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USPAS
June 13-24, 2011
Melville, NY

Injectors and longitudinal physics -- I

1. Fluid equations
2. Child-Langmuir Law
(Reiser 2.5.2, Appendix 1)
3. Pierce electrodes
4. Transients in injectors
5. Injector choices

I) FLUID EQUATIONS

START WITH VLASOV EQUATION FOR $f(x, p, t)$

$$\frac{\partial f(x, p, t)}{\partial t} + \underline{\dot{x}} \cdot \frac{\partial f(x, p, t)}{\partial \underline{x}} + \underline{\dot{p}} \cdot \frac{\partial f(x, p, t)}{\partial \underline{p}} = 0$$

$$\text{HERE } \underline{\dot{x}} = \frac{d\underline{x}}{dt} = \frac{\underline{p}}{\gamma m}$$

$$\underline{\dot{p}} = \frac{d\underline{p}}{dt} = q(\underline{E}(x, t) + \frac{\underline{p}}{\gamma m} \times \underline{B}(x, t))$$

$$\gamma^2 = (p/mc)^2 + 1$$

INTEGRATE OVER MOMENTUM AND MULTIPLY BY POWERS OF \underline{p}

a) CONTINUITY EQUATION

$$\int d^3 p \left\{ \frac{\partial f}{\partial t} + \underline{\dot{x}} \cdot \frac{\partial f}{\partial \underline{x}} + (q \underline{E}(x, t) + \frac{q \underline{p}}{\gamma m} \times \underline{B}(x, t)) \cdot \frac{\partial f}{\partial \underline{p}} \right\} = 0 \quad \text{①}$$

$$\text{DEFINE } n(x, t) = \int f(x, p, t) d^3 p$$

$$n(x, t) \underline{v}(x, t) = \int \frac{\underline{p}}{\gamma m} f(x, p, t) d^3 p$$

① FIRST INTEGRAL

$$\int d^3 p \frac{\partial f}{\partial t} = \frac{\partial}{\partial t} \int f d^3 p = \frac{\partial n(x, t)}{\partial t}$$

② SECOND INTEGRAL

$$\begin{aligned} \int d^3 p \underline{\dot{x}} \cdot \frac{\partial f}{\partial \underline{x}} &= \int d^3 p \frac{\underline{p}}{\gamma m} \cdot \frac{\partial f}{\partial \underline{x}} = \frac{\partial}{\partial \underline{x}} \cdot \int d^3 p \frac{\underline{p}}{\gamma m} f d^3 p \\ &= \frac{\partial}{\partial \underline{x}} \cdot n \underline{v} \end{aligned}$$

(3) THIRD INTEGRAL

$$\int d^3p \left(q \underline{E} + \frac{q}{\gamma m} \underline{p} \times \underline{B} \right) \frac{\partial f}{\partial \underline{p}}$$

\downarrow
 $qE f \Big|_{p=-\infty}^{p=\infty} = 0$

$$\int \frac{q}{\gamma m} (p_y B_z - p_z B_y) \frac{\partial f}{\partial p_x} \downarrow p_x \downarrow \frac{1}{\gamma} \downarrow \dots$$

$$\int \frac{q}{\gamma^3 m^3 c^2} (p_y B_z - p_z B_y) p_x$$

$$+ \frac{q}{\gamma^3 m^3 c^2} (p_z B_x - p_x B_z) p_y$$

$$+ \frac{q}{\gamma^3 m^3 c^2} (p_x B_y - p_y B_x) p_z \cdot f d^3p$$

$= 0 !$

$$\int_a^b uv' dx = uv \Big|_a^b - \int_a^b u'v dx$$

$$u = \frac{q}{\gamma m} (p_y B_z - p_z B_y)$$

$$u' = \frac{q}{\gamma^3 m} (p_y B_z - p_z B_y) \frac{\partial \gamma}{\partial p_x}$$

$$v = p_x$$

$$v' = \frac{\partial p_x}{\partial p_x}$$

$$\gamma^2 = \frac{p_x^2 + p_y^2 + p_z^2}{m^2 c^2} + 1$$

$$\Rightarrow 2\gamma \frac{\partial \gamma}{\partial p_x} = \frac{2 p_x}{m^2 c^2}$$

So $\int d^3p \left\{ \frac{\partial f}{\partial t} + \underline{\dot{x}} \cdot \frac{\partial f}{\partial \underline{x}} + \underline{\dot{p}} \cdot \frac{\partial f}{\partial \underline{p}} \right\} = 0$

$$\Rightarrow \left[\frac{\partial n(\underline{x}, t)}{\partial t} + \frac{\partial}{\partial \underline{x}} \cdot n(\underline{x}, t) \underline{v}(\underline{x}, t) = 0 \right]$$

CONTINUITY EQUATION \uparrow $q n(\underline{x}, t) \underline{v}(\underline{x}, t) = \underline{J}(\underline{x}, t)$
 CURRENT DENSITY \uparrow

ALTERNATIVELY $\frac{\partial \rho}{\partial t} + \nabla \cdot \underline{J} = 0$

BALANCED
LUND

b) MOMENTUM EQUATION

(FOR SIMPLICITY: ASSUME NON-RELATIVISTIC)

$$\dot{\underline{x}} = \frac{\underline{p}}{m} \quad \dot{\underline{p}} = q(\underline{E}(x,t) + \frac{\underline{p}}{m} \times \underline{B}(x,t))$$

MULTIPLY BY $\dot{\underline{x}}$ & INTEGRATE OVER MOMENTUM ($\int d^3p$)

$$\int d^3p \left\{ \dot{\underline{x}} \frac{\partial f}{\partial t} + \dot{\underline{x}} \dot{\underline{x}} \cdot \frac{\partial f}{\partial \underline{x}} + \dot{\underline{x}} \left(q\underline{E} + \frac{\underline{p}}{m} \times \underline{B} \right) \frac{\partial f}{\partial \underline{p}} \right\} = 0$$

DEFINE $\underline{P} \equiv m \int d^3p (\underline{x} - \underline{v})(\underline{x} - \underline{v}) f(x, p, t)$

(\underline{P} = pressure tensor)

$$\begin{aligned} &= m \int d^3p \dot{\underline{x}} \dot{\underline{x}} f - 2m \underline{v} \int \dot{\underline{x}} f d^3p + m \underline{v} \underline{v} \int f d^3p \\ &= m \int d^3p \dot{\underline{x}} \dot{\underline{x}} f - m n \underline{v} \underline{v} \end{aligned}$$

