Beam Bunchers, Re-Bunchers, DeBunchers

RF acceleration requires bunched beams.

Sources Often DC Beam $\Rightarrow$ Require Bunching

Also, there are transitions to other RF accelerator structures with differing frequencies etc. Also, when beam propogates without RF focusing particles will spread out due to the spread in longitudinal momentum.

Re-Buncher $\Rightarrow$ Reduce longitudinal spread

De-Buncher $\Rightarrow$ Enhance longitudinal spread.

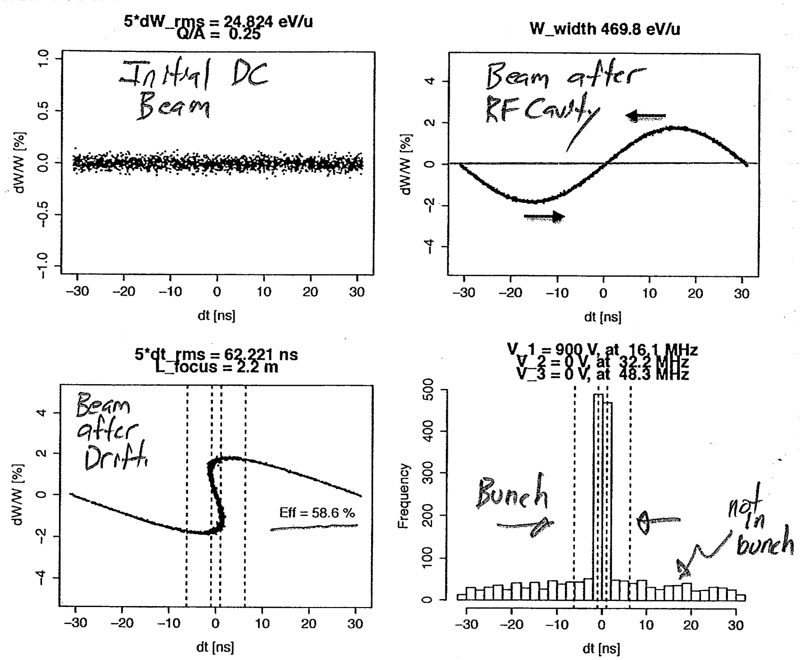
Run at synchronous phase $\phi_s = -\pi/2$

* Maximize synchrotron wavenumber for enhanced focusing strength.
* Maximize phase range for focusing.

Exa'e: Buncher Feeding RFQ: Alt MSU Thesis

Bunching using fundamental RF Harmonic

$V(t) = V_1 \sin(2\pi f t)$ Bunching Potential



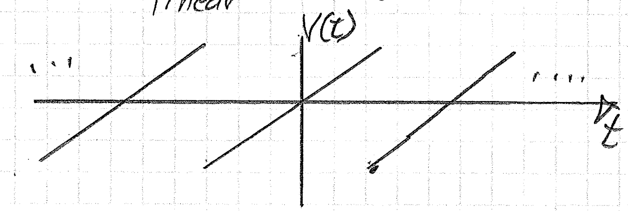
★ Simple but a good fraction of particles (~40%) not bunched.

USPAS: Alt Thesis, Syphers Fundamentals

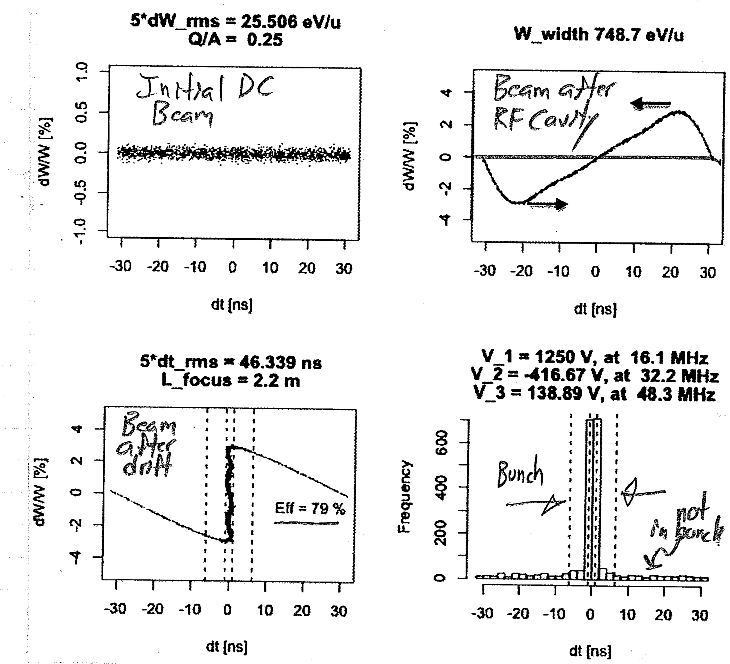
Bunching using multiple RF Harmonics

$$V(t) = V_1 \sin(2\pi f t) + V_2 \sin(4\pi f t) + V_3 \sin(6\pi f t) + V_4 \sin(8\pi f t) + \ldots$$

