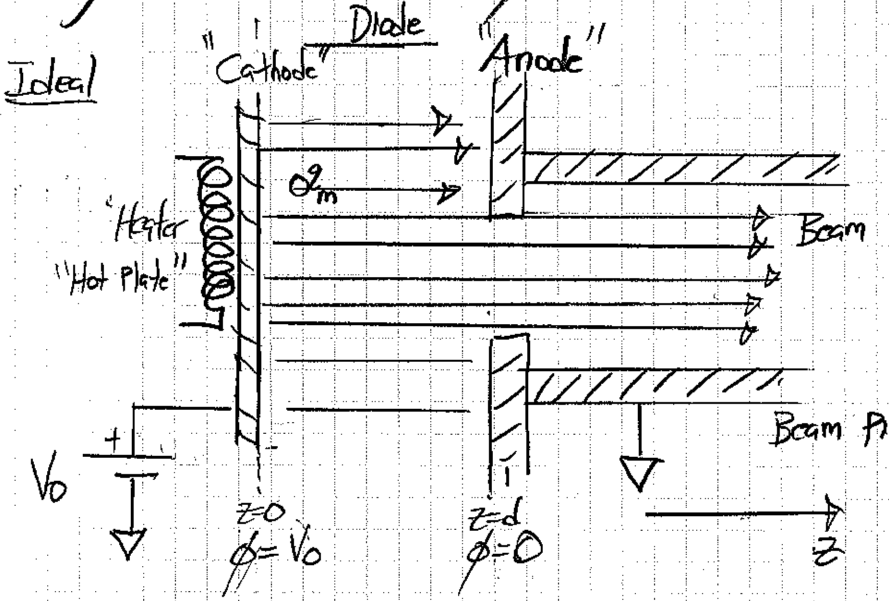
Injectors

Ideally want distribution of monoenergetic particles with high current density and minimal Phase-space size.

Ideal



Accelerator
Focusing
Optics

V_0 = Source bias

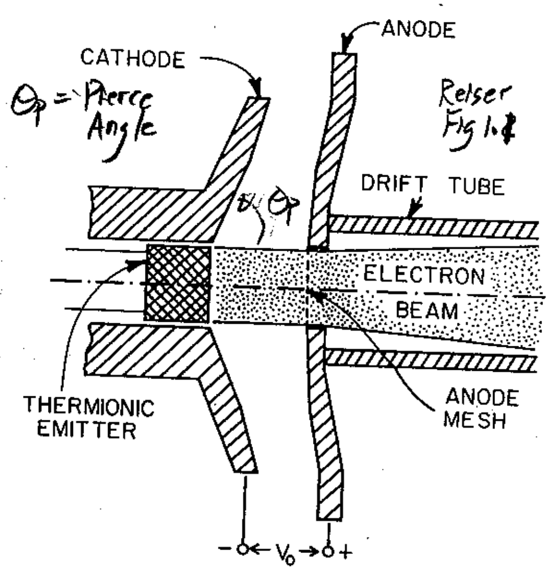
q = particle charge

m = particle mass

R = Radius Apeture

d = Anode - Cathode Seperation

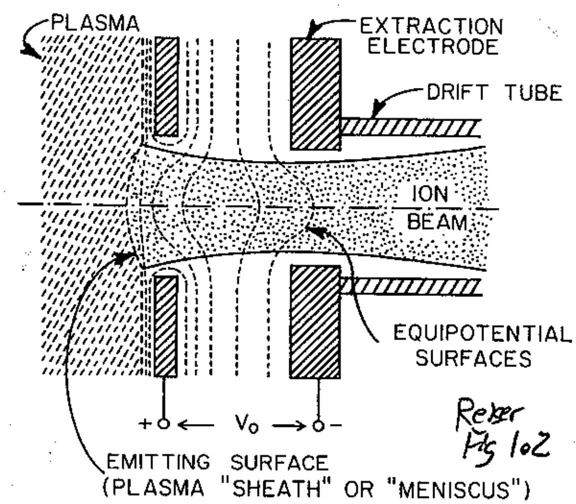
Electrons



Plasma
- ECR
- Discharge ...

