USPAS – *Simulation of Beam and Plasma Systems*

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Lecture: Electron Bunch Compression

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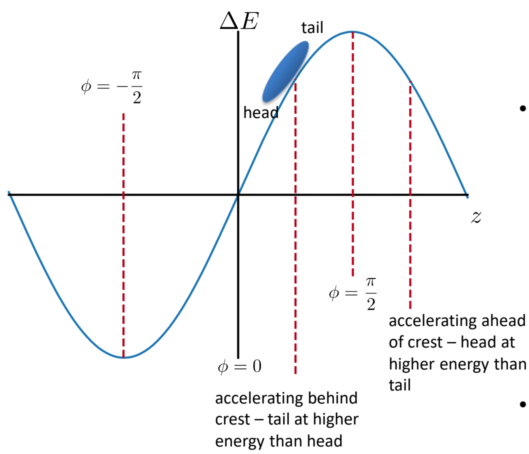
Goals

- Brief review of rf cavity phase
  - how it affects longitudinal phase space of accelerated beam
  - need to understand longitudinal phase space conventions of Elegant
- Brief review of linear optics and R-matrix
- Brief discussion of simple chicane concept
- Why do we need electron bunch compression?
  - increase luminosity in a collider
  - increase peak current in a free-electron laser

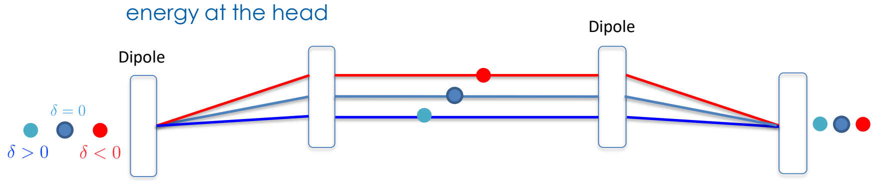
rf Cavity



- Following elegant, use a sine convention for the rf wave
  $$E_f(z) = E_0 + eV sin(k_{rf} z + \phi)$$
- Energy gain will be determined by voltage V and phase ϕ
  - Acceleration
    $$0 \leq \phi \leq \pi$$
  - Zero-crossing
    $$\phi = 0, \pi$$
  - Deceleration
    $$-\pi \leq \phi \leq 0$$
- Convention for longitudinal variable z (i.e. t) in elegant:
  $$z_{head} < z_{tail}$$

Chicane

- For relativistic particles, energy differences are not velocity differences
  - all particles are moving with velocities very close to c
- Hence, magnetic fields can be used for longitudinal compression
  - Particles with different energies will take different paths through a dipole
  - If we correlate particle energy with longitudinal position we can use a sequence of dipoles to perform compression
  - We need higher energy particles at the back of the bunch and lower energy at the head



Nominal Bending Angle:

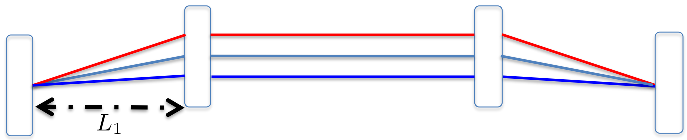
$$\theta_0 = \frac{l_{dip}}{\rho} = \frac{eB}{p_0} l_{dip}$$

Bending Angle for off-energy particles:

$$\theta(\delta) = \frac{\theta_0}{1+\delta}$$

- Higher-energy particles are bent less
  - they take a shorter path
- Lower-energy particles are bent more
  - they take a longer path
- The high-energy tail shifts forward; the low-energy head shifts back → *Compression*!

Bunch Compression



Path distance for one leg $$s_{leg} = \frac{L_1}{cos(\theta)}$$

Bending angle for off-energy particles:
$$\theta(\delta) = \frac{\theta_0}{1+\delta}$$

The difference in path length for an on energy and off-energy particle will be:
$$\Delta s = \frac{L_1}{cos(\theta)} - \frac{L_1}{cos(\theta_0)}$$

Using this expression and the bending angle, we get a first-order approximation for the energy difference corresponding to a given path length difference:
$$\Delta s \approx -2L_1\theta^2\delta$$

Transport Matrix Notation (R-Matrix)

Express particles in term of 6D phase space coordinates:
$$\vec{x} = \begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta \end{pmatrix}$$

For linear optics, we can use matrices to represent components of an accelerator beamline. This R-matrix transforms the phase space state from initial to final:

$\vec{x}_0$ [accelerator beamline $R(s_0 \rightarrow s_1)$] $\vec{x}_1$

The R matrix for a beamline is the concatenation of R matrices for individual elements:
$$\vec{x}_1 = R_i ... R_2 R_1 \vec{x}_0$$
$$\Downarrow$$
$$\vec{x}_1 = R\vec{x}_0$$

For nonlinear effects, like chromatic aberrations or emittance growth, one can generalize the R-matrix to a transfer map, which often takes the form of a Taylor series up to arbitrary order:
$$x_i(1) = \sum_{j=1}^{6} R_{ij}x_j(0) + \sum_{j=1}^{6}\sum_{k=1}^{6} T_{ijk}x_j(0)x_k(0) + \sum_{j=1}^{6}\sum_{k=1}^{6}\sum_{l=1}^{6} U_{ijkl}x_j(0)x_k(0)x_l(0) + ...$$

An Ideal Chicane

- The R-Matrix for an ideal chicane has the special form shown below
  - no transverse coupling
  - no acceleration
  - bending only in the horizontal plane

$$R = \begin{pmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & R_{34} & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- 4 elements must be zero to enforce:
  - no horizontal dispersion
  - no dependence on horizontal position

$$R_{16} = R_{26} = R_{51} = R_{52} = 0$$

- This leaves only one component to determine longitudinal position:

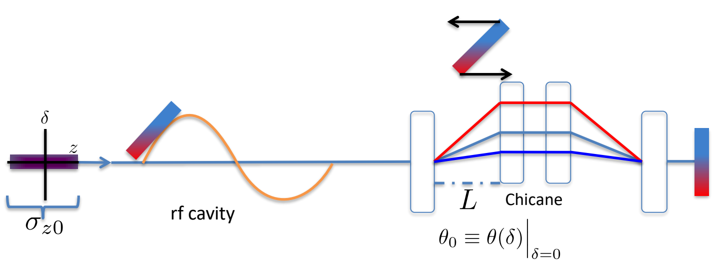
$$z_f = (1 + R_{56}) z_i$$

What is R_{56} ? $R_{56} \equiv \frac{\Delta z}{\delta}$

Using our calculation on slide #5, we have:

$$R_{56} = -2 L_1 \theta^2$$

Putting it all together – Linac and Chicane



Use acceleration off-crest in the rf cavity to correlate energy with longitudinal position.

Referred to as energy chirp:

$$h \equiv \frac{dE}{E_0 dz} \approx \frac{k_{rf} eV cos(\phi)}{E_0 + eV sin(\phi)}$$

Calculate the expected compression (to first-order) based on h and R_{56}:

$$C = \frac{\sigma_{z0}}{\sigma_{zf}} = \frac{1}{|1 + hR_{56}|}$$

Wrap up:

- Any questions?
- For the computer lab, we will construct a linac + chicane + focusing optics to demonstrate a basic bunch compressor.
- This is more involved than previous Sirepo/elegant exercises, so we'll start now