∞ terms with right V_n for linear

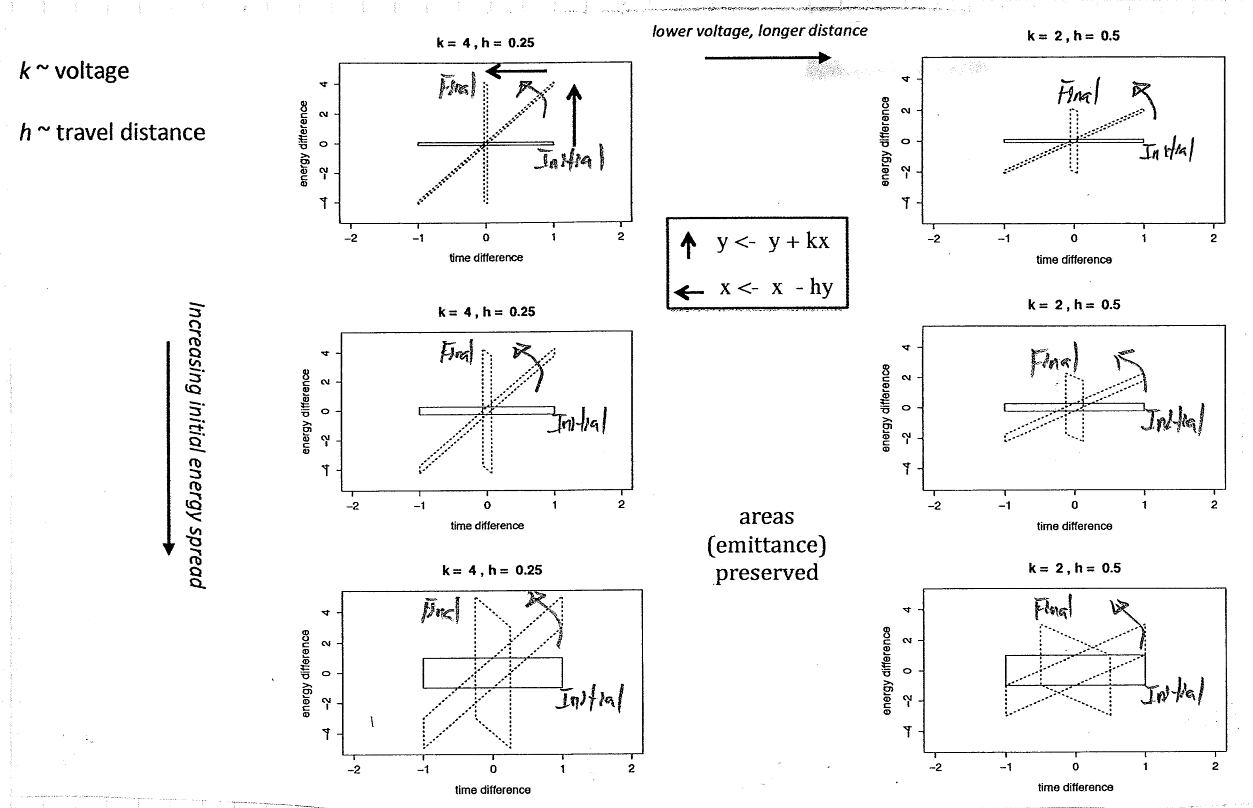


truncate to 3 terms



★ Better but not perfect
- cost of more RF control

Bunching will conserve statistical phase-space area when $V(t)$ linear.



"Emittance" Areas Preserved, but:

height ~ ΔW

thickness ~ Δt

Vary with RF voltage applied.

RF Acceleration in a Ring:

Conte & Mackay, "An Intro to the Physics of Particle Accelerators" Chapter 7

We will now modify the form of our RF longitudinal focusing and acceleration equations to a form appropriate for a particle accelerating in a ring. With bends, the path length varies with the value of the momentum p and the structure of the lattice.

See lecture notes 09.lecture.pdf on "slip factor"

ω = particle angular freq in ring
τ = particle period to cycle ring
p = particle axial momentum

$\omega = \frac{2\pi}{\tau}$

Subscript "s" ⇒ Previous "0" Ref. Orbit "Synchronous"

$$\boxed{\frac{d\omega}{\omega_s} = -\frac{d\tau}{\tau_s} = \eta_s \frac{dp}{p_s} \qquad \eta_s = \frac{1}{\gamma_s^2} - \frac{1}{\gamma_{tr}^2} \qquad \gamma_{tr} = \text{Transition Gamma (property lattice)}}$$

$\eta_s > 0$ Below Transition $\gamma_s < \gamma_{tr}$ $\Rightarrow$ $\frac{d\tau}{\tau_s} = -|\eta_s| \frac{dp}{p_s}$

Probably natural expectation → More energetic particle orbits <u>more</u> quickly than less energetic

$\eta_s < 0$ Above Transition $\gamma_s > \gamma_{tr}$ $\Rightarrow$ $\frac{d\tau}{\tau_s} = |\eta_s| \frac{dp}{p_s}$

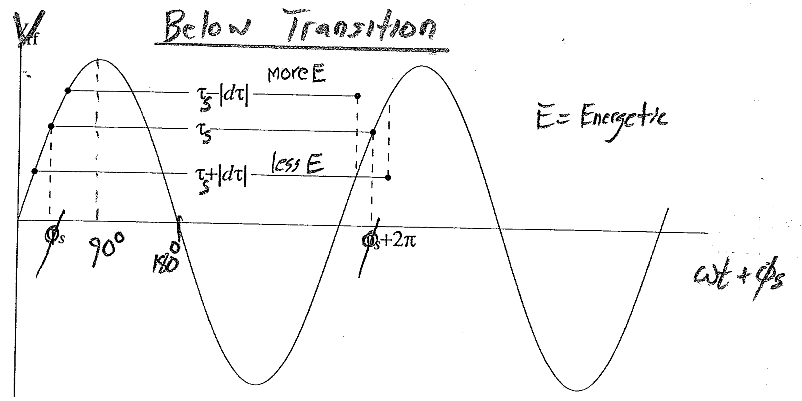
More energetic particle orbits <u>less</u> quickly than less energetic

Can seem counter-intuitive.

For a harmonically oscillating RF voltage; for phase stability and synchronous particle energy gain:

$$V(t) = \begin{matrix}\text{Cavity Energy}\\ \text{Gain or}\\ \text{RF Voltage}\end{matrix} = V_{rf} \sin(\omega t + \phi_s)$$

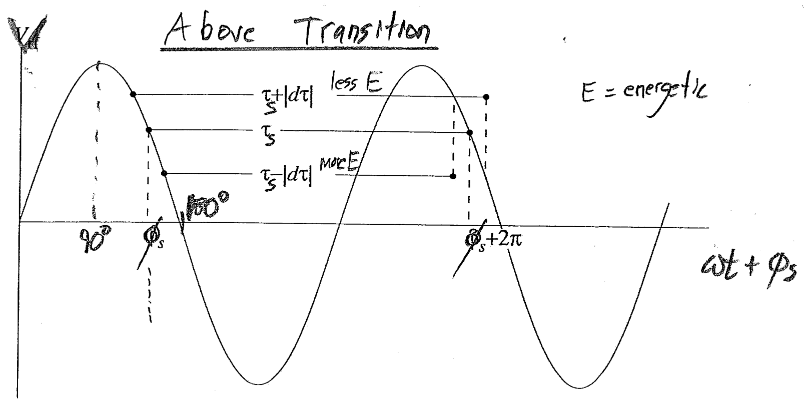
Comment: Here taking $\sin(\cdots)$ variation rather than $\cos(\cdots)$ to match most common textbook forms for rings.



$$\frac{d\tilde{\tau}}{\tilde{\tau}_s} = -|\eta_s| \frac{dp}{p_s}$$

More energetic particle orbits more quickly: operate on + slope of V for phase stability

$$0 < \phi_s < 90°$$



$$\frac{d\tilde{\tau}}{\tilde{\tau}_s} = |\eta_s| \frac{dp}{p_s}$$

More energetic particle orbits less quickly: operate on − slope of V for phase stability

$$90° < \phi_s < 180°$$

Note:
When accelerating through transition RF must have a phase jump from ϕ_s to $\pi - \phi_s$.

