v \, G(f) = \text{const.}$$

This so-called "generalized entropy" measure with G specified can be used to check Vlasov simulations. For example:

$$\begin{aligned} G(f) = f: &\quad \int d^3x \int d^3v \, f = \text{const.} \to \text{charge conservation} \\ G(f) = f^2: &\quad \int d^3x \int d^3v \, f^2 = \text{const.} \to \text{“enstropy” conservation} \\ G(f) = f \ln f: &\quad \int d^3x \int d^3v \, f \ln f = \text{const.} \to \text{entropy conservation} \end{aligned}$$

Problem 2 - Installing your own python module

You will now set up the solver classes from yesterday and today as a python module and install it on your system.

- Make sure that you have separate python files for both the `EulerSolver` and the `RKSolver` classes (e.g. `euler_solver.py` and `rk_solver.py`).
- Create the appropriate folder structure, `setup.py`, and `__init__.py` files to install a module called "uspas_solvers" (see class notes). i.e. in your module folder, create a `setup.py` and a subfolder `uspas_solvers`. Put the two solver files (`euler_solver.py` and `rk_solver.py`) inside this subfolder and create the `__init__.py` file inside the subfolder as well.
- You are now ready to install your module using pip.
- Open up ipython and import your module (be careful that you are not opening ipython from within the module folder, otherwise you might get a false positive).
- To receive credit on this problem, find Remi or Daniel and show them the working import of the module.

Problem 3 - Version Control

a) In the Computer Lab this afternoon, you accomplished the following:

- fork the rsbeams repo to your own GitHub account
- clone this forked repo to your laptop or desktop
- document each of the following with an issue:
  - run the existing tests
  - create a branch
    - create a new example, based on one of the existing tests
    - merge the branch back into 'master'

Repeat the exercise independently (but you don't have to rerun the tests). Write a brief description of what you did, including the URL of your GitHub repository, the name of your branch, links to the issues you created, which test you chose to work from, and a link to your new Python example file on GitHub.

b) During today's lecture, several links were provided to online git documentation. Follow one of those links (your choice which one) and read the documentation.

Then go to StackOverflow (`https://stackoverflow.com`) and enter the following into the search box: `"git your_command_goes_here"`

The search results will let you know how this git command is being used, and problems that developers are having with it.

Write a single paragraph about how this git command can be useful and/or problematic.