04. Kinetic Energy Scaling*

Prof. Steven M. Lund
Physics and Astronomy Department
Facility for Rare Isotope Beams (FRIB)
Michigan State University (MSU)

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Steven M. Lund and Yue Hao

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Axial Particle Kinetic Energy

Relativistic particle kinetic energy is:

$$\mathcal{E} = (\gamma - 1)mc^2$$

$$\gamma = \frac{1}{\sqrt{1 - \mathbf{v}^2/c^2}}$$

$$\mathbf{v} = (\beta_b + \delta\beta_z)c\hat{\mathbf{z}} + \beta_\perp c\hat{\mathbf{x}}_\perp$$
$$= \text{Particle Velocity (3D)}$$

For a directed paraxial beam with motion primarily along the machine axis the kinetic energy is essentially the axial kinetic energy $\mathcal{E}_b$:

$$\mathcal{E} = (\gamma_b - 1)mc^2 + \Theta\left(\frac{|\delta\beta_z|}{\beta_b}, \frac{\beta_\perp^2}{\beta_b^2}\right)$$

$$\gamma_b \equiv \frac{1}{\sqrt{1 - v_z^2/c^2}}$$

$$\mathcal{E} \simeq \mathcal{E}_b \equiv (\gamma_b - 1)mc^2$$

In nonrelativistic limit: $\beta_b^2 \ll 1$

$$\begin{aligned}\mathcal{E}_b \equiv (\gamma_b - 1)mc^2 &= \frac{1}{2}m\beta_b^2c^2 + \frac{3}{8}m\beta_b^4c^2 + \cdots \\ &\simeq \frac{1}{2}m\beta_b^2c^2 + \Theta(\beta_b^4)\end{aligned}$$

Convenient units:

Electrons:

$$m = m_e = 511\ \frac{\text{keV}}{c^2}$$

Electrons rapidly relativistic due to relatively low mass

Ions/Protons:

$$m = (\text{atomic mass}) \cdot m_u$$

$$m_u \equiv \text{Atomic Mass Unit} = 931.49 \ \frac{\text{MeV}}{c^2}$$

Note:

$$m_p = \text{Proton Mass} = 938.27 \ \frac{\text{MeV}}{c^2}$$

$$m_n = \text{Neutron Mass} = 939.57 \ \frac{\text{MeV}}{c^2}$$

$$m_p \simeq m_n \simeq 940 \frac{\text{MeV}}{c^2}$$

Approximate roughly for ions:

$$m \simeq A m_u \qquad A = \text{Mass Number (Number of Nucleons)}$$

$$m_u \gg m_e$$

Protons/ions take much longer to become relativistic than electrons

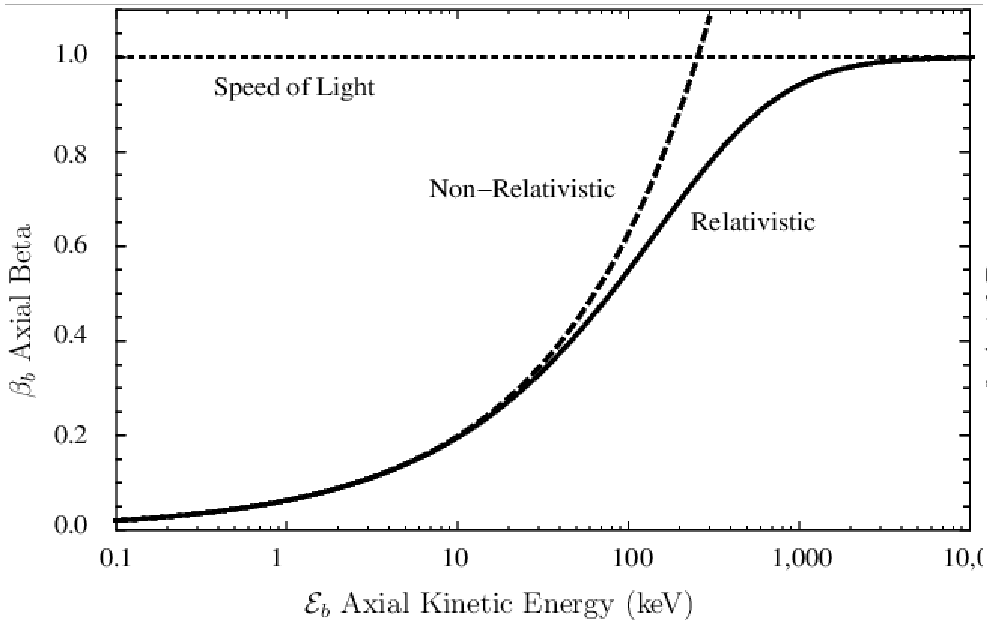
$m_p, m_n > m_u$ due to nuclear binding energy