Transit Time Factor

Proceed analogously to RF linac case for a single gap:

Use z as longitudinal coordinate in cavity (straight section, short gap)

$t=0$ particle at gap center

$$V = q V_{rf} \sin\phi \quad = \text{energy gain of cavity in ring}$$

$$= \int_{gap} \vec{E}\cdot d\vec{l} = q\int_{-L/2}^{L/2} E(0,z)\sin[\omega t(z)+\phi]\,dz$$

$$= q\int_{-L/2}^{L/2} E(0,z)\left[\underbrace{\sin(\omega t(z))\cos\phi}_{\approx 0 \text{ typically}} + \cos(\omega t(z))\sin\phi\right]dz$$

(neglect energy gain during transit)

$$= q(E_0 L T)\sin\phi$$

where:

$$V_{rf} \equiv E_0 L T \quad \sim \text{definition RF Voltage}$$

$$E_0 = \int_{-L/2}^{L/2} E(0,z)\,dz$$

$$T \equiv \frac{\int_{-L/2}^{L/2} E(0,z)\cos(\omega t(z))\,dz}{\int_{-L/2}^{L/2} E(0,z)\,dz}$$

same as before with Linac phase convention

Panofsky equation:

$$\boxed{\text{Energy Change Transit gap} = \Delta W \equiv V = q V_{rf}\sin\phi}$$

ϕ = RF phase particle at center gap.

To illustrate formulation for ring, make simplifying assumptions:

1) Single cavity in ring

2) RF angular frequency is an integer multiple of design (synchronous) particle angular revolution frequency ω_s in the ring:

$$\boxed{\omega = h\,\omega_s}$$

ω_s = Design (synchronous) particle revolution frequency in ring.

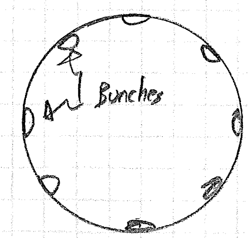
h = harmonic number (integer)

$h = 1$ lowest possible

~100s or 1000's for large ring possible.

Example LHC: $h = 35{,}640$ used.

★ h corresponds to the number (max) of bunches that can circulate in the ring



$h=8 \Rightarrow$ 8 bunches.

filling ring uniformly spaced.

★ As particles gain energy, if NR, then cavity frequency must change with energy gain to maintain synchronous form.

— Cavities adjust with energy gain in heavy ion synchrotron, Magnetic alloy loaded broadband RF (with low Q) are often used.

3) Take RF phase conditions such that the energy gain of the particle transiting the cavity is

$$\boxed{\Delta W = V = V_{rf}\sin\phi \qquad \phi = \text{RF phase} \qquad V_{rf} = E_0 L T}$$

Under these conditions we have:

RF phase general particle, nth pass through cavity

$$\phi_n = \phi_{n-1} + \omega\, d\tilde{T}_{n-1\to n} \qquad d\tilde{T}_{n-1} = \text{time from } (n-1)\text{'th to } n\text{'th cavity transit.}$$

$$= \phi_{n-1} + \omega\, \tilde{T}_{s,n-1} \frac{d\tilde{T}_{n-1}}{\tilde{T}_{s,n-1}} \qquad \tilde{T}_{s,n-1} = \text{synchronous particle transit time, } n-1\text{'th lap.}$$

But $\dfrac{d\tilde{T}_{n-1}}{\tilde{T}_{s,n-1}} = -\eta_{s,n-1} \dfrac{dp_{n-1}}{p_{s,n-1}}$ $\qquad \eta_{s,n-1} = \dfrac{1}{\gamma_{s,n-1}^2} - \dfrac{1}{\gamma_{tr}^2}$ Slip Factor on $n-1$'th lap

$$\Rightarrow \boxed{\phi_n = \phi_{n-1} - \omega\, \eta_{s,n-1} \tilde{T}_{s,n-1} \frac{dp_{n-1}}{p_{s,n-1}}}$$

But

$E = \gamma mc^2 = \sqrt{(mc^2)^2 + (pc)^2}$ = Total Energy $\qquad$ not $W = (\gamma-1)mc^2$ Kinetic Energy

$E = W + mc^2$

$\Rightarrow dE = dW$

$\Rightarrow E^2 = (mc^2)^2 + (pc)^2$

$2E\,dE = 2c^2 p\,dp \rightarrow \dfrac{dp}{p} = \dfrac{E^2}{c^2p^2}\dfrac{dE}{E}$ $\qquad$ But $p = \gamma\beta mc$, $E = \gamma mc^2$

$= \dfrac{\gamma^2 m^2 c^4}{c^4 m^2 \gamma^2 \beta^2} \dfrac{dE}{E} = \dfrac{1}{\beta^2}\dfrac{dE}{E}$

Note: d differentials are relative to sync. particle here!

Showing

$$\boxed{\frac{dp_{n-1}}{p_{s,n-1}} = \frac{1}{\beta_{s,n-1}^2}\frac{dE_{n-1}}{E_{s,n-1}} = \frac{1}{\gamma_{s,n-1}\beta_{s,n-1}^2}\frac{dW_{n-1}}{mc^2} = \frac{W_{n-1} - W_{s,n-1}}{\gamma_{s,n-1}\beta_{s,n-1}^2 mc^2}}$$

Also, note that

$$\boxed{\omega \tilde{T}_{s,n-1} = h\,\omega_{s,n-1} \tilde{T}_{s,n-1} = 2\pi h}$$

since $\omega = h\,\omega_s$ tuned for synchronous (slowly vary)

Inserting these:

$$\boxed{\phi_n - \phi_{n-1} = -\frac{2\pi h\,\eta_{s,n-1}}{\gamma_{s,n-1}\beta_{s,n-1}^2}\,\frac{\overline{W}_{n-1} - \overline{W}_{s,n-1}}{mc^2} = -\frac{2\pi h\,\eta_{s,n-1}}{\gamma_{s,n-1}\beta_{s,n-1}^2}\cdot\frac{\Delta\overline{W}_{n-1}}{mc^2}} \quad \text{—(1)}$$

$\Delta\overline{W} \equiv \overline{W} - \overline{W}_s$

Examining the energy gain Panofsky Eqn:

$$\overline{W}_n - \overline{W}_{n-1} = q\overline{V}_{rf}(\beta_n)\sin\phi_n$$

← $\overline{V}_{rf}$ is a function of β due to embeded transit-time factor.

for synchronous particle:

$$\overline{W}_{s,n} - \overline{W}_{s,n-1} = q\overline{V}_{rf}(\beta_{s,n})\sin\phi_s.$$

← ϕ_s same for all n (same cavity)

subtract and take $\overline{V}_{rf}(\beta_n) \simeq \overline{V}_{rf}(\beta_{s,n})$

$$(\overline{W}_n - \overline{W}_{s,n}) - (\overline{W}_{n-1} - \overline{W}_{s,n-1}) = q\overline{V}_{rf}(\beta_{s,n})\left[\sin\phi_n - \sin\phi_s\right]$$

or

$$\boxed{\Delta\overline{W}_n - \Delta\overline{W}_{n-1} = q\overline{V}_{rf}(\beta_{s,n})\left[\sin\phi_n - \sin\phi_s\right]} \quad \text{—(2)}$$

Also, we will need to advance for the synchronous particle

$$\boxed{\overline{W}_{s,n} - \overline{W}_{s,n-1} = q\overline{V}_{rf}(\beta_{s,n})\sin\phi_s} \quad \text{—(3)}$$

Equations (1) – (3) describe the longitudinal particle dynamics in a ring.