① FIRST INTEGRAL:

$$\int d^3p \dot{\underline{x}} \frac{\partial f}{\partial t} = \frac{\partial}{\partial t} \int d^3p \dot{\underline{x}} f = \frac{\partial}{\partial t} n \underline{v}$$

② SECOND INTEGRAL:

$$\begin{aligned} \int d^3p \dot{\underline{x}} \dot{\underline{x}} \cdot \frac{\partial f}{\partial \underline{x}} &= \frac{\partial}{\partial \underline{x}} \cdot \int d^3p \dot{\underline{x}} \dot{\underline{x}} f \\ &= \frac{1}{m} \frac{\partial}{\partial \underline{x}} \cdot \underline{P} + \frac{\partial}{\partial \underline{x}} \cdot n \underline{v} \underline{v} \\ &= \frac{1}{m} \frac{\partial}{\partial \underline{x}} \cdot \underline{P} + \left(\frac{\partial}{\partial \underline{x}} \cdot n \underline{v} \right) \underline{v} + n \underline{v} \cdot \frac{\partial \underline{v}}{\partial \underline{x}} \end{aligned}$$

③ THIRD INTEGRAL:

$$\begin{aligned} &\int d^3p \frac{\underline{p}}{m} \left(q\underline{E} + q \frac{\underline{p}}{m} \times \underline{B} \right) \cdot \frac{\partial f}{\partial \underline{p}} \\ &= \underbrace{f \frac{\underline{p}}{m} (q\underline{E} + q \frac{\underline{p}}{m} \times \underline{B})}_{=0} \Big|_{-\infty}^{\infty} - \int d^3p \left(q\underline{E} + q \frac{\underline{p}}{m} \times \underline{B} \right) f \\ &= -\frac{n q \underline{v}(t)}{m} (q\underline{E} + q \underline{v} \times \underline{B}) \end{aligned}$$

$$\begin{aligned} u &= \frac{\underline{p}}{m} (q\underline{E} + \frac{\underline{p}}{m} \times \underline{B}) \\ u' &= \frac{1}{m} (q\underline{E} + \frac{\underline{p}}{m} \times \underline{B}) \\ v' &= \frac{\partial f}{\partial \underline{p}} \end{aligned}$$

BALANCED
LUND

b) MOMENTUM EQUATION

(FOR SIMPLICITY: ASSUME NON-RELATIVISTIC)

$$\dot{\underline{x}} = \frac{\underline{p}}{m} \quad \dot{\underline{p}} = q(\underline{E}(x,t) + \frac{\underline{p}}{m} \times \underline{B}(x,t))$$

MULTIPLY BY $\dot{\underline{x}}$ & INTEGRATE OVER MOMENTUM ($\int d^3p$)

$$\int d^3p \left\{ \dot{\underline{x}} \frac{\partial f}{\partial t} + \dot{\underline{x}} \dot{\underline{x}} \cdot \frac{\partial f}{\partial \underline{x}} + \dot{\underline{x}} \left(q\underline{E} + \frac{\underline{p}}{m} \times \underline{B} \right) \frac{\partial f}{\partial \underline{p}} \right\} = 0$$

DEFINE $\underline{P} \equiv m \int d^3p (\underline{x} - \underline{v})(\underline{x} - \underline{v}) f(x, p, t)$

(\underline{P} = pressure tensor)

$$\begin{aligned} &= m \int d^3p \dot{\underline{x}} \dot{\underline{x}} f - 2m \underline{v} \int \dot{\underline{x}} f d^3p + m \underline{v} \underline{v} \int f d^3p \\ &= m \int d^3p \dot{\underline{x}} \dot{\underline{x}} f - m n \underline{v} \underline{v} \end{aligned}$$

① FIRST INTEGRAL:

$$\int d^3p \dot{\underline{x}} \frac{\partial f}{\partial t} = \frac{\partial}{\partial t} \int d^3p \dot{\underline{x}} f = \frac{\partial}{\partial t} n \underline{v}$$

② SECOND INTEGRAL:

$$\begin{aligned} \int d^3p \dot{\underline{x}} \dot{\underline{x}} \cdot \frac{\partial f}{\partial \underline{x}} &= \frac{\partial}{\partial \underline{x}} \cdot \int d^3p \dot{\underline{x}} \dot{\underline{x}} f \\ &= \frac{1}{m} \frac{\partial}{\partial \underline{x}} \cdot \underline{P} + \frac{\partial}{\partial \underline{x}} \cdot n \underline{v} \underline{v} \\ &= \frac{1}{m} \frac{\partial}{\partial \underline{x}} \cdot \underline{P} + \left(\frac{\partial}{\partial \underline{x}} \cdot n \underline{v} \right) \underline{v} + n \underline{v} \cdot \frac{\partial \underline{v}}{\partial \underline{x}} \end{aligned}$$

③ THIRD INTEGRAL:

$$\begin{aligned} &\int d^3p \frac{\underline{p}}{m} \left(q\underline{E} + q \frac{\underline{p}}{m} \times \underline{B} \right) \cdot \frac{\partial f}{\partial \underline{p}} \\ &= \underbrace{f \frac{\underline{p}}{m} (q\underline{E} + q \frac{\underline{p}}{m} \times \underline{B})}_{=0} \Big|_{-\infty}^{\infty} - \int d^3p \left(q\underline{E} + q \frac{\underline{p}}{m} \times \underline{B} \right) f \\ &= -\frac{n q \underline{v}(t)}{m} (q\underline{E} + q \underline{v} \times \underline{B}) \end{aligned}$$

$$\begin{aligned} u &= \frac{\underline{p}}{m} (q\underline{E} + \frac{\underline{p}}{m} \times \underline{B}) \\ u' &= \frac{1}{m} (q\underline{E} + \frac{\underline{p}}{m} \times \underline{B}) \\ v &= \frac{\partial f}{\partial \underline{p}} \\ v' &= \frac{\partial f}{\partial \underline{x}} \end{aligned}$$

ADDING THE INTEGRALS TOGETHER:

$$\frac{\partial}{\partial t} n\bar{v} + \left(\frac{\partial}{\partial x} \cdot n\bar{v} \right) \bar{v} + n\bar{v} \cdot \frac{\partial \bar{v}}{\partial x} = n \frac{q}{m} (\underline{E} + \bar{v} \times \underline{B}) - \frac{1}{m} \frac{\partial}{\partial x} \cdot \underline{P}$$

$$n \frac{\partial \bar{v}}{\partial t} + \underbrace{\frac{\partial n}{\partial t} \bar{v} + \left(\frac{\partial}{\partial x} \cdot n\bar{v} \right) \bar{v}}_{= 0 \text{ BY CONTINUITY EQUATION}} + n\bar{v} \cdot \frac{\partial \bar{v}}{\partial x} = \quad \quad \quad "$$