Hot Plate
- Doped Tungsten
- Al - Silicate

Ions



Reiser Fig 1.2

Electrons

Emitted from Maxwellian tail of Fermi-Dirac distribution of conduction electrons with current density

$$\boxed{J = A T^2 e^{-W/(k_B T)} \qquad A = \frac{4\pi e m k_B^2}{h^3} = 1.2\times10^6 \frac{\text{Amp}}{\text{meter}^2\ {}^\circ K^2}}$$

Richardson - Dushman Eqn.
Phil. Mag. 28 633 (1914)
Phys Rev. 21 623 (1923)

T = cathode temp.
W = work function (few eV)
k_B = Boltzmann's const (°K units) $= 8.6175\times10^{-5}\ \frac{eV}{{}^\circ K}$
m = e^- mass $= 9.11\times10^{-31}$ kg
e = e^- charge $= 1.6\times10^{-19}$ Coulomb
h = Planck's constant $= 6.63\times10^{-34}$ J·sec

- In lab A often ~ 2x lower.
- Thermionic cathode fabrication difficult.
  - Low W
  - Long life at high temp.
  - Smooth surface

$\Big\}$ Tungsten common
$W = 4.5$ eV
$T \sim 2500$ °K $\Rightarrow k_B T \sim 0.2$ eV
$J \sim 10-20\ \frac{\text{Amp}}{\text{cm}^2}$

+ Photoemitters (not covered)
often used for short pulse e^- sources for XFEL facilities and e-microscopes.

Ions

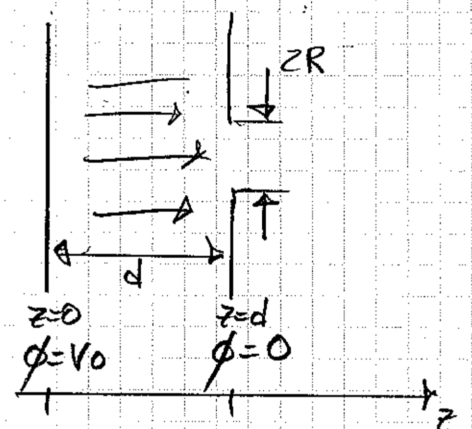
Generally more complex than electron sources.
Many different technologies
- Light ions
- Heavy ions
- Negative ions (H^-)

$\Big\}$ Plasma often used or "Hot Plate" emitter

Often born in magnetic field that confines plasma.

Analyze a simple "hot plate" source with fluid equations to identify scales.

Model ions as a cold fluid:



$n(z,t)$ = Density

$V_z(z,t)$ = Flow Velocity

m = ion mass

q = ion charge

Charge Conservation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0$$

$\rho = qn$

$\vec{J} = qnV_z\hat{z}$

$$\Rightarrow \boxed{\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(nV_z) = 0}$$ Continuity Equation

Momentum (Non Rel. + Cold Fluid)

$$n\frac{d}{dt}(m\vec{V})_z = qn\left[\vec{E} + \vec{V}\times\vec{B}\right]_z - \cancel{\nabla(P)_z}^{\,0}$$

convective Der. z-comp

Field Coupling

$\vec{E}$ = Elec. Field

$\vec{B}$ = Mag Field = 0

Cold Fluid No Pressure

$$\boxed{mn\frac{\partial V_z}{\partial t} + mnV_z\frac{\partial V_z}{\partial z} = qnE_z}$$ Force Equation

Field (Electrostatic)

$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{E} = -\nabla\phi$$

$$\boxed{\frac{\partial^2\phi}{\partial z^2} = \frac{-qn}{\varepsilon_0}}$$ Poisson Equation

Analyze equations assuming steady $\partial/\partial t = 0$ flow. $n = n(z)$, $V_z = V_z(z)$ $\quad q\ \rho(z)$

1) Continuity Equation

$$\cancel{\frac{\partial}{\partial t} n}^{\,0} + \frac{\partial}{\partial z}(n V_z) = 0 \quad \Rightarrow \quad \boxed{q n V_z = \text{const} \equiv J} \quad (1)$$

- Notice this implies $n \to \infty$ as $V_z \to 0$ (near emitter). This is idealized, but still expect very large n near emitter.

