# USPAS Accelerator Physics
# Problem Set 6 - 90 pts.

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## Problem 1

### P012 Normalized Emittance 20 pts.

Consider a distribution of particles evolving according to the particle equation of motion:

$$x'' + \frac{(\gamma\beta)'}{\gamma\beta}x' + \kappa(s)x = 0$$

Denote an average over the distribution as $\langle ... \rangle$.
A statistical measure of beam phase-space area is provided by the normalized RMS emittance:

$$\varepsilon \equiv (\gamma\beta)\left[\left\langle x^2\right\rangle\left\langle x'^2\right\rangle - \left\langle xx'\right\rangle^2\right]^{\frac{1}{2}}$$

Show directly using the equation of motion that $\varepsilon$ is constant.
Would you expect $\varepsilon$ to be conserved if the equation of motion had non-linear terms?

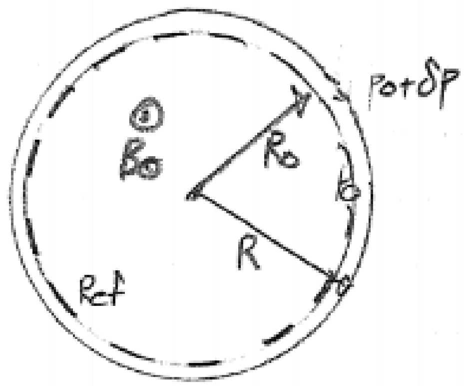
$$x'' + \frac{(\gamma\beta)'}{\gamma\beta}x' + \kappa(s)x = F(x)$$

Explain why. It is not necessary to rework the problem.
Hint: It is easier to show that $\frac{\mathrm{d}}{\mathrm{d}s}\varepsilon^2 = 0$

## Problem 2

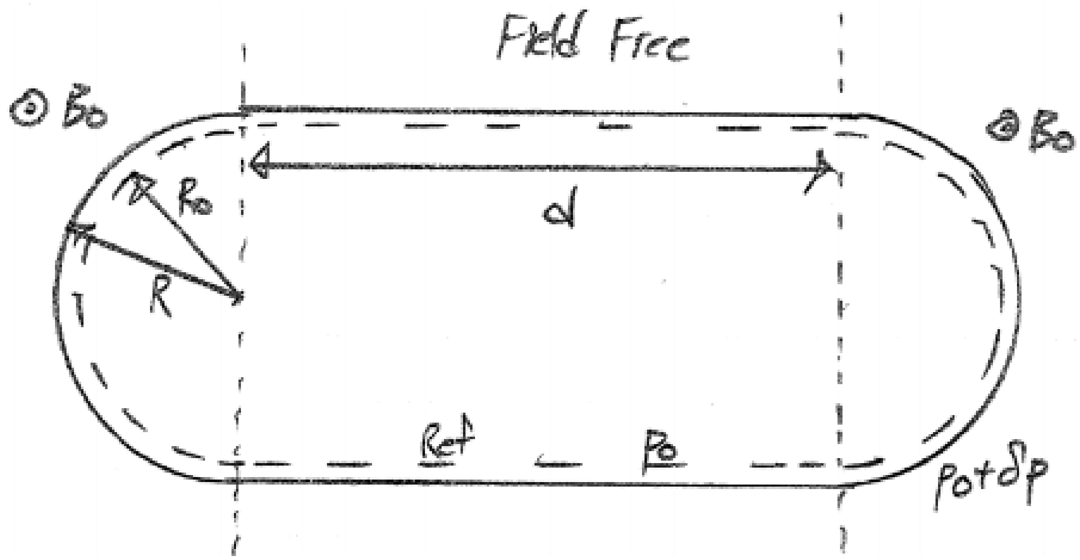
### P023 Slip Factor 20 pts.

a) 5 pts: Consider a circular accelerator/storage ring composed of a uniform magnetic field, $\mathbf{B}_y = B_0\hat{y} =$ constant. The ideal reference path for a particle of momentum $p_0$ has radius $R_0$ in the plane perpendicular to $\mathbf{B}_y$. A particle with momentum $p = p_0 + \delta p$ will have a different closed path and radius $R$.




Calculate the slip factor, $\eta$, in terms of $\gamma_0$ for this situation.

b) 5 pts: Next, repeat part (a) for a racetrack accelerator with two uniform dipole bends separated by a field free drift of length $d$. Calculate the slip factor, $\eta$, in terms of $\gamma$ and $d/R_0$.

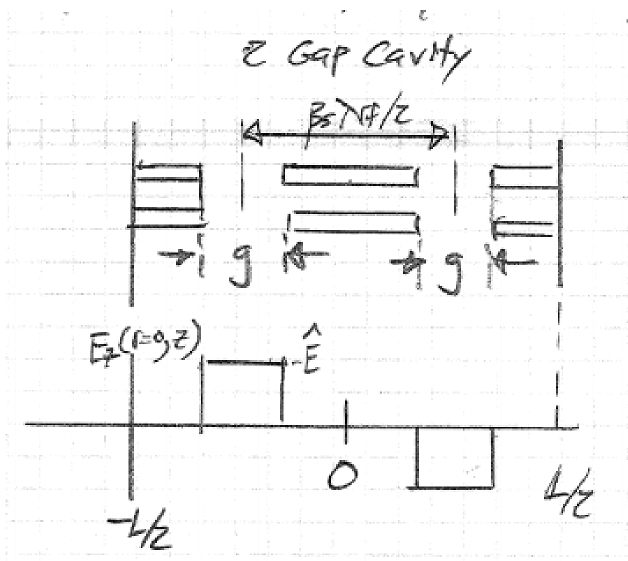



c) 10 pts: For $d = 2R_0$ in part (c), plot $\eta$ as a function of $\gamma$ and note where it changes sign. Is this the "transition $\gamma$"? What speed in $\beta = v/c$ does this correspond to?

## Problem 3

## P019 Transit Time Factor 50 pts.

Many RF cavities are multi-gap. They can be modeled by the usual Panofsky equation if an appropriate transit time factor, $T$, is employed.




$$E_z = E_z(r,z)\cos(\omega t + \phi)$$
$$E_z \approx \text{uniform in in gaps}$$

The energy gain of a particle traversing the cavity is:

$$\Delta W = q\int_{-L/2}^{L/2} E(0,z)\sin\left(\frac{2\pi z}{\beta\lambda_{\text{rf}}} + \phi\right)\mathrm{d}z$$

when approximating $\beta \simeq$ constant in the cavity.

a) 15 pts: For this structure, derive a transit time factor, $T$, to show that:

$$\Delta W = qE_0 LT\cos\phi$$

with

$$E_0 = \frac{1}{L}\int_{-L/2}^{L/2} |E(0,z)|\mathrm{d}z = \text{average magnitude of field over the cell}$$
$$T = \frac{\sin\left[\pi g/(\beta\lambda_{\text{rf}})\right]}{\pi g/(\beta\lambda_{\text{rf}})}\sin\left(\frac{\pi\beta_s}{2\beta}\right)$$

b) 30 pts: Assuming that the length of each gap is $g = \frac{1}{8}\beta_s\lambda_{\text{rf}}$, plot $T$ vs. $\beta$ for the follow four cases:

1) $f_{\text{rf}} = 80.5$ MHz, $\beta_s = 0.041$
2) $f_{\text{rf}} = 80.5$ MHz, $\beta_s = 0.085$
3) $f_{\text{rf}} = 322$ MHz, $\beta_s = 0.29$
4) $f_{\text{rf}} = 322$ MHz, $\beta_s = 0.53$

For each case, estimate the approximate range of $\beta$ for $T > 0.65$ corresponding to efficient RF acceleration.

Use any graphics package you want to make plots. Please no hand plots.

c) 5 pts: Explain why this two gap transit time factor shows more variation in $\beta$ than for a one gap model. Why can $T$ be zero for some values of $\beta$? Qualitative answers only.