DIVIDING BY n :

$$p(x,t) = m n(x,t)$$

$$\frac{\partial \bar{v}}{\partial t} + \bar{v} \cdot \frac{\partial \bar{v}}{\partial x} = \frac{q}{m} (\underline{E} + \bar{v} \times \underline{B}) - \frac{1}{\rho} \frac{\partial}{\partial x} \cdot \underline{P}$$

↑ MOMENTUM EQUATION ↑

(NON-RELATIVISTIC)

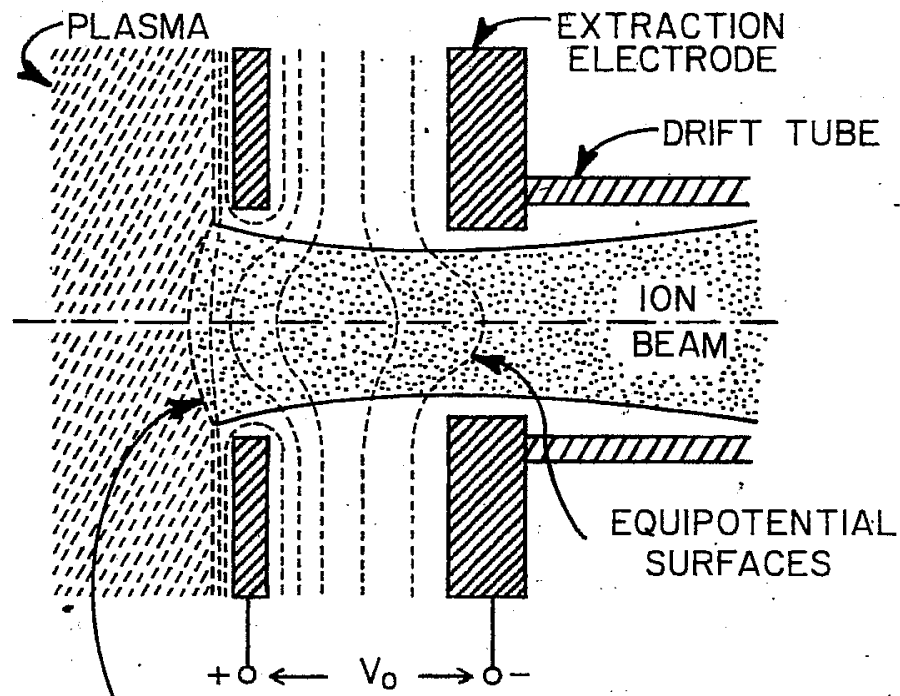
NOTE THAT $\frac{\partial \bar{v}}{\partial t} + \bar{v} \cdot \frac{\partial \bar{v}}{\partial x} = \frac{d}{dt} \bar{v}$ ALONG A TRAJECTORY

$$\Rightarrow \frac{d\bar{v}}{dt} = \frac{q}{m} (\underline{E} + \bar{v} \times \underline{B}) - \frac{1}{\rho} \frac{\partial}{\partial x} \cdot \underline{P}$$

(NON-RELATIVISTIC)

INJECTORS

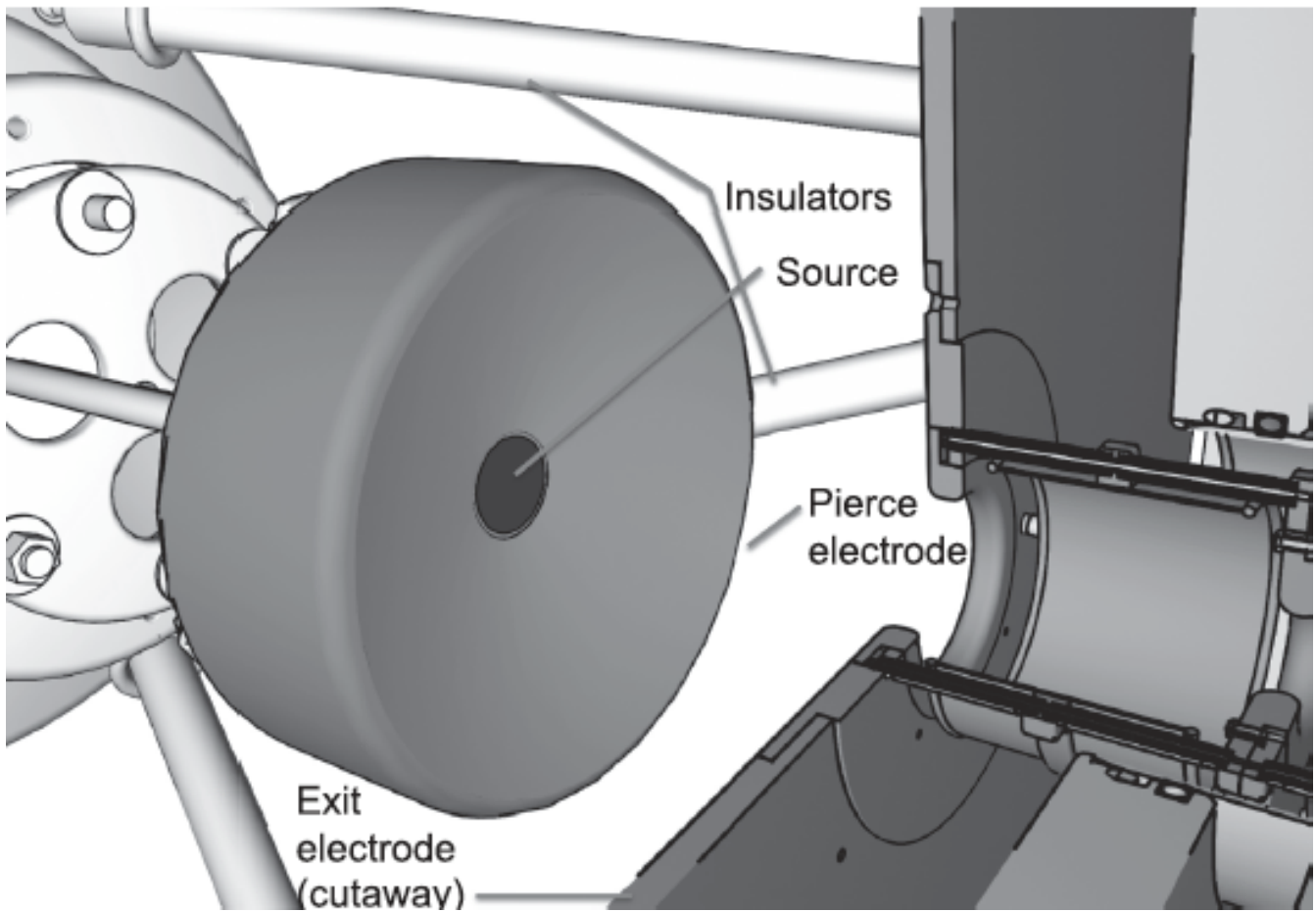
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EMITTING SURFACE
(PLASMA "SHEATH" OR "MENISCUS")
OR "HOT PLATE"