Contrast Ring and Linac difference equations:

Ring

$$\phi_n - \phi_{n-1} = \frac{-2\pi h\, \eta_{s,n-1}}{\gamma_{s,n-1}\, \beta^2_{s,n-1}} \frac{\Delta W_{n-1}}{mc^2} \quad ①$$

$$\Delta W_n - \Delta W_{n-1} = q V_{rf}(\beta_{s,n}) \left[\sin\phi_n - \sin\phi_s \right] \quad ②$$

$$W_{s,n} - W_{s,n-1} = q V_{rf}(\beta_{s,n}) \sin\phi_s \quad ③$$

$$\Delta W = W - W_s$$

$$\eta_{s,n-1} = \frac{1}{\gamma^2_{s,n-1}} - \frac{1}{\gamma^2_{tr}}$$

$$V_{rf}(\beta_{s,n}) = q E_0 L\, T_n(\beta_{s,n})$$

Linac

$$\Delta\phi_n - \Delta\phi_{n-1} = \frac{-2\pi N}{\gamma^3_{s,n-1}\, \beta^2_{s,n-1}} \frac{\Delta W_{n-1}}{mc^2}$$

$$\Delta W_n - \Delta W_{n-1} = q E_{0,n} L_n T_n(\beta_{s,n}) \left[\cos\phi_n - \cos\phi_{s,n} \right]$$

$$W_{s,n} - W_{s,n-1} = q E_{0,n} L_n T_n(\beta_{s,n}) \cos\phi_{s,n}$$

$$\Delta W = W - W_s$$
$$\Delta\phi = \phi - \phi_s$$
$$N = \begin{cases} 1 & \text{0-Mode Structure} \\ 1/2 & \pi\text{-Mode Structure} \end{cases}$$

Let's show the Ring equations are essentially the same as the Linac equations by manipulating the Ring equations into Linac form:

1st note for synchronous particle
$$\phi_n = \phi_{n-1} = \text{const} \equiv \phi_s$$
$$\Delta W_{s,n} = 0$$

$$① \Rightarrow \phi_{s,n} = \phi_{s,n-1}$$

subtract sync eqn from ① and use $\Delta\phi = \phi - \phi_s$

$$\Rightarrow \Delta\phi_n - \Delta\phi_{n-1} = \frac{-2\pi h\, \eta_{s,n-1}}{\gamma_{s,n-1}\, \beta^2_{s,n-1}} \frac{\Delta W_{n-1}}{mc^2} \quad ①$$

Take also for consistency with linac definitions

$h = 1$; only fundamental harmonic makes sense in linac context.

$\phi_n \rightarrow \phi_n + \pi/2$; for equivalent phase def.

$$\eta_{s,n-1} = \frac{1}{\gamma_{s,n-1}^2} - \frac{1}{\gamma_{tr}^2} \rightarrow \frac{1}{\gamma_{s,n-1}^2}$$ since $\gamma_{tr} \rightarrow \infty$ for linac, (no transition)

Then ①-③ for a ring become:

$$\boxed{\begin{aligned} \Delta\phi_n - \Delta\phi_{n-1} &= \frac{-2\pi}{\gamma_{s,n-1}^3 \beta_{s,n-1}^2} \frac{\Delta W_{n-1}}{mc^2} \quad ① \\ \Delta W_n - \Delta W_{n-1} &= q E_0 L T_n(\beta_{s,n}) \left[\cos\phi_n - \cos\phi_s\right] \quad ② \\ W_{s,n} - W_{s,n-1} &= q E_0 L T_n(\beta_{s,n}) \cos\phi_s \quad ③ \end{aligned}}$$

Same as Linac equations:

$$\boxed{\begin{aligned} \Delta\phi_n - \Delta\phi_{n-1} &= \frac{-2\pi N}{\gamma_{s,n-1}^3 \beta_{s,n-1}^2} \frac{\Delta W_{n-1}}{mc^2} \\ \Delta W_n - \Delta W_{n-1} &= q E_{0,n} L_n T_n(\beta_{s,n}) \left[\cos\phi_n - \cos\phi_{s,n}\right] \\ W_{s,n} - W_{s,n-1} &= q E_{0,n} L_n T_n(\beta_{s,n}) \cos\phi_{s,n} \end{aligned}}$$

since for a single cavity (limit context) periodically repeated

$N = 1$ in "0" mode

$$\left.\begin{aligned} \phi_{s,n} &= \phi_s \\ E_{0,n} &= E_0 \\ L_n &= L \end{aligned}\right\}$$ all cavities same.

Using these correspondances, most of our developments for longitudinal dynamics in an RF linac are straightforward to apply to rings.

Elaborate on several issues though for enhanced clarity: See also Edwards and Syphers: Intro to the Physics of High Energy Accelerators.

Continuous Limit:

☆ Should be well respected in high energy rings:
- Energy gain per lap small.
- Synchrotron frequency low rel to RF freq, which is a harmonic (h) of bunch revolution frequency in ring.

☆ Can be convenient to take n as a continuous variable: n = lap number.

Discreet

$$\boxed{\begin{aligned} \phi_n - \phi_{n-1} &= \frac{-2\pi h \eta_{s,n-1}}{\gamma_{s,n-1}\beta^2_{s,n-1}} \frac{\Delta W_{n-1}}{mc^2} \\ \Delta W_n - \Delta W_{n-1} &= q V_{rf}\left[\sin\phi_n - \sin\phi_s\right] \end{aligned}}$$

n continuous lap number $\Rightarrow$

$\beta_s = \beta_s(n)$ etc.

Continuous

$$\boxed{\begin{aligned} \frac{d\phi}{dn} &= \frac{-2\pi h \eta_s}{\gamma_s \beta_s^2} \frac{\Delta W}{mc^2} \\ \frac{d\Delta W}{dn} &= q V_{rf}\left[\sin\phi - \sin\phi_s\right] \end{aligned}}$$

Synchrotron Oscillations: Small Amplitude Phase Oscillations about Synchronous Particle.

