Simulation of Beam and Plasma Systems
Homework 4

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Problem 1 - Python script for 1D EM solver in vacuum

In the 1D case and in vacuum ($\boldsymbol{j} = \mathbf{0}$), the discretized Maxwell equations for E_x and B_y are:

$$\frac{B_{y}{}_{k+1/2}^{n+1/2} - B_{y}{}_{k+1/2}^{n-1/2}}{\Delta t} = -\left(\frac{E_{x}{}_{k+1}^{n} - E_{x}{}_{k}^{n}}{\Delta z}\right)$$

$$\frac{E_{x}{}_{k}^{n+1} - E_{x}{}_{k}^{n}}{\Delta t} = -c^2\left(\frac{B_{y}{}_{k+1/2}^{n+1/2} - B_{y}{}_{k-1/2}^{n+1/2}}{\Delta z}\right)$$

Download the file `em_pic_1d.py` from:

`http://raw.githubusercontent.com/RemiLehe/uspas_exercise/master/em_pic_1d.py`

It is an incomplete implementation of a field solver, based on these equations (with periodic boundaries)

a) Read the code and try to understand it (you are encouraged to run the code to see what it produces). Answer the following questions:

- What does `np.zeros` do? What type of object are `self.Ex` and `self.By`?
- In the method `plot_fields`, explain why we use:
  - `self.z` in the line
    `plt.plot( self.z, self.Ex[1:-1],'o-')`
  - but `self.z+0.5*self.dz` in the line
    `plt.plot( self.z+0.5*self.dz, self.By[1:-1], 'o-' ).`
- With the current version of the code, does the initial pulse propagate with time? Why?

b) Rewrite the above Maxwell equations in the form $B_{y}{}_{k+1/2}^{n+1/2} = ...$ and $E_{x}{}_{k}^{n+1} = ...$ These are the update equations that need to be implemented in `em_pic_1d.py`.

In the code, find the lines that are tagged with the text `ASSIGNEMENT` and write the correct update equations, using Python syntax. Note that, as explained in the text of the code, at a given iteration n, the element `Ex[k]` represents $E_{x}{}_{k}^{n}$ and the element `By[k]` represents $B_{y}{}_{k+1/2}^{n-1/2}$ (thus be mindful of using the correct indices for the array `Ex` and `By`).

c) In order to validate your implementation, check that the pulse propagates to the right. At what speed should it propagate physically? From the images, can you evaluate at which speed it propagated?

Problem 2 - Courant limit in 1D

For the Yee scheme in 1D, the discrete propagation equation is:

$$\frac{1}{c^2}\,\frac{E_{x\,\ell}^{\,n+1} - 2E_{x\,\ell}^{\,n} + E_{x\,\ell}^{\,n-1}}{\Delta t^2} = \frac{E_{x\,\ell+1}^{\,n} - 2E_{x\,\ell}^{\,n} + E_{x\,\ell-1}^{\,n}}{\Delta z^2}$$

As explained in class, for $c\Delta t > \Delta z$ this equation has unstable solutions.

a) When assuming that E_x is of the form $E_0 e^{ikz-i\omega t}$, what is the dispersion relation that corresponds to the above discrete propagation equation? (See today's lecture slides.)

b) In the case $c\Delta t > \Delta z$, what is the maximum value of k (between 0 and $\pi/\Delta z$) for which there is a real solution ω to the dispersion equation?

c) For values of k above this threshold, we will assume that E_x is of the form:

$$E_{x\,\ell'}^{\,n} = (-1)^n E_0 e^{ik\,\ell'\Delta z + n\Gamma\Delta t}$$

By inserting this ansatz into the discrete propagation equation, find the corresponding dispersion relation which links Γ and k. From this relation, extract the expression of Γ as a function of k (use the function argch, which is sometimes also denoted as $\cosh^{-1}$).

d) For which value of the wavevector k does the instability growth rate reach its highest value?

e) Download the script `em_unstable.py` from:

`https://raw.githubusercontent.com/RemiLehe/uspas_exercise/master/unstable.py`

As can be seen from the section `if __name__ =='__main__':`, this script is a slight modification of the script `em_pic_1d.py` from the previous assignement, where Δt has been set to:

$$\Delta t = 1.01\frac{\Delta z}{c}$$

Run the script and look at plots of the fields in the folder `diagnostics`. Is the observed evolution consistent with the answers to the previous questions? Why?

Problem 3 - Slice Energy Spread

Will be handed out by David.