REISER, FIGURE 1.2

- DOPED TUNGSTEN
- ALUMINO-SILICATE

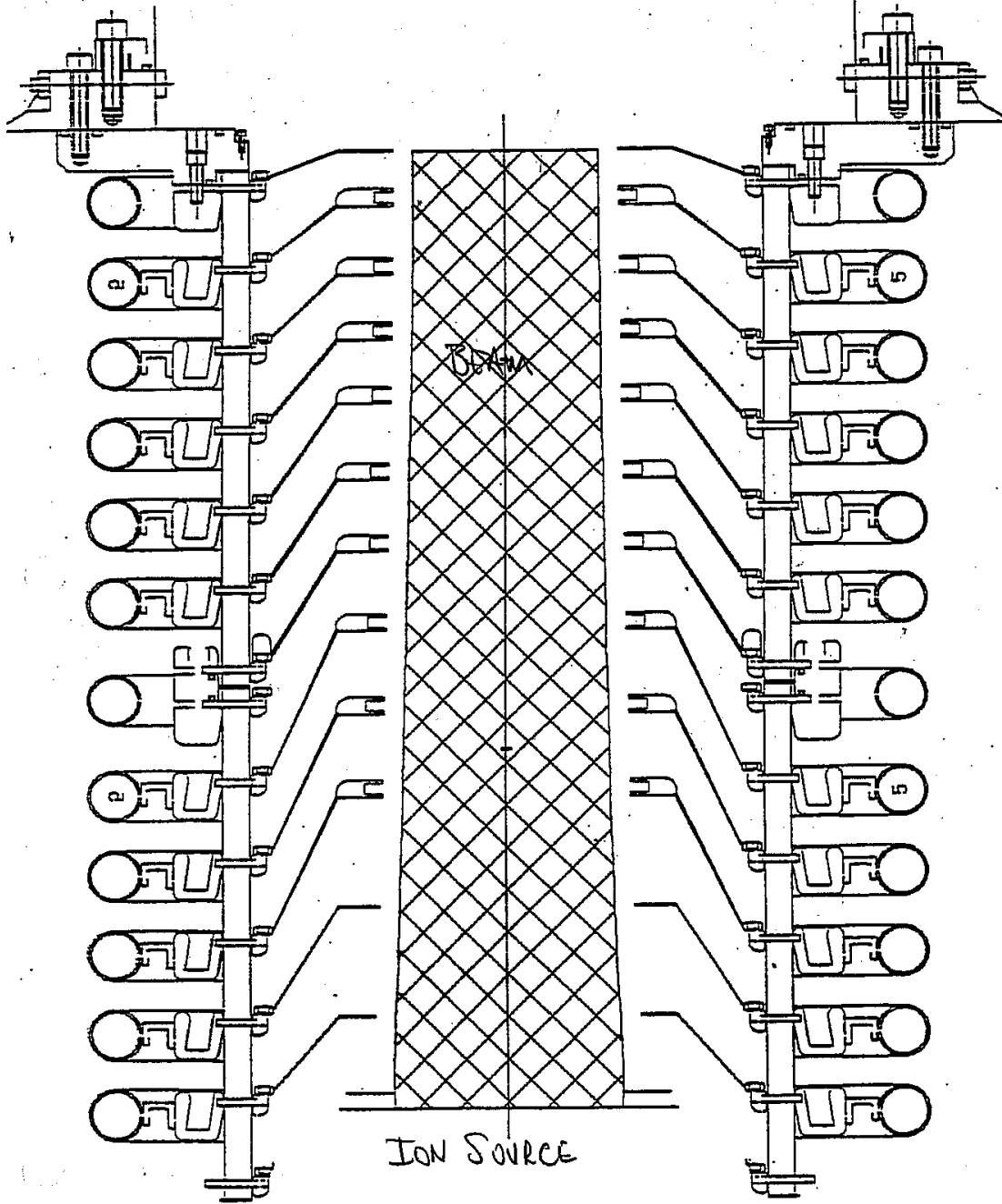


A mechanical drawing of a hot-plate diode used on the NDCX-1 experiment at LBNL. One quarter of the exit electrode is cut away for viewing the source geometry

PIERCE COLUMN

$V \sim z^{4/5}$

$E \sim z^{1/3}$



~~Derive~~ ^{Derive} WE A THE PARAXIAL RAY EQUATION FOR PARTICLES IN AXISYMMETRIC SYSTEMS:

$$r'' + \underbrace{\frac{\gamma'}{\beta^2 \gamma}}_{\text{INERTIAL}} r' + \underbrace{\frac{\gamma''}{2\beta^2 \gamma}}_{E_r} r + \underbrace{\left(\frac{\omega_L}{2\gamma\beta c}\right)^2}_{V_0 B_z - \text{CENTRIFUGAL}} r - \underbrace{\left(\frac{p_0}{\gamma\beta mc}\right)^2}_{\text{CENTRIFUGAL}} \frac{1}{r^3} - \underbrace{\frac{q}{\gamma^3 m v_z^2} \frac{\lambda(r)}{2\beta^2 \gamma}}_{\text{SELF-FIELD}} = 1$$

$$\theta' = \frac{p_0}{\gamma m r^2 \beta c} - \frac{\omega_L}{2\gamma\beta c} \quad \leftarrow \text{CONSTANCY + DEFINITION OF CANONICAL MOMENTUM}$$

ENVELOPE EQUATION FOR AXISYMMETRIC BEAM

$$r_b'' + \frac{\gamma' r_b'}{\beta^2 \gamma} + \frac{\gamma''}{2\beta^2 \gamma} r_b + \left(\frac{\omega_L}{2\gamma\beta c}\right)^2 r_b - \frac{4\langle p_0 \rangle^2}{(\gamma m \beta c)^2 r_b^3} - \frac{E_r}{V_b^3} - \frac{Q}{r_b} = 0$$

$$E_r \equiv 4(\langle r^2 \rangle \langle \omega_L^2 \rangle - \langle r \omega_L \rangle^2 + \langle v^2 \rangle \langle r^2 \theta'^2 \rangle - \langle r^2 \theta' \rangle^2)$$