Approximate in continuous limit formulation:

$$\phi = \phi_s + \Delta\phi \quad ; \quad \Delta\phi \text{ small}$$

$$\Rightarrow \sin\phi - \sin\phi_s = \sin[\phi_s + \Delta\phi] - \sin\phi_s = \sin\phi_s \underbrace{\cos\Delta\phi}_{\simeq 1} + \cos\phi_s \underbrace{\sin\Delta\phi}_{\simeq \Delta\phi} - \sin\phi_s \simeq \cos\phi_s \,\Delta\phi$$

Contrast for RF linac

$$\cos\phi - \cos\phi_s = \cos(\phi_s + \Delta\phi) - \cos\phi_s \simeq \sin(-\phi_s)\,\Delta\phi \xrightarrow[\phi_s \to \phi_s + \frac{\pi}{2}]{} \sin(-\phi_s + \tfrac{\pi}{2})\,\Delta\phi = \cos\phi_s\,\Delta\phi$$

to convert phase choice.

✓ same.

In terms of this formulation:

$$\frac{d}{dn}\Delta\phi = \frac{-2\pi h \eta_s}{\gamma_s \beta_s^2}\frac{\Delta W}{mc^2}$$

$$\frac{d}{dn}\Delta W = q V_{rf} \cos\phi_s \Delta\phi$$

$$\longrightarrow \frac{d^2 \Delta\phi}{dn^2} + \frac{2\pi h \eta_s}{\gamma_s \beta_s^2}\frac{q V_{rf}}{mc^2}\cos\phi_s \, \Delta\phi$$

$$\eta_s = \frac{1}{\gamma_s^2} - \frac{1}{\gamma_{tr}^2} \quad \text{Slip Factor}$$

γ_{tr} = Transition Gamma.

$$\boxed{\frac{d^2\Delta\phi}{dn^2} + (2\pi\nu_s)^2 \Delta\phi = 0}$$

$$\nu_s = \sqrt{\frac{h\eta_s}{2\pi\gamma_s\beta_s^2}\frac{qV_{rf}}{mc^2}\cos\phi_s}$$

$\equiv$ Synchrotron Tune:

\# Longitudinal Synchrotron Oscillations per lap in ring

★ Can define synchrotron wavenumber as:

$$k_s = \frac{2\pi\nu_s}{C} \qquad C = \text{circumfrance ring.}$$

– straightforward to show form is as should be expected from definition in RF linac.

★ Need

$$\eta_s \cos\phi_s > 0 \quad \text{for stability.}$$

$$\begin{cases} \eta_s > 0 & \text{below transition } \gamma_s < \gamma_{tr} \Rightarrow -\frac{\pi}{2} < \phi_s < \frac{\pi}{2} \\ \eta_s < 0 & \text{above transition } \gamma_s > \gamma_{tr} \Rightarrow \frac{\pi}{2} < \phi_s < \pi \end{cases}$$

<u>Transition</u>

There is no "transition" in Linacs. But in rings,

$$\boxed{\text{Slip Factor} = \eta_s \equiv \frac{1}{\gamma_s^2} - \frac{1}{\gamma_{tr}^2}}$$

For linac $\gamma_{tr} \to \infty$

$\eta_s = \frac{1}{\gamma_s^2} > 0$ Always.

changes sign for

$\gamma_s < \gamma_{tr}$ <u>Below Transition</u>: $\eta_s > 0$

$\gamma_s > \gamma_{tr}$ <u>Above Transition</u>: $\eta_s < 0$

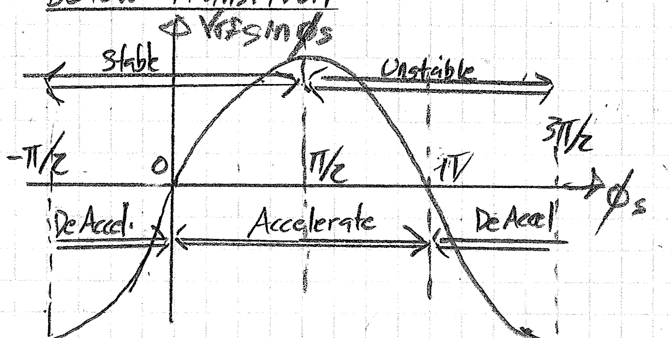
So for ring, the energy being above / below transition has important implications for choice of the synchronous RF phase ϕ_s for longitudinal stability.

★ At transition: $\eta_s = 0 \Rightarrow$ no synchrotron oscillations

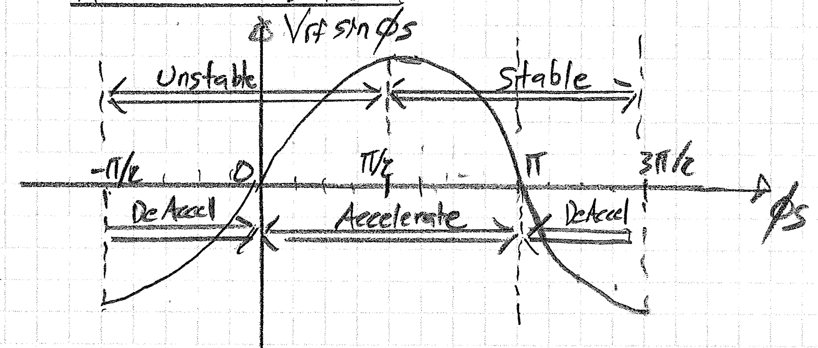
- <u>Not really true</u>: higher order terms dropped in calculation of η_s now matter and dynamics becomes complicated.
- Generally desireable to go through transition as quickly as possible in acceleration cycle or avoid via design choices (energy always above / below).

Cases of Acceleration / Stability

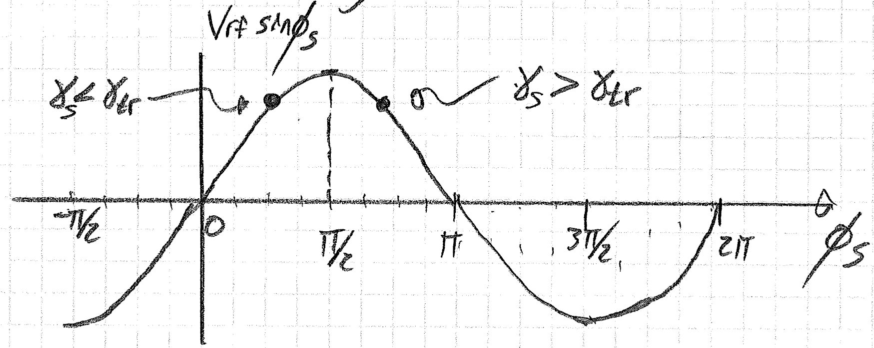
<u>Below Transition</u>



<u>Above Transition</u>



Will want the RF phase control to rapidly jump as the particles in a ring accelerate through transition.