$$\frac{\mathcal{E}_b/A}{m_u c^2} \simeq \gamma_b - 1 \quad \Longrightarrow \quad \gamma_b = 1 + \frac{\mathcal{E}_b/A}{m_u c^2}, \quad \beta_b = \sqrt{1 - 1/\gamma_b^2}$$

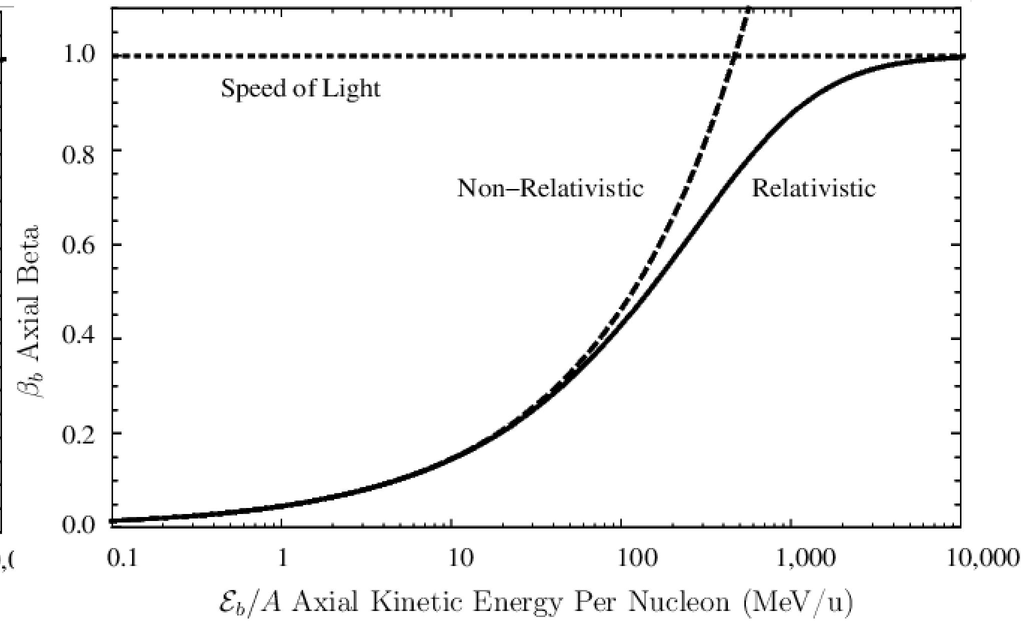
Energy/Nucleon $\mathcal{E}_b/A$ fixes β_b to set phase needs of RF cavities

Contrast beam relativistic β_b for electrons and protons/ions:

Electrons



Ions (and approx Protons)



Notes: 1) plots do not overlay, scale changed
2) Ion plot slightly off for protons since $m_u \neq m_p$

- Electrons become relativistic easier relative to protons/ions due to light mass
- Space-charge more important for ions than electrons (see later course notes)
  - Low energy ions near injector expected to have strongest space-charge

Corrections and suggestions for improvements welcome!

These notes will be corrected and expanded for reference and for use in future editions of US Particle Accelerator School (USPAS) and Michigan State University (MSU) courses. Contact:

Prof. Steven M. Lund
Facility for Rare Isotope Beams
Michigan State University
640 South Shaw Lane
East Lansing, MI 48824

lund@frib.msu.edu
(517) 908 – 7291 office
(510) 459 - 4045 mobile

Please provide corrections with respect to the present archived version at:

https://people.nscl.msu.edu/~lund/uspas/ap_2018/

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