RETURNING TO PARAXIAL ENVELOPE EQUATION:

$$(for \beta \ll 1) \quad v_b'' + \frac{\beta'}{\beta} v_b' + \left[\frac{1}{2} \frac{\beta'^2}{\beta^2} + \frac{1}{2} \frac{\beta''}{\beta} \right] v_b - \frac{Q}{v_b} = 0$$

$$for \quad v_b'' = \frac{\beta'}{\beta} v_b' = 0$$

$$if \quad \Phi = v_b \left(\frac{z}{d} \right)^{1/3}$$

$$v = C z^{1/3}$$

$$v' = \frac{1}{3} C z^{-2/3}$$

$$v'' = -\frac{2}{9} C z^{-5/3}$$

$$\Rightarrow \left[\frac{1}{2} \frac{\beta'^2}{\beta^2} + \frac{1}{2} \frac{\beta''}{\beta} \right] v_b^2 = Q$$

$$\left[\frac{2}{9} \frac{1}{z^2} \quad -\frac{1}{9} \frac{1}{z^2} \right]$$

$$\Rightarrow Q(z) = \frac{1}{9} \frac{v_b^2}{z^2}$$

So Child-Langmuir flow satisfies the

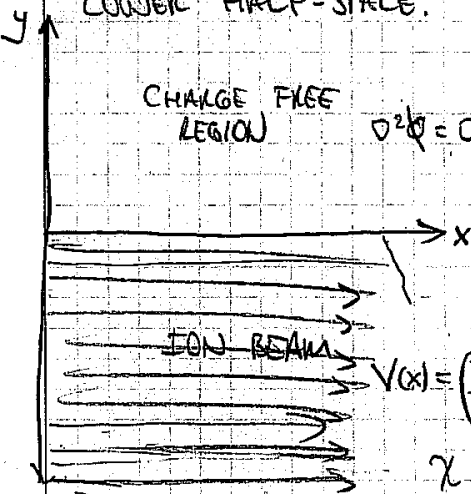
PARAXIAL ENVELOPE EQUATION FOR

A CONSTANT BEAM CURRENT (AS IT SHOULD!)

PIERCE'S ELECTRODES: GOING BEYOND PARAXIAL APPROXIMATION

CONSIDER THE CASE A BEAM WHICH FILLS THE LOWER HALF-SPACE.

42-182 100 SHEETS
National Brand
Made in U.S.A.



$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

FIND SOLUTION

SUCH THAT

$$\frac{\partial \phi}{\partial y}(x, y=0) = 0$$

$$\phi(x, y=0) = V(x)$$

PIERCE'S SOLUTION: LET THE POTENTIAL BE THE REAL PART

OF
$$\phi + iW = V(x+iy) \equiv V(z) \quad z = x+iy$$

NOTE THAT FOR ANY $V(z)$ WITH DERIVATIVES THAT EXIST INDEPENDENT OF DIRECTION (ANALYTIC) THE REAL PART OF $V(z)$ SATISFIES LAPLACE'S EQUATION:
$$\frac{\partial^2 \text{Re}[V]}{\partial x^2} + \frac{\partial^2 \text{Re}[V]}{\partial y^2} = 0$$

$$\frac{\partial \phi}{\partial x} = \text{Re} \left[\frac{\partial V}{\partial z} \right]$$

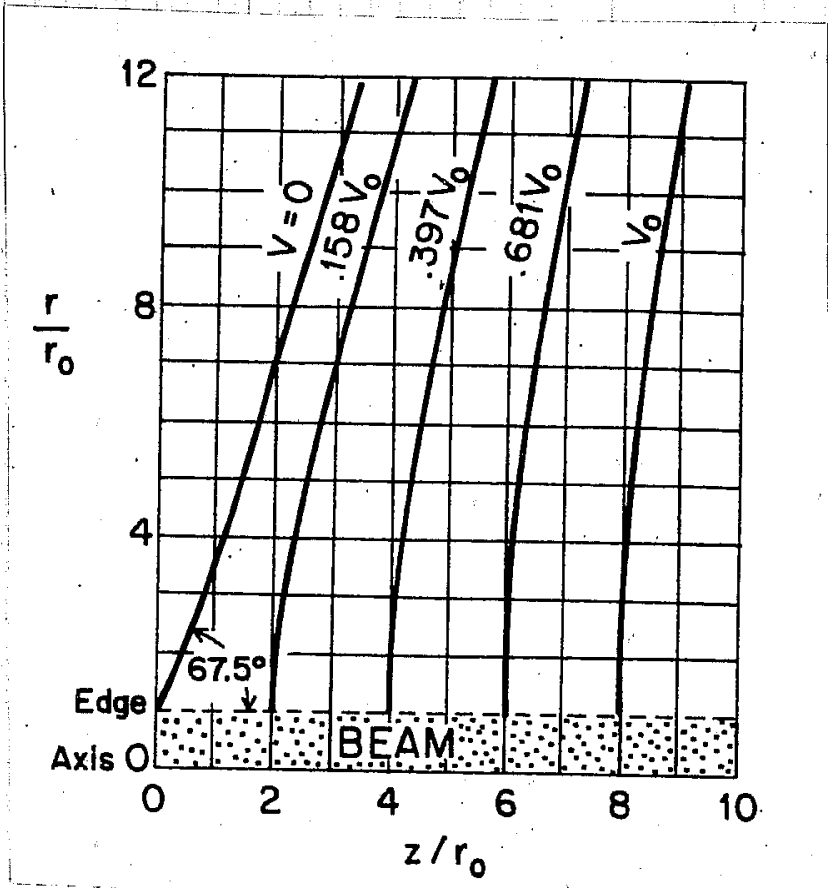
$$\frac{\partial \phi}{\partial y} = \text{Re} \left[i \frac{\partial V}{\partial z} \right]$$

$$\frac{\partial^2 \phi}{\partial x^2} = \text{Re} \left[\frac{\partial^2 V}{\partial z^2} \right]$$

$$\frac{\partial^2 \phi}{\partial y^2} = -\text{Re} \left[\frac{\partial^2 V}{\partial z^2} \right]$$

$$\Rightarrow \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \text{Re} \left[\frac{\partial^2 V}{\partial z^2} \right] - \text{Re} \left[\frac{\partial^2 V}{\partial z^2} \right] = 0$$

PIERCE ELECTRODES FOR CIRCULAR BEAMS



- SOLUTION is similar, but must be done numerically
- $\phi = 0$ is same as planar case