2) Force Equation

$$m \cancel{\frac{\partial V_z}{\partial t}}^{\,0} + m V_z \frac{\partial}{\partial z} V_z = q E_z = -q \frac{\partial \phi}{\partial z}$$

$$\frac{1}{2} m \frac{\partial}{\partial z} V_z^2 = -q \frac{\partial \phi}{\partial z} \quad \Rightarrow \quad \frac{1}{2} m V_z^2 \Big|_{z=0}^{z} = -q \phi \Big|_{z=0}^{z}$$

$z=0$: $\phi = V_0$, $\Phi = 0$ $\qquad$ $z=d$: $\phi = 0$, $\Phi = V_0$

$V_z(0) \simeq 0 \Rightarrow$ particles emitted near rest. $\quad \phi(0) = V_0$

$$\frac{1}{2} m V_z^2 = q V_0 - q\phi \quad \Rightarrow \quad \boxed{\frac{1}{2} m V_z^2 = q \Phi} \quad (2) \qquad \Phi \equiv V_0 - \phi$$

convenient ref choice

3) Poisson Equation

$$\frac{\partial^2 \phi}{\partial z^2} = \frac{-qn}{\varepsilon_0} \quad \Rightarrow \quad \frac{\partial^2 \Phi}{\partial z^2} = \frac{qn}{\varepsilon_0} \overset{\text{Use (1)}}{=} \frac{J}{\varepsilon_0 V_z} \overset{\text{Use (2)}}{=} \frac{J}{\varepsilon_0 \left(\frac{2q\Phi}{m}\right)^{1/2}}$$

$$\boxed{\frac{\partial^2 \Phi}{\partial z^2} = \frac{J\, \Phi^{-1/2}}{\varepsilon_0 (2q/m)^{1/2}}}$$

$$\underset{\text{soluc.}}{\Rightarrow} \times \frac{\partial \Phi}{\partial z} \Rightarrow \frac{\partial \Phi}{\partial z} \frac{\partial^2 \Phi}{\partial z^2} = \frac{J}{\varepsilon_0 (2q/m)^{1/2}} \frac{\partial \Phi}{\partial z} \Phi^{-1/2}$$

$$\Rightarrow \frac{\partial}{\partial z}\left[\frac{1}{2}\left(\frac{\partial \Phi}{\partial z}\right)^2\right] = \frac{J}{\varepsilon_0 (2q/m)^{1/2}} \frac{\partial}{\partial z}\left(2 \Phi^{1/2}\right) \Rightarrow \frac{1}{2}\left(\frac{\partial \Phi}{\partial z}\right)^2 \Big|_{z=0}^{z} = \frac{J}{\varepsilon_0 (2q/m)^{1/2}} 2 \Phi^{1/2} \Big|_{z=0}^{z}$$

But $\quad \Phi(z=0) = V_0 - \phi(0) = 0$

$$\frac{\partial \Phi}{\partial z}(z=0) = -\frac{\partial \phi}{\partial z}\Big|_{z=0} = E_z(0) = 0$$

Space-Charge Limited Current: as much charge drawn off as possible for $E_z = 0$ before steady solution reached at stagnation.

$$\Rightarrow \boxed{\frac{\partial \Phi}{\partial z} = \left(\frac{4J}{\varepsilon_0}\right)^{1/2} \left(\frac{m}{2q}\right)^{1/4} \Phi^{1/4}}$$

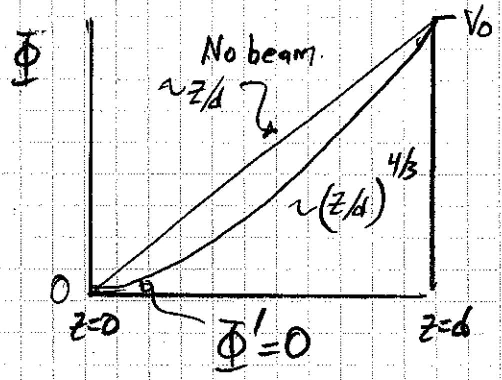
Integrate Poisson's Equation:

$$\frac{d\Phi}{\Phi^{1/4}} = \left(\frac{4J}{\varepsilon_0}\right)^{1/2}\left(\frac{m}{Zq}\right)^{1/4} dz$$

$$\frac{4}{3}\Phi^{3/4}\Big|_{z=0}^{z} = \left(\frac{4J}{\varepsilon_0}\right)^{1/2}\left(\frac{m}{Zq}\right)^{1/4} z \quad \Rightarrow \quad \boxed{\Phi(z) = \left(\frac{3}{4}\right)^{4/3}\left(\frac{4J}{\varepsilon_0}\right)^{2/3}\left(\frac{m}{Zq}\right)^{1/3} z^{4/3}}$$

