03. lecture.pdf
PHY 905
Spring 2016

Magnetic Field Calculations

Example: Iron Dominated Dipole Magnet

Maxwells' Eqns In Media:

$\nabla \cdot \vec{D} = \rho$ Coulombs' Law

$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ Ampres' Law

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ Faradays' Law

$\nabla \cdot \vec{B} = 0$ No Magnetic Monopoles

$\vec{B} = \mu \vec{H}$

$\vec{D} = \varepsilon \vec{E}$

Free space $\mu = \mu_0$, $\varepsilon = \varepsilon_0$

$\mu_0 = 4\pi \times 10^{-7} \frac{N \cdot sec^2}{Coulomb^2}$

$\mu_0 \varepsilon_0 = \frac{1}{c^2}$

c = speed of light.

For iron magnet analysis, reduce equations:

For steady magnets:

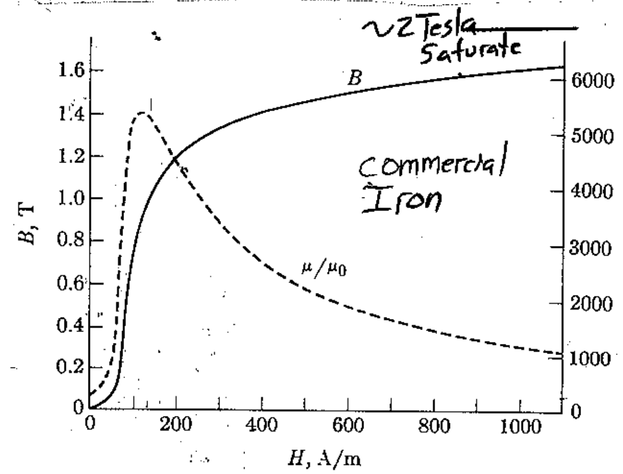
$\partial_t = 0$, $\vec{D}, \rho = 0$

$$\boxed{\begin{aligned} \nabla \times \vec{H} &= \vec{J} \\ \nabla \cdot \vec{B} &= 0 \end{aligned}}$$