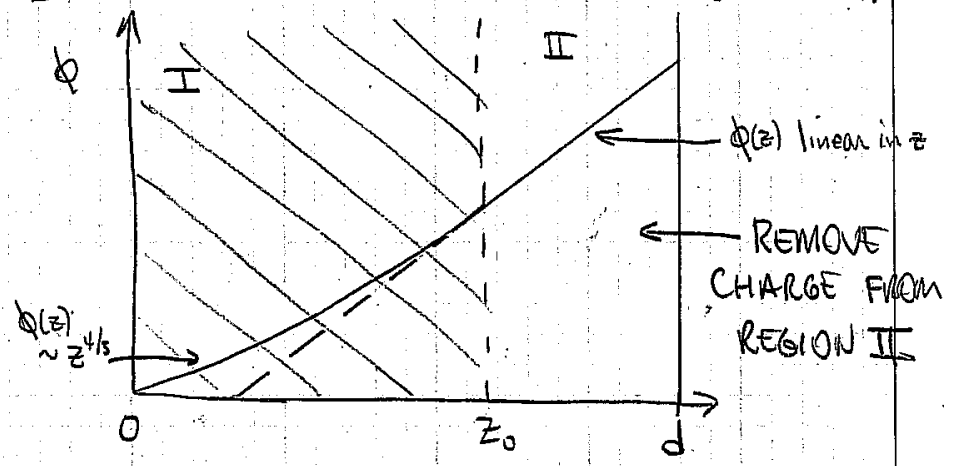
FIGURES FROM A.T. FORRESTER,
 "LARGE ION BEAMS,"
 Wiley, 1988

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TRANSIENTS IN INJECTORS (LAMPET & TIEFENBACH, Appl. Phys. Lett, 43, 57, 1983)

DURING TURN-ON THERE IS NO SPACE CHARGE IN FRONT OF BEAM, SO FIELDS MAY NOT BE GIVEN BY CHILD-LANGMUIR LAW. \Rightarrow CURRENT SPIKES POSSIBLE \Rightarrow ADVERSE TRANSVERSE COUPLING.

SOLUTION: ADJUST VOLTAGE ON DIODE SUCH THAT C-L FIELD OCCURS EVERYWHERE THERE IS BEAM.



(POSITION OF BEAM HEAD) $E(z_0) = \frac{4}{3} \frac{V_0}{d} \left(\frac{z_0}{d}\right)^{1/3}$

$$\begin{aligned} \Phi(d) &= \int_0^{z_0} E(z) dz + \int_{z_0}^d E(z) dz \\ &= V_0 \left(\frac{z_0}{d}\right)^{4/3} + \frac{4}{3} \frac{V_0}{d} \left(\frac{z_0}{d}\right)^{1/3} (d - z_0) \\ &= V_0 \left[\frac{4}{3} \left(\frac{z_0}{d}\right)^{1/3} - \frac{1}{3} \left(\frac{z_0}{d}\right)^{4/3} \right] \end{aligned}$$

(NOTE V_0 IS THE DESIRED STEADY STATE VOLTAGE ACROSS DIODE)

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INJECTOR CHOICES (cf. Kwan et al, NIMPR, 464, 379 (2001))

CHILD-LANGMUIR $\Rightarrow J = \chi \frac{V^{3/2}}{d^2}$

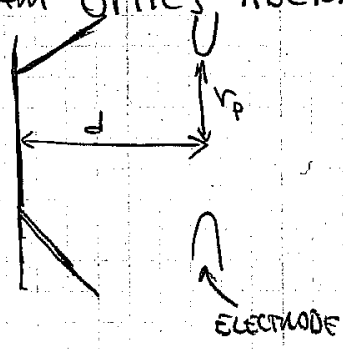
where $\chi = \frac{4}{9} \epsilon_0 \left(\frac{2q}{m}\right)^{1/2}$

SOME CONSTRAINTS:

(1) VOLTAGE BREAKDOWN

EMPIRICALLY $V \leq \sim 100 \text{ kV} \begin{cases} \left(\frac{d}{1 \text{ cm}}\right) & \text{for } d \leq 1 \text{ cm} \\ \left(\frac{d}{1 \text{ cm}}\right)^{1/2} & \text{for } d \geq 1 \text{ cm} \end{cases}$

(2) BEAM OPTICS ABERRATIONS: $d \geq \sim \frac{3}{4} r_p$ (TYPICALLY)



NOTE THAT

$J \sim \frac{V^{3/2}}{d^2} \sim \begin{cases} V^{-1/2} & \leftarrow d < 1 \text{ cm} \\ V^{-3/2} & \leftarrow d > 1 \text{ cm} \end{cases} \quad I \sim \pi r_p^2 J \sim \begin{cases} V^{3/2} \end{cases}$

Thus current density decreases with size and voltage, but I increases.

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A_{iv}
 $V = 24.5d^{7/5}$

from A. Faltens:

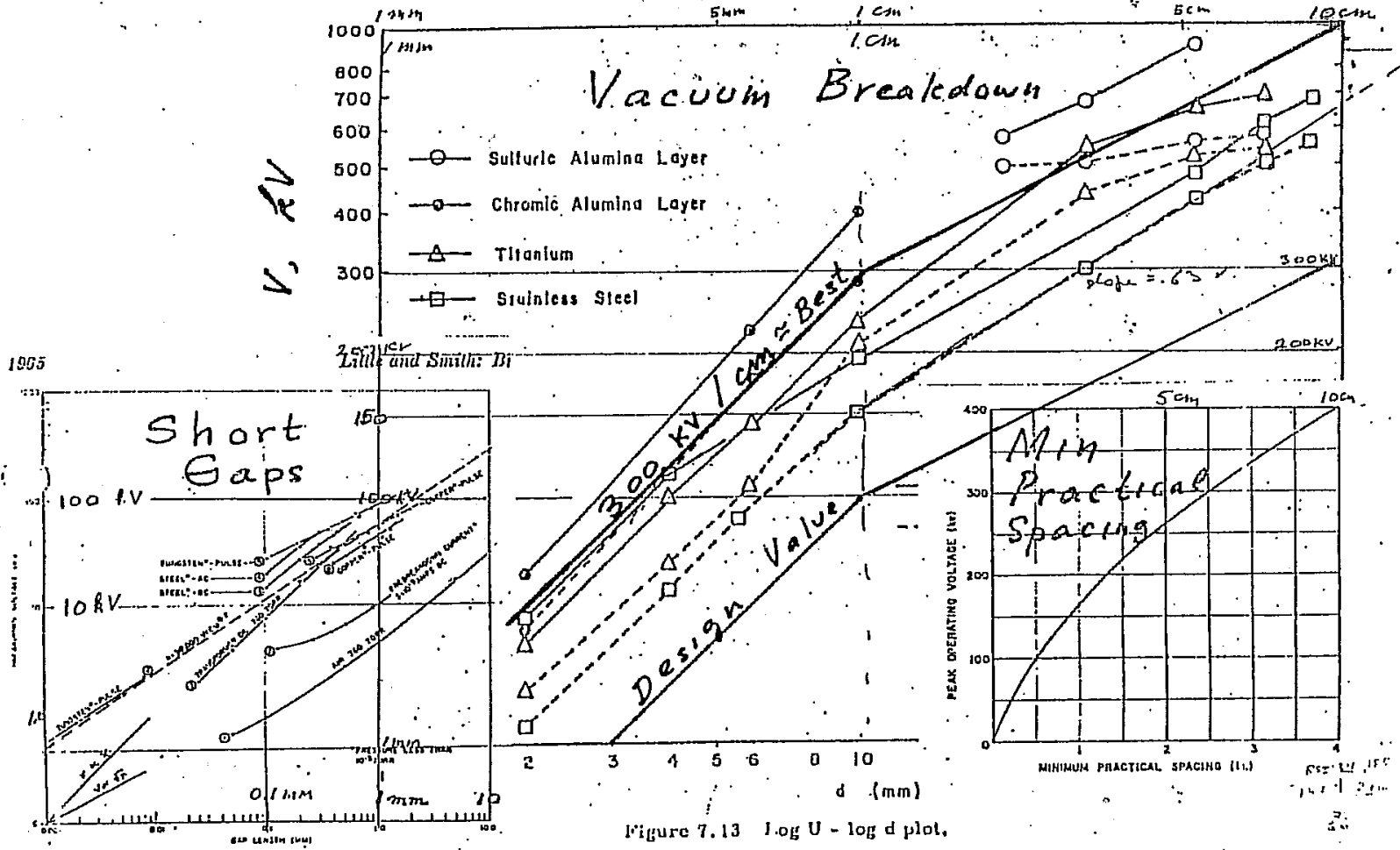


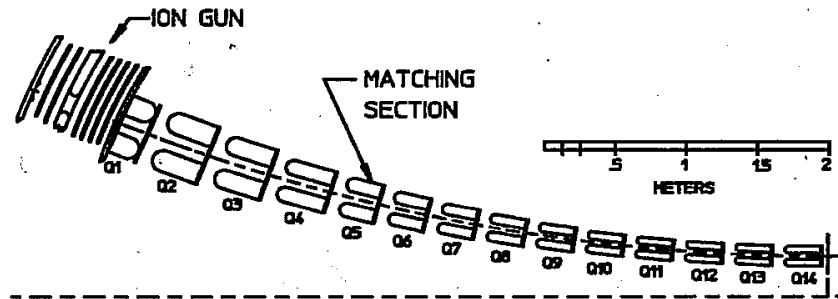
Figure 7.13 Log U - log d plot.

Fig. 1. Breakdown voltage-vs.-gap length for uniform-field and non-uniform-field geometry. Numbers on curves indicate the

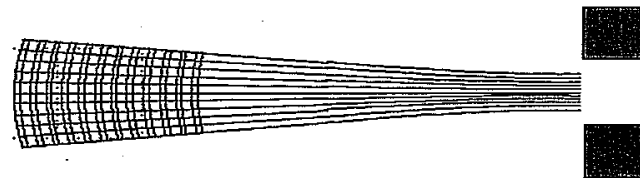
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MULTIPLE BEAMLET INJECTORS CAN HAVE HIGHER CURRENT DENSITY
 DECREASING SIZE OF INJECTOR

traditional
 design using
 single large
 diameter source



advanced
 design using
 multiple
 beamlets



Each beamlet carries higher current density; But merging beamlets increases thermal spread.

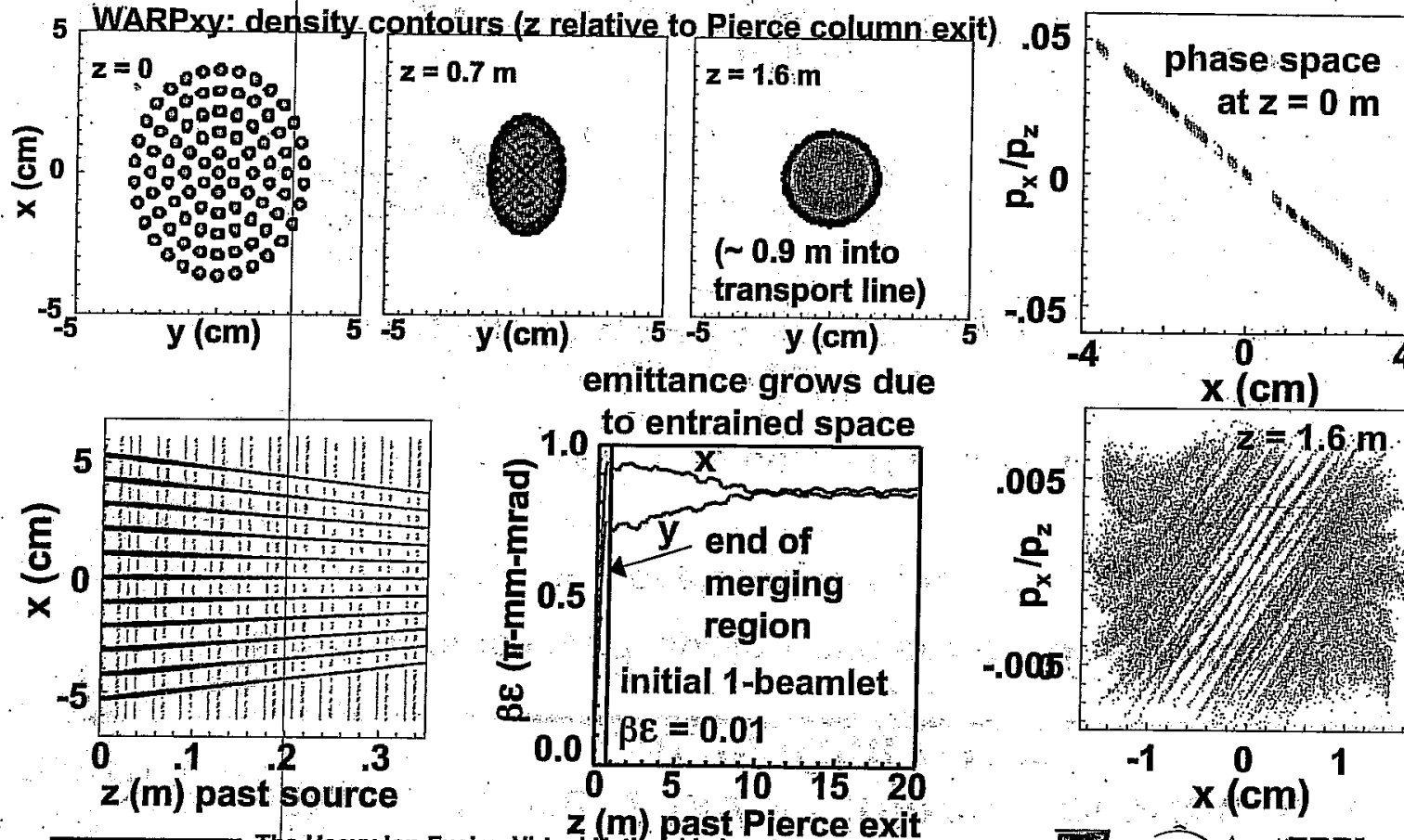
Child-Langmuir $J_{CL} \propto \frac{V^{3/2}}{d^2}$

Breakdown limit $V \propto d^{1.0 \text{ to } 0.5}$

$J \propto V^{-1/2 \text{ to } -5/2} \propto d^{-1/2 \text{ to } -5/4}$

Merge and match
 beamlets into an
 ESQ channel

Simulations of merging-beamlet injector



The Heavy Ion Fusion Virtual National Laboratory.

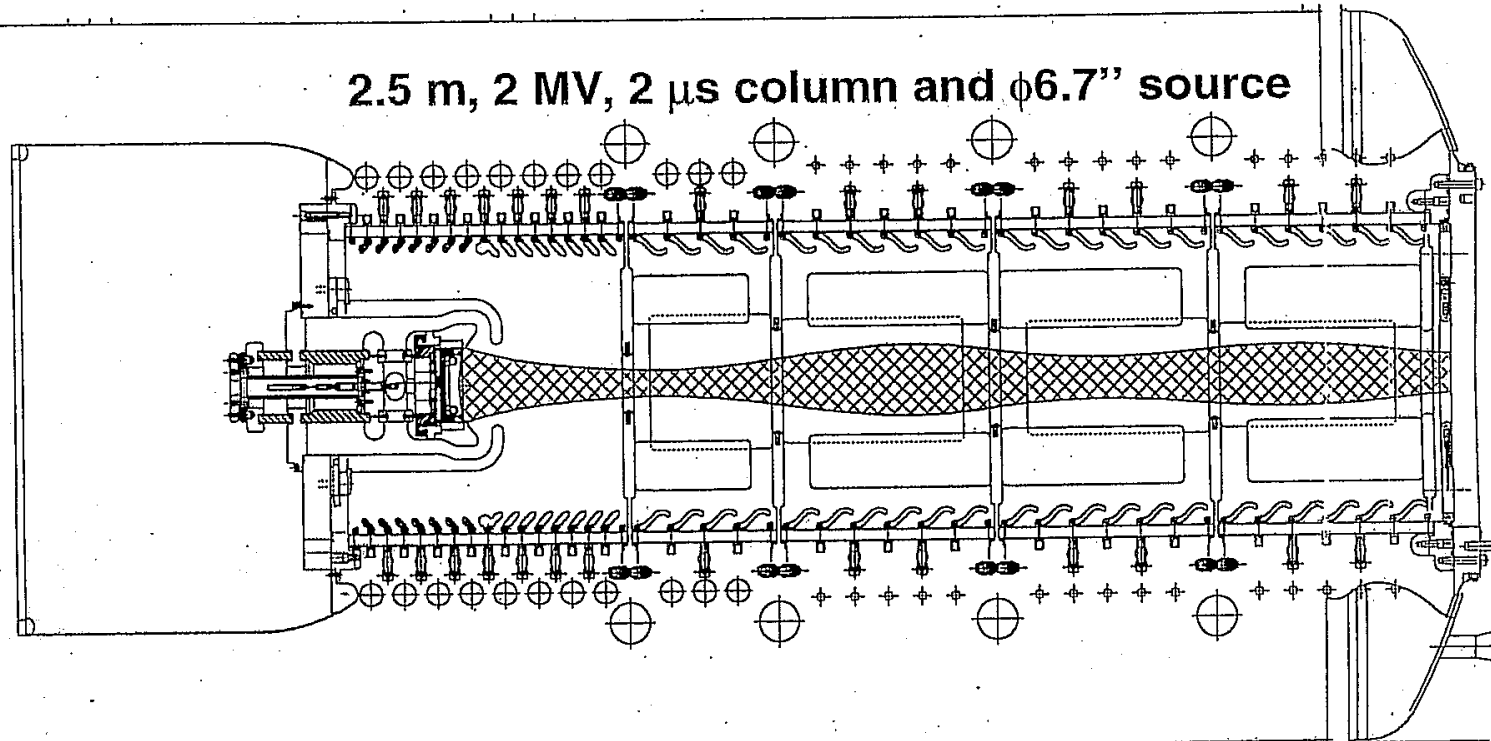
from D.I. GROTE, E. HENBESTLOZA, J.W. KWAN, "DESIGN & SIMULATION OF THE MULTIBEAMLET INJECTOR FOR A HIGH CURRENT ACCELERATOR" SUBMITTED TO IACAT (2007)

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0.8 Ampere, 2 MV K^+ Injector produced a $\lambda=0.25\mu C/m$ beam

Electrostatic Quadrupole Accelerator for simultaneous focusing and acceleration of ion beams to 2 MV.



LAWRENCE BERKELEY NATIONAL LABORATORY

File: no for each of reference

SCALING OF BRIGHTNESS IN INJECTORS

$$G_N = 4 \beta \langle x^2 \rangle^{1/2} \langle x'^2 \rangle^{1/2} = \frac{4}{c} \left(\frac{v_b}{2} \right) \langle v_x^2 \rangle^{1/2}$$

$$C_1 = 2 \pi \beta \sqrt{\frac{kT}{mc^2}}$$

$$\frac{1}{2} m v_x^2 = \frac{1}{2} kT$$

$$\Rightarrow B = \frac{I}{\epsilon_N^2} = \frac{\pi J}{4(kT/mc^2)} \sim \frac{J}{T}$$

\Rightarrow FOR HIGH BRIGHTNESS & HIGH CURRENT
 MAY WISH TO ACCELERATE MANY BEAMLETS
 AND THEN MERGE TO FORM SINGLE BEAM.

MANY ISSUES NOT DISCUSSED HERE!

- SOURCES
- ELECTRON TRAPPING
- CONVERGING BEAMS
- MATCHING TO AN ESQ (e.g.)
- rf
- ...