Require:

$$\Phi(d) = V_0 \quad \Rightarrow \quad \Phi = V_0\left(\frac{z}{d}\right)^{4/3} \qquad V_0 = \left(\frac{3}{4}\right)^{4/3}\left(\frac{4J}{\varepsilon_0}\right)^{2/3}\left(\frac{m}{Zq}\right)^{1/3} d^{4/3}$$



Solve for J:

$$\left(\frac{4J}{\varepsilon_0}\right)^{2/3} = \left(\frac{4}{3}\right)^{4/3}\left(\frac{Zq}{m}\right)^{1/3}\frac{V_0}{d^{4/3}}$$

$$J = \frac{\varepsilon_0}{4}\left(\frac{4}{3}\right)^{4/2}\left(\frac{Zq}{m}\right)^{1/2}\frac{V_0^{3/2}}{d^2}$$

– or –

$$= \frac{4}{9}\varepsilon_0\left(\frac{Zq}{m}\right)^{1/2}\frac{V_0^{3/2}}{d^2}$$

Current Density

$$J = \text{const}\,\frac{V_0^{3/2}}{d^2} = \text{Child-Langmuir Current Density}$$

Also called "Child-Langmuir Law"

$$\text{const} = \frac{4}{9}\varepsilon_0\left(\frac{Zq}{m}\right)^{1/2}$$

Current

$$I = \pi R^2 J = \frac{4\sqrt{2}\,\pi}{9}\varepsilon_0\left(\frac{q}{m}\right)^{1/2}\left(\frac{R}{d}\right)^2 V_0^{3/2} = \begin{cases} 0.232\left(\frac{R}{d}\right)^2\left(V_0\,[\text{kV}]\right)^{3/2}\,\text{A} & \text{electrons (sign flip)} \\ 5.43\left(\frac{Q}{M}\right)^{1/2}\left(\frac{R}{d}\right)^2\left(\frac{V_0}{1\,\text{kV}}\right)^{3/2}\,\text{mA} & \text{ions} \end{cases}$$

R = Radius Beam Aperture

$m = A\,m_u$
$q = Qe$

Want for source: $J = \text{const} \cdot V_0^{3/2}/d^2$

J high $\Rightarrow$ V_0 large, d small

But must also suppress:

1) <u>Voltage Breakdown</u>

$$V_0 \lesssim 100\,\text{kV} \begin{cases} \left(\frac{d}{1\text{cm}}\right) & \text{for } d \lesssim 1\text{cm} \\ \left(\frac{d}{1\text{cm}}\right)^{1/2} & \text{for } d > 1\text{cm} \end{cases}$$

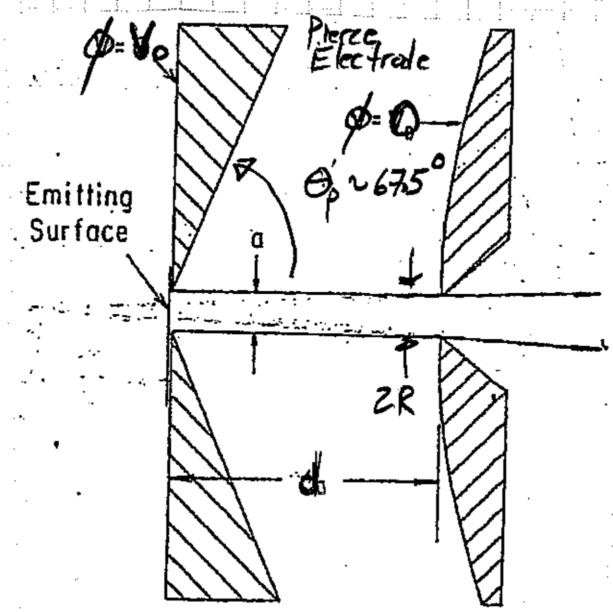
Suppress to avoid distruction of hardware

J. Kwan, LBNL

Practical observation based with real hardware.

2) <u>Optical Abberations</u>

Nonlinear fields must be suppressed to preserve good beam quality.



Angle helps suppress radial beam expansion due to space charge for finite radial extent beam.

A.T. Forrester
Wiley, 1988

Real Beam 3D <u>not</u> 1D.

Find angled "Pierce Electrodes" can be used to keep beam approx 1D. But even with detailed code optimization, And must have

$$d \gtrsim (3 \text{ to } 4)\, R$$

R = beam apeture radius.

Note that:

$$J = \text{const}\,\frac{V_0^{3/2}}{d^2} \sim \begin{cases} V_0^{-1/2} \sim d^{-1/2} & d \lessapprox 1\,\text{cm} \\ V_0^{-5/2} \sim d^{-5/4} & d > 1\,\text{cm} \end{cases}$$

$$I = \pi R^2 J \sim R^2 \frac{V_0^{3/2}}{d^2} \sim V_0^{3/2}$$

$$R \sim d$$

$\therefore$ J <u>decreases</u> as source size increases

But I <u>increases</u> as source size increases

Also, want sources that deliver maximum current in phase-space volume measures of the beam.

Will find later that normalized rms x- and y-emittances measure $\perp$ phase-space areas of the beam:

$$\varepsilon_{nx} = (\gamma\beta)\left[\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2\right]^{1/2}$$

$$\varepsilon_{ny} = (\gamma\beta)\left[\langle y^2\rangle\langle y'^2\rangle - \langle yy'\rangle^2\right]^{1/2}$$

$\perp$ Phase-Space Volume $\sim \varepsilon_{nx}\,\varepsilon_{ny}$

1. Beam is not converging or diverging:

$$\langle xx' \rangle = 0$$

For ~ uniform density beam in aperture:

$$R = 2\langle x^2 \rangle^{1/2}$$

For NR ions born at temperature T:

$$\gamma \simeq 1$$

$$\beta^2 \langle x'^2 \rangle = \frac{1}{c^2}\langle v_x^2 \rangle \qquad \frac{1}{2} m \langle v_x^2 \rangle = \frac{1}{2} k_B T \qquad \frac{1}{2} k_B T \text{ per DOF}$$

$$\beta \langle x'^2 \rangle^{1/2} = \frac{1}{c}\left(\frac{k_B T}{m}\right)^{1/2}$$

Thus

$$\varepsilon_{nx} = \gamma\beta\left[\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2\right]^{1/2} \simeq \beta \langle x^2 \rangle^{1/2} \langle x'^2 \rangle^{1/2} = \frac{R}{2}\left(\frac{k_B T}{mc^2}\right)^{1/2}$$

$$\boxed{\text{Phase Volume} \sim \varepsilon_{nx}\varepsilon_{ny} = \frac{R^2}{4}\frac{k_B T}{mc^2}}$$

Assume x and y planes equivalent.

This leads naturally to a measure of source performance called "Brightness" defined as

$$J \leq \frac{4}{9}\varepsilon_0 \left(\frac{2q}{m}\right)^{1/2} \frac{V_0^{3/2}}{d^2}$$

$$B \equiv \text{Brightness} = \frac{\text{Current}}{\text{Phase-Space Vol.}} = \frac{I}{\varepsilon_{nx}\varepsilon_{ny}} = \frac{\pi R^2 J}{\frac{R^2}{4}\frac{k_B T}{mc^2}}$$

$$\boxed{B = \frac{16\pi}{9}\varepsilon_0\left(\frac{2q}{m}\right)^{1/2}\frac{V_0^{3/2}}{d^2\left(k_B T/mc^2\right)}} \propto \frac{V_0^{3/2}}{d^2 T}$$

Note:
- B independent of source radius R
- Want ions as low a temp T as possible! Hard to do

Summary: Sources

- Well functioning sources crucial for accelerator:
  - Bright: compact phase-space $\Leftrightarrow$ compact beam
    - smaller (more economical) structures
    - lower losses to mitigate damage
- Improving sources allows running higher intensity to get more out of accelerator.
  - Upgrade source $\Leftrightarrow$ Upgrade accelerator
- Need also high reliability for long periods of time
  - Source not operational, then expensive machine will not work.
- Sources high leverage, but also difficult:
  - Technology and physics strongly coupled
  - Rely on difficult materials science, plasma physics, etc.

  Possible to teach a whole course on just one type of source technology.
- Many types of sources. Only scratched surface in this lecture:

electrons	Ions	Radioactive	Antiparticle
Thermionic emitters	hot plate (low Q)	Capture / cool	Accel. drives source - capture, cool, inject
Field Emission (point)	plasma (higher Q) discharge ECR, ... laser driven e beam driven		
Photocathodes			

- Source projects great for PhDs: rich physics, improvements high leverage