solve with
$B = \mu(H) H$
materials specification

$[\vec{H}] = \frac{\text{Amps}}{\text{meter}}$

$\vec{J}$ = Transport Current Density



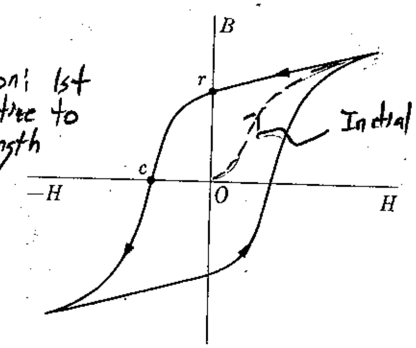
Reitz Milford, Christy
E&M Theory

$\vec{B} = \mu(H) \vec{H}$

Isotropic iron

Hysterisis Loop

Implication: 1st Demagnetize to set strength reliably



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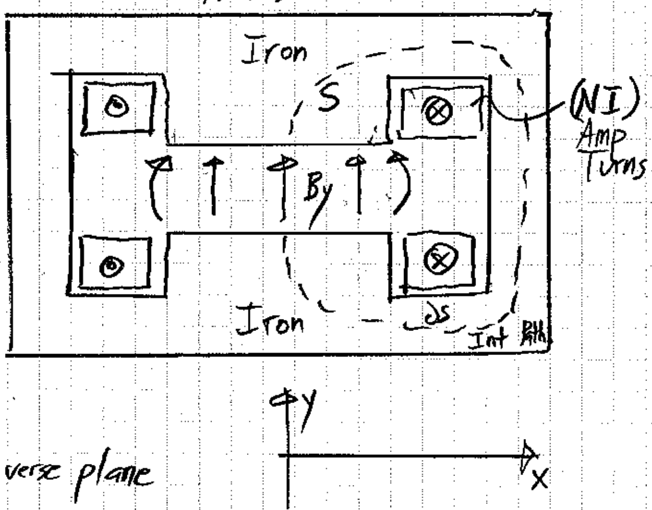
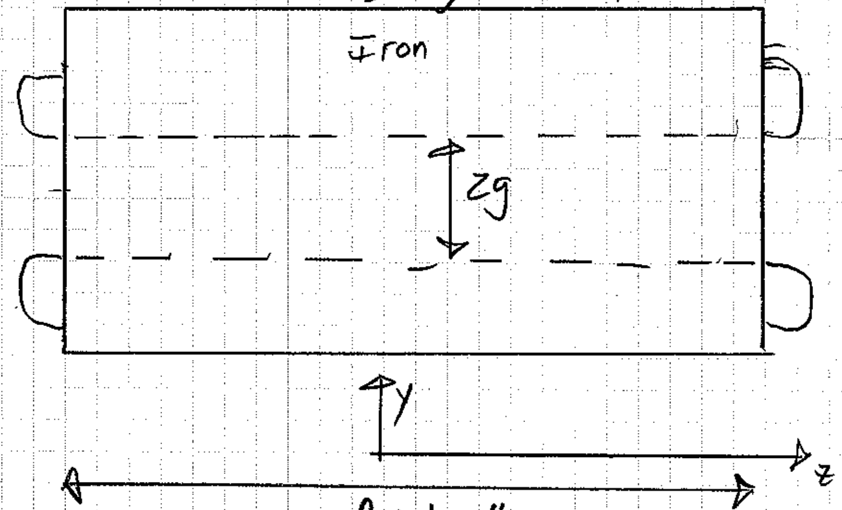
Examine 2D Field of an <u>iron dipole magnet</u> for beam bending

Transverse



Transverse plane

Longitudinal



$2g$ = Magnet Pole Gap

ℓ = length

$\ell >> 2g$ for ~2D field.

$$\nabla \times \vec{H} = \vec{J}$$

$$\int_{\text{Surface}} \nabla \times \vec{H} \cdot \hat{z}\, d^2x = \int_{\text{Surface}} \vec{J} \cdot \hat{z}\, d^2x \quad \underset{\text{Theorem}}{\overset{\text{Stokes}}{\Longrightarrow}} \quad \oint_{\partial S} \vec{H} \cdot d\vec{\ell} = \int_S J_z\, d^2x$$

Surface ~ see sk

$H \approx 0$ in iron for μ large when not saturated

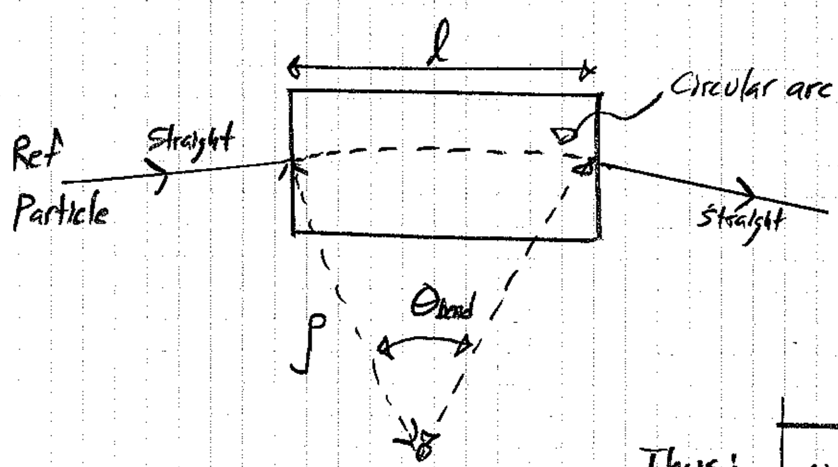
$$\oint_{\partial S} \vec{H} \cdot d\vec{\ell} = H_{gap} \cdot (2g) + \int_{\text{Dashed}} \overbrace{\frac{\vec{B}}{\mu}}^{0} \cdot d\vec{\ell} = \int_S J_z\, d^2x = 2(NI)$$

$$\Rightarrow \quad \boxed{B_y = \mu_0 H_{gap} = \mu_0 \frac{(NI)}{g}}$$

Will derive analogous formula on homework to specify iron magnet quadrupole excitation (NI) to achieve field gradient.

We will show that the bend radius ρ of a dipole satisfies:

$$\frac{1}{\rho} = \frac{B_y}{(B\rho)} \qquad (B\rho) = \frac{p}{q} = \frac{\gamma m v}{q} = \frac{\text{momentum}}{\text{charge}} = \text{Rigidity}$$



θ_{bend} small, ρ large:

$$\Rightarrow \quad l = \rho\,\theta_{bend}$$

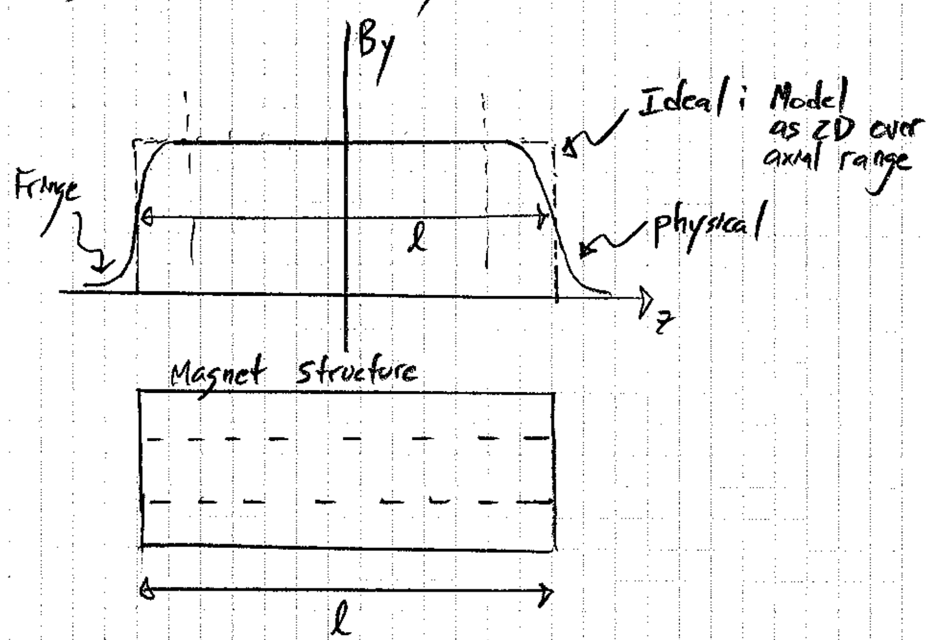
$$\frac{1}{\rho} = \frac{B_y}{(B\rho)} \qquad B_y = \frac{\mu_0 (NI)}{g}$$

Thus:

$$\boxed{(NI) = g\frac{B_y}{\mu_0} = g\frac{(B\rho)}{\mu_0 \rho} = \left(\frac{g}{l}\right)\frac{\theta_{bend}}{\mu_0}}$$

gives needed magnet excitation (NI) to realize required bend.

Magnets also really 3D:



3D design much harder and often relies on codes.

- Poisson: r-z, xy
- Tosca / Vector Fields, Maxwell 3D

Fringe can impact magnet excitation (NI) needed.

- Effect larger when magnet short (g/l larger)
- In lab often adjust till bend angle correct to account for fringe + calibration errors: base on beam diagnostics

Summary

- Magnet design large and diverse topic : can teach whole courses on

  Iron Dominated   Superconductors   Permanent Magnet   Hybrid

  Iron Free   Pulsed

- Topic often exploits large codes for detailed design iteration and relies on technology and materials limit knowledge
  - Iron saturation
  - Critical currents of superconductors
  - Quench / Failure protection due to large stored energy

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- Results and limits impact choices for beam manipulations.

  - Focusing / Bending limits
  - Aperture sizes
  - Beam transfer (eg. inserting / extracting from ring)
  - Final focusing

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- This lecture only constitutes a brief intro to a small area of the field to illustrate simple estimate