$$\eta_s = \frac{1}{\gamma_s^2} - \frac{1}{\gamma_{tr}^2}$$

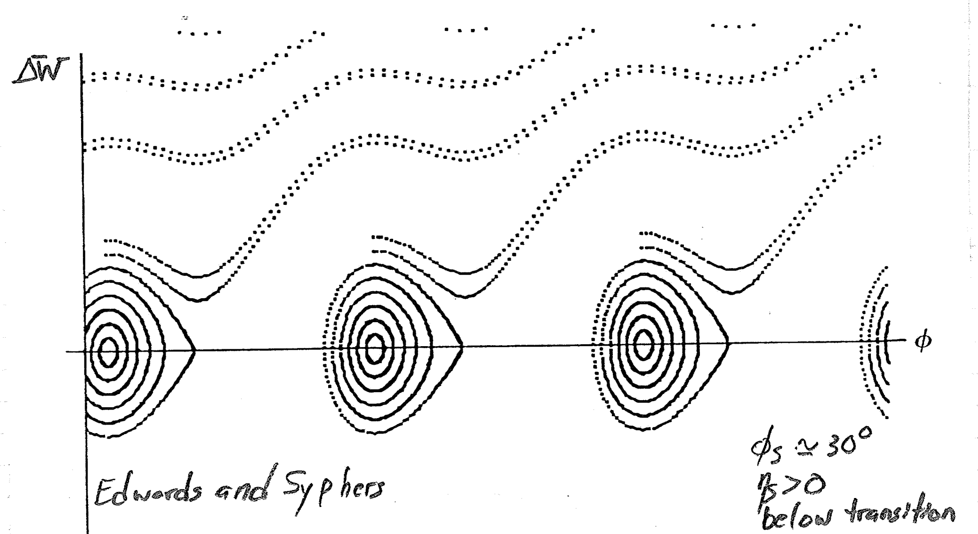
* No phase stability when $\gamma = \gamma_{tr}$
  - Momentum spread gets large near transition

* Best to <u>accelerate as quickly as possible</u> through transition.

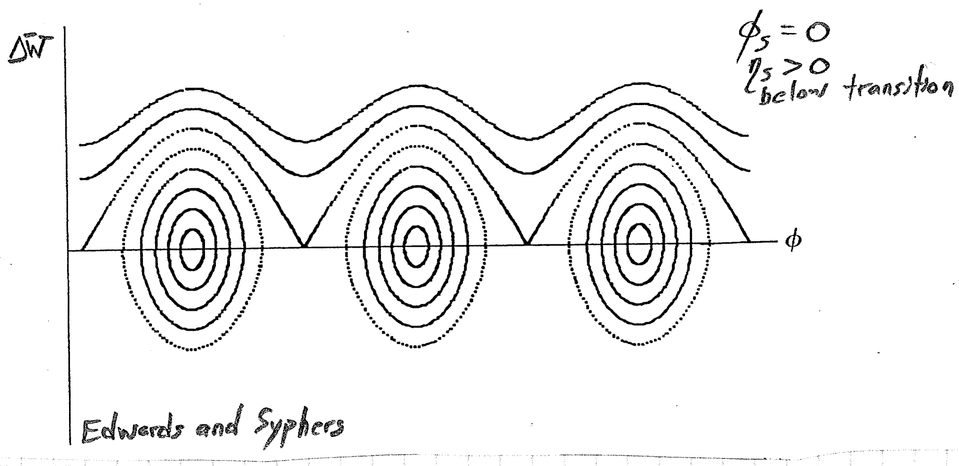
Longitudinal Dynamics Examples.

Essentially same as for RF linac. Examine difference equations for a variety of initial conditions to delineate trapped/stable conditions.

$$\boxed{\begin{aligned} \phi_n - \phi_{n-1} &= \frac{-2\pi h\, \eta_{s,n-1}}{\gamma_{s,n-1}\,\beta^2_{s,n-1}} \frac{\Delta W_{n-1}}{mc^2} \qquad\qquad W_{s,n} - W_{s,n-1} = q V_{rf} \sin\phi_s \\ \Delta W_n - \Delta W_{n-1} &= q V_{rf}\left[\sin\phi_n - \sin\phi_s\right] \end{aligned}}$$



Edwards and Syphers



Edwards and Syphers

Longitudinal Bunch Manipulations

Particularly in rings, the long path length and large number of potential synchrotron oscillations over many laps

Machine Cycle: (Synchrotron)

$$\text{Inject} \downarrow \text{Accelerate} \downarrow \boxed{\text{Bunch / Hold}} \downarrow \text{Collide / Extract}$$

Machine Cycle Storage Ring

$$\text{Inject} \downarrow \boxed{\text{Bunch / Hold}} \downarrow \text{Extract.}$$

Bunching phase typically done with $\phi_s = 0$ or π below or above transition, for max focus strength.

opens prospects for for many longitudinal phase-space manipulations.

- Adiabatic Bunching (covered already) Adiabatic also done <u>with</u> acceleration.
- Fast Bunch Rotation / Compression
- Bunch Coalescing
- Cogging
- Slip Stacking
- Barrier Buckets

⋮

Will Only briefly overview

Cavities for RF acceleration in ring are often broadband magnetic alloy resonators which are similar to induction acceleration technology with a harmonic drive. These may be operated in an <u>amplifier like mode</u> to allow considerable flexibility in tayloring the RF <u>Voltage</u> V_{rf} as a function of time to enable many bunch manipulations.

Example Ferrite loaded cavities for RF Accel.

* ★ Broad band, low Q amplifier: Allows great flexability in pulse shape.
* ★ Allows retuning resonant freq. and shaping pulse. : Magnetic field used to alter ferrite properties
* ★ Design similar to induction accel. with RF drive rather than pulse power.

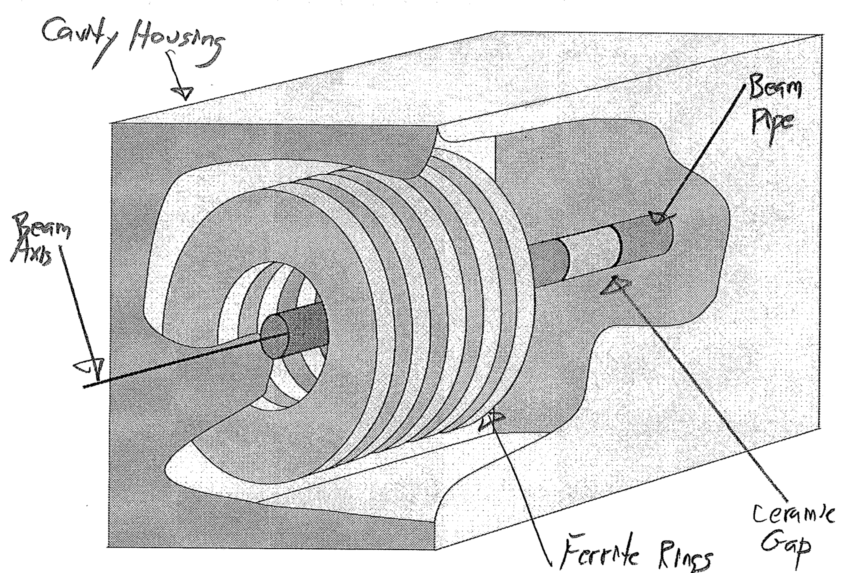


Fig. 2: Simplified 3D sketch of a ferrite-loaded cavity

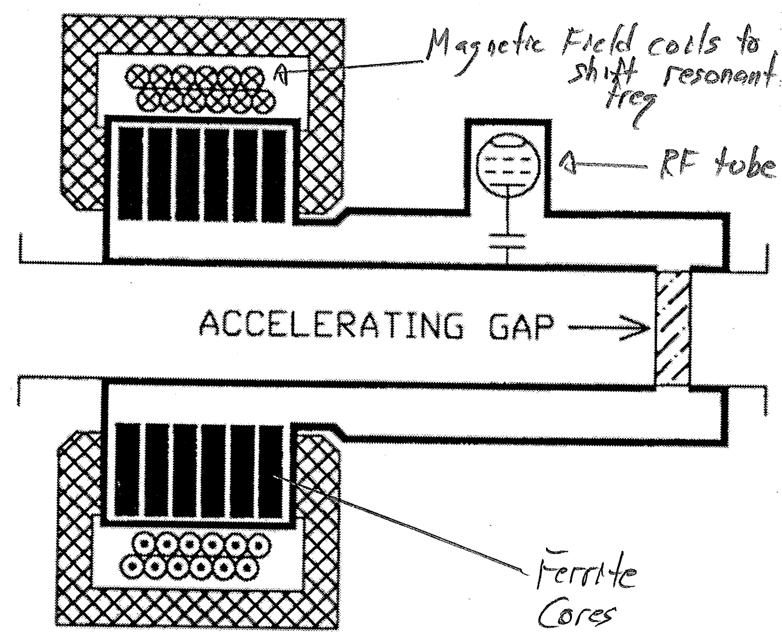


Fig. 20 Los Alamos / TRIUMF cavity

From GSI documentation.
by Harald Klingbeil.

RF Bunch Manipulations

RF Buckets can be filled or not filled (empty) to allow for many bunch arrangements

- ★ Adjust spacing
- ★ Adjust arrangement

We will briefly overview a few concepts to clarify.

1/ Adiabatic. $\phi_s \neq 0, \pi$ Need acceleration to damp phase excursions.

V_{rf} changes slowly relative to synchrotron frequency.

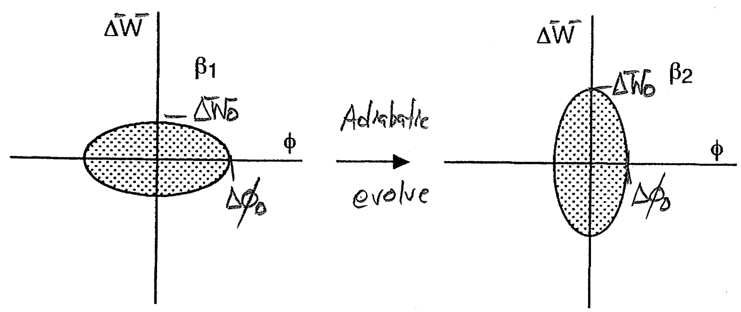
$$\left(\frac{\Delta\phi_{0f}}{\Delta\phi_{0i}}\right) = \left(\frac{(\gamma_s\beta_s)_i}{(\gamma_s\beta_s)_f}\right)^{3/4} \qquad \frac{\Delta\bar{W}_{0f}}{\Delta\bar{W}_{0i}} = \left(\frac{(\gamma_s\beta_s)_f}{(\gamma_s\beta_s)_i}\right)^{3/4}$$

i = initial
f = final

$\Delta\phi_0$ = phase width
$\Delta\bar{W}_0$ = Energy width

$$\nu_s = \sqrt{\frac{h\eta_s}{2\pi\gamma_s\beta_s^2} \cdot \frac{qV_{rf}\cos\phi_s}{mc^2}}$$

= synchrotron tune



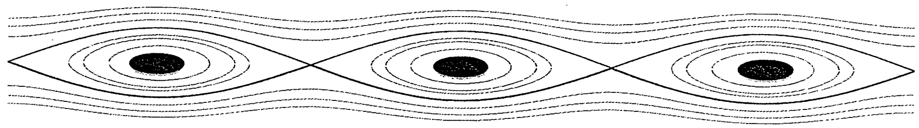
★ Easy to do this in ring since evolution can take place over many laps.
★ Change in $\gamma_s\beta_s$ should occur over $N_{lap} > \frac{1}{\nu_s}$ for slow changes.

2/ Fast Rotation: Same as with Bunching / Debunching.

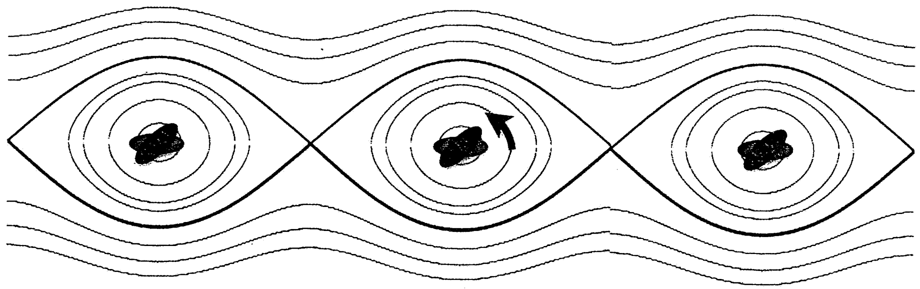
RF Voltage rapidly changed.

Bunch Rotation

start:
Initial "Match"



instantly raise RF voltage... bunches will begin to rotate in phase space:



when rotated by 90° can rapidly switch to a higher-harmonic RF system in order to maintain the shorter bunch length; *or*, for example, extract the beam and send to a target!

Syphers, USPAS

k_s = synchrotron wavenumber

$k_s(\text{length}) = \frac{\pi}{2}$

for 90° rotation.

or for ring: ν_s = synchrotron tune.

$2\pi \nu_s N_{lap} = \frac{\pi}{2}$

$\rightarrow \boxed{N_{lap} = \frac{1}{4\nu_s}}$

* Rings again ideal since path length long and RF rotation voltage applied each lap.

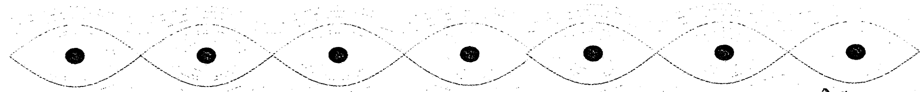
* Can also work in linacs with Large V_0 jump.
  - Same as buncher physics: large kick + drift.

$$\nu_s = \sqrt{\frac{h\eta_s}{2\pi\gamma_s\beta_s^2}\,\frac{qV_{rf}\cos\phi_s}{mc^2}}$$

3/ Bunch Coalescing

Bunch Coalescing

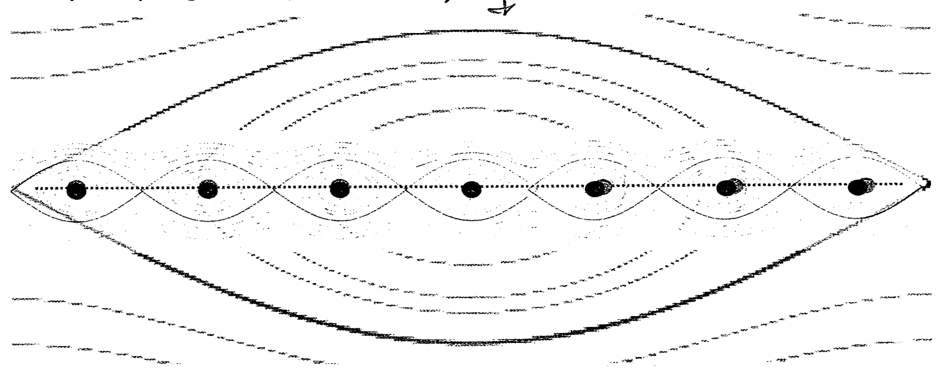
similar to bunch rotation, but also involves a change in RF frequency (harmonic)

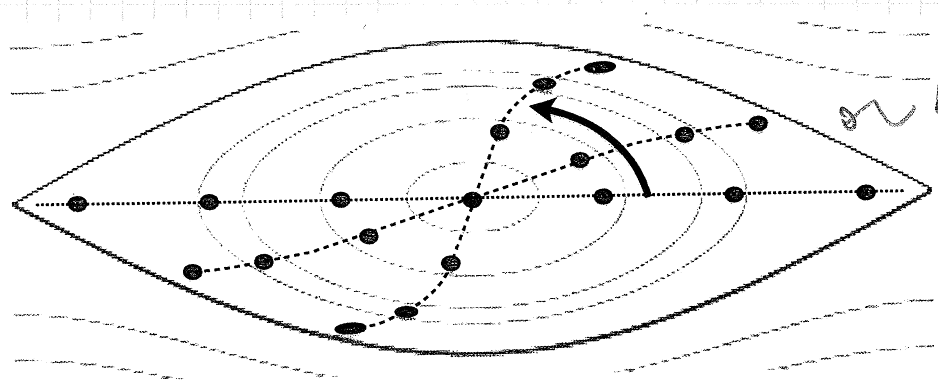


Initial Matched Bunch Train

switch off high frequency, low voltage system,
switch on low frequency, high voltage system...

Large bucket.

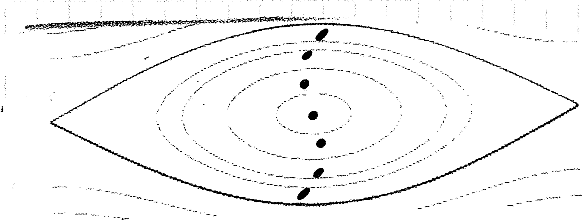




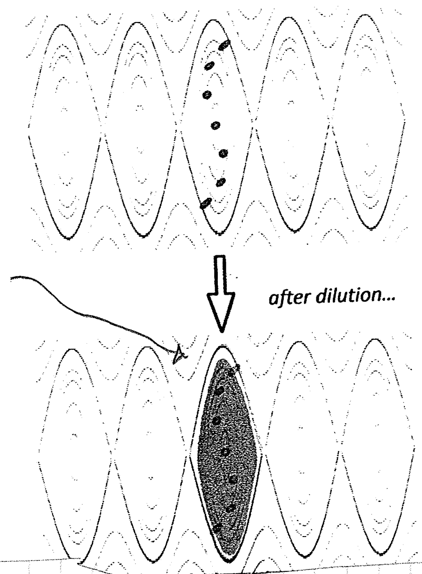
bunches rotate in larger bucket.

Can use coalescing to take bunched beam from one accelerator, make intense bunches and then inject in another accelerator to increase throughput of particles.

* Statistical longitudinal emittance can increase due to nonlinearity



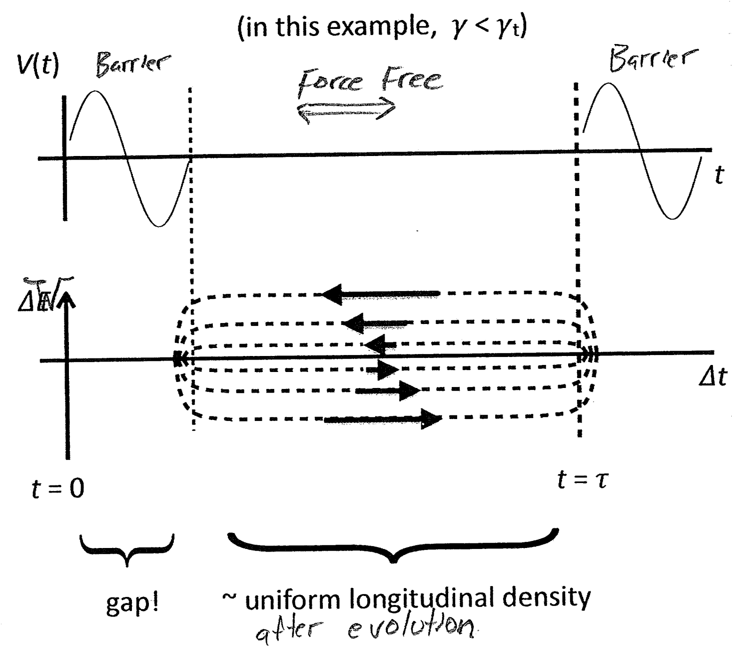
then, recapture with the original harmonic system @ higher voltage



differential evolution mixes (nonlinear)

4/ Barrier Buckets

Use a pulsed RF waveform to produce a longitudinal "barrier" potential, to contain or exclude beam in lengths/intervals of the circulating beam in a ring.



Syphers USPAS

☆ Gaps between uniform beam intervals can allow time for:

- kicker magnets to energize, for extraction etc.
- "reset" of barrier waveform potential in magnetic alloy cavity.
- injection of more particles after compression.

⋮

* Can adjust pulse separation voltages adiabatically so phase space area is conserved as barrier intervals are adjusted.

Adjustments should occur over

$$N_{lap} > \frac{1}{\nu_s} \quad ; \quad \nu_s = \sqrt{\frac{h\eta_s}{2\pi\gamma_s\beta_s^2}\frac{qV_{rf}\cos\phi_s}{mc^2}}$$

Also:

5/ Cogging
6/ Slip Stacking

} Useful to fill